CSE504 Compiler Design
Type Checking

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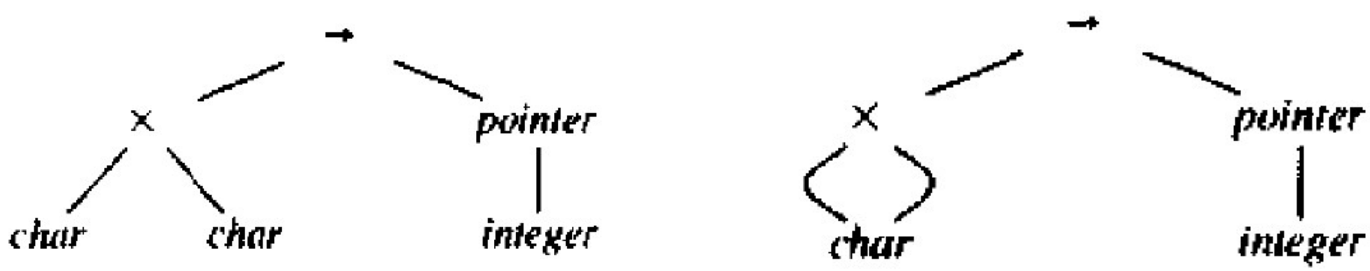
Type Expressions

- Type Expressions
  - Basic type or constructed type by applying type constructors to type expressions.
- Basic type:
  - `boolean, integer, real, type_error, void,…`
- Type name is a type expression
- Type constructor
  - Arrays: if T is a type expression, then array(I,T) is a type expression (I is the size of the array)
    - `array[10] of integer:  array(10, integer)`
  - Products: if T1 and T2 are type expressions, then T1 * T2 is a type expression (Cartesian product)
    - `integer * integer`

Type Expressions

- Records: similar to products, but the fields have names
  - `record( (row * integer) * (column * integer) )`
- Pointers: if T is a type, then pointer(T) is a type
  - `pointer(integer)`
- Functions: if D and R are types, D -> R is a type
  - `integer * integer -> integer`

- Type expressions may contain variables whose values are type expressions.
- Tree and dag representation of
  `(char * char) -> pointer(integer)`



Type Systems

- Type system
  - A collection of rules for assigning type expressions to the various parts of a program
  - A type checker implements a type system
- Type checking
  - Static type checking: checking is done by a compiler
  - Dynamic type checking: checking is done when the target program runs
  - Sound type system: if a type system assigns a type to a program part, type error shouldn't occur for the program part.
  - A language is strongly typed: static type checking guarantees that type errors cannot occur during the runtime.
    - Difficult to achieve

```
int a[100];
a[ a_very_complex_function() ] = 1;
```

A Simple Type Checker

- A Simple Language

```
P -> D ; E
D -> D ; D | id : T
T -> char | integer
     | array [ num ] of T
     | ^ T
E -> literal | num | id
     | E mod E
     | E [ E ]
     | E ^
```

Type of an Id

- A Part of Translation Scheme

```
P -> D ; E
D -> D ; D
D -> id : T  { addtype(id.entry, T.type) }
T -> char    { T.type := char }
T -> integer { T.type := integer }
T -> ^ T1    { T.type := pointer(T1.type) }
T -> array [ num ] of T1 {
              T.type := array(num.val, T1.type) }
```

Type Checking of Expressions

```
E -> literal    { E.type := char }
E -> num        { E.type := integer }
E -> id         { E.type := lookup(id.entry) }
E -> E1 mod E2 { E.Type :=
    if E1.type = integer and E2.type = integer
    then integer
    else type_error }
E -> E1 [ E2 ] { E.type :=
    if E2.type = integer and E1.type = array(s,t)
    then t
    else type_error }
E -> E1^        { E.type :=
    if E1.type = pointer(t)
    then t
    else type_error }
```

Type Checking of Statements

```
S -> id := E        { S.type :=
    if id.type = E.type
    then void else type_error }
S -> if E then S1  { S.type :=
    if E.type = boolean
    then S1.type else type_error }
S -> while E do S1 { S.type :=
    if E.type = boolean
    then S1.type else type_error }
S -> S1 ; S2        { S.type :=
    if S1.type = void and S2.type = void
    then void else type_error }
```

Type Checking of Functions

- Type expression for functions

```
T -> T1 '->' T2 {
    T.type := T1.type -> T2.type }
```

- Rules for checking a function application

```
E -> E1 ( E2 ) { E.type :=
    if E2.type = s and E1.type = s->t
    then t else type_error }
```

- For more than one arguments
  - `T1,… ,Tn` can be views as a single argument type `T1*… *Tn`

Equivalence of Type Expressions

- “Two type expressions are equal” what does that mean?
- Structural Equivalence
  - Two expressions are either the same basic type,
  - They are formed by applying the same constructor to structurally equivalent types.

Structural Equivalence

```
function sequiv(s, t): boolean
begin
   if s and t are the same basic type then
      return true
   else if s = array(s1,s2) and t = array(t1,t2) then
      return s1 = t1 and sequiv(s2,t2)
   else if s = s1*s2 and t = t1*t2 then
      return sequiv(s1,t1) and sequiv(s2,t2)
   else if s = pointer(s1) and t = pointer(t1) then
      return sequiv(s1,t1)
   else if s = s1->s2 and t = t1->t2 then
      return sequiv(s1,t1) and sequiv(s2,t2)
   else
      return false
end
```

Names for Type Expressions

- In some languages types can have names

```
type link = ^cell;
var next : link;
    last : link;
    p    : ^cell;
    q, r : ^cell;
```

  - Allow type expressions to be named, and allow the names to appear in type expressions
  - `p:  pointer(cell)` type, `next: link` type
- Name Equivalence
  - Views each type name as a distinct type.
  - `last` and `p` are not the same type
- Structural Equivalence
  - Names are replaced by the type expressions they define.
  - `next, last, p, q`, and `r` are the same type

Type Conversions

- Coercions: Implicit conversion from one type to another by the compiler

```
E -> num        { E.type := integer }
E -> num . num  { E.type := real }
E -> id         { E.type = lookup(id.entry) }
E -> E1 op E2   { E.type :=
   if E1.type = integer and E2.type = integer then
      integer
   else if E1.type = real and E2.type = integer then
      real
   else if E1.type = integer and E2.type = real then
      real
   else if E1.type = real and E2.type = real then
      real
   else
      type_error }
```

Overloading

- An overloaded symbol has different meanings depending on its context
  - E.g. +: integer addition, real addition, complex addition, string concatenation, …
  - Overloading is said to be resolved if a unique meaning is determined.

Set of Possible Types

- It is not always possible to resolve overloading immediately.
  - E.g.)

```
function "*" (i, j: integer) return integer;
function "*" (i, j: integer) return complex;
function "*" (i, j: complex) return complex;
```

  - 3*5 : either integer or complex
  - 2*(3*5) : 3*5 must be integer because * takes same arg. types
  - z*(3*5) : 3*5 must be complex if z is a complex type.

```
E -> E1 { E.types := E1.types }
E -> id { E.types := lookup(id.entry) }
E -> E1(E2) { E.types :=
   { t | s->t ∊ E1.types and s ∊ E2.types }}
```

Narrowing the Set of Possible Types

- Two depth-first traversals
  - Synthesize types during the first path
  - Update type during the second path

`E -> E1`	`E.types := E1.types`
	`E.type  := if E1.types = {t}`
	`            then t else type_error`

`E -> id`	`E.types := lookup(id.entry)`

`E -> E1(E2)`	`E.types :=`
	`     {r|s in E2.types and s->r in E1.types}`
	`t := E.type`
	`S := {s|s in E2.types and s->t in E1.types}`
	`E2.type := if S={s}`
	`            then s else type_error`
	`E1.type := if S={s}`
	`            then s->t else type_error`

Polymorphic Functions

- Polymorphic functions
  – The statements in the body can be executed with arguments of different types.
- Type Variables
  – Variables representing type expressions.
  – Allow us to talk about unknown types.
  – Checking consistent usage of identifiers that don’t need to be declared.

Polymorphic Functions

- Type Inference
  - Problem of determining the type of a language construct from the way it is used.
  - E.g.

```
function deref(p)
begin
  return p^
end;
```

    - From `deref(p)`, assume that the type of `p` is β
    - From `p^`, infer that the type of `p` is β `= pointer(`α`)`
    - The type of `deref` is `pointer(`α`) ->` α

Language with Polymorphic Functions

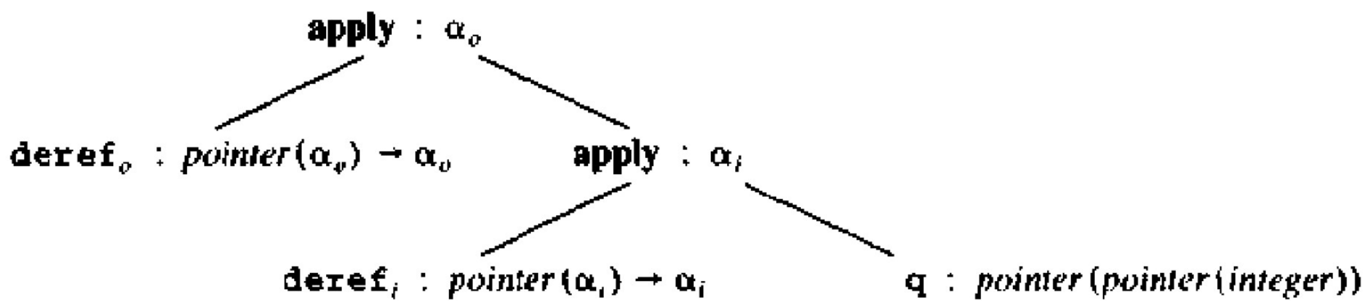
- Grammar for language with polymorphic functions

```
P -> D ; E
D -> D ; D | id : Q
Q -> ∀ type_var . Q | T
T -> T '->' T
   | T * T
   | unary ( T )
   | basic_type
   | type_var
   | ( T )
E -> E ( E ) | E, E | id
```

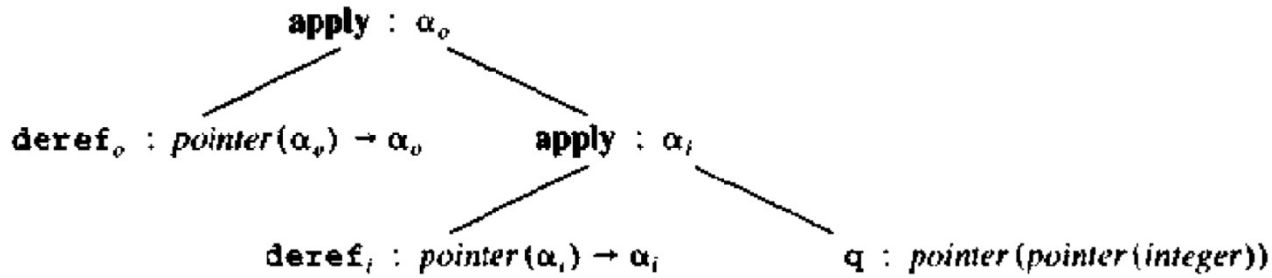
Language with Polymorphic Functions

- Example Program

```
deref: ∀α.pointer(α) -> α;
q    : pointer(pointer(integer));
deref(deref(q))
```



Type-Checking Polymorphic Functions



- Distinct occurrences of a polymorphic function may have different types
  – deref$_o$ and deref$_i$ have different types
- Equivalence of type need to be updated
  – Unification: make s and t structurally equivalent by replacing the type variables in s and t by type expressions
- Record the effect of unifying two expressions
  – If after unification a type variable α represents a type t then it should keep represent t through out type-checking.

Substitution

- Substitution:
  - `S(α)`: mapping from type variables to type expressions
  - `S(t)`: consistent replacement of type variables with their mapped type expressions (= `subst(t)`)

```
function subst(t:type_expr): type_expr
begin
   if t is a basic type then return t
   if t is a variable then return S(t)
   if t is t1->t2 then
           return subst(t1)->subst(t2)
end
```

Instance

- `S` is the substitution function, `S(t)` is an instance of `t`.
- We write `s < t` to indicate that `s` is an instance of `t`
  - `pointer(integer) < pointer(α)`
  - `integer -> integer < α -> α`
  - `pointer(α) < β`
  - `α < β`

  - `integer ? real`
  - `integer -> real ? α -> α`
  - `integer -> α ? α -> α (all occurrences of α must be replaced)`

Unification

- Two type expressions `t1` and `t2` **unify** if there is a substitution `S` such that `S(t1) = S(t2)`
- A substitution is the **most general unifier** if
  - `S(t1) = S(t2)`
  - for any other unifier `S'`, `S'` is an instance of `S` (for any `t, S'(t) < S(t)`)

Checking Polymorphic Functions

```
E -> E1 ( E2 ) {
    p := mkleaf(newtypevar)
    unify(E1.type, mknode(->, E2.type, p))
    E.type := p
}
E -> E1, E2 {
    E.type := mknode(*, E1.type, E2.type)
}
E -> id {
    E.type := fresh(id.type)
}
```

- Type Checking rule for `E -> E1(E2)`
  - If `E1.type` = α and `E2.type` = β, then `E.type` = β -> γ

Unification Algorithm

```
boolean unify(Node m, Node n) {
        s = find(m);  t = find(n);
        if ( s = t ) return true;
        else if ( nodes s and t represent the same basic type ) return true;
        else if (s is an op-node with children s1 and s2 and
                       t is an op-node with children t1 and t2) {
              union(s, t);
              return unify(s1, t1) and unify(s2, t2);
        }
        else if s or t represents a variable {
              union(s, t);
              return true;
        }
        else return false;
}
```

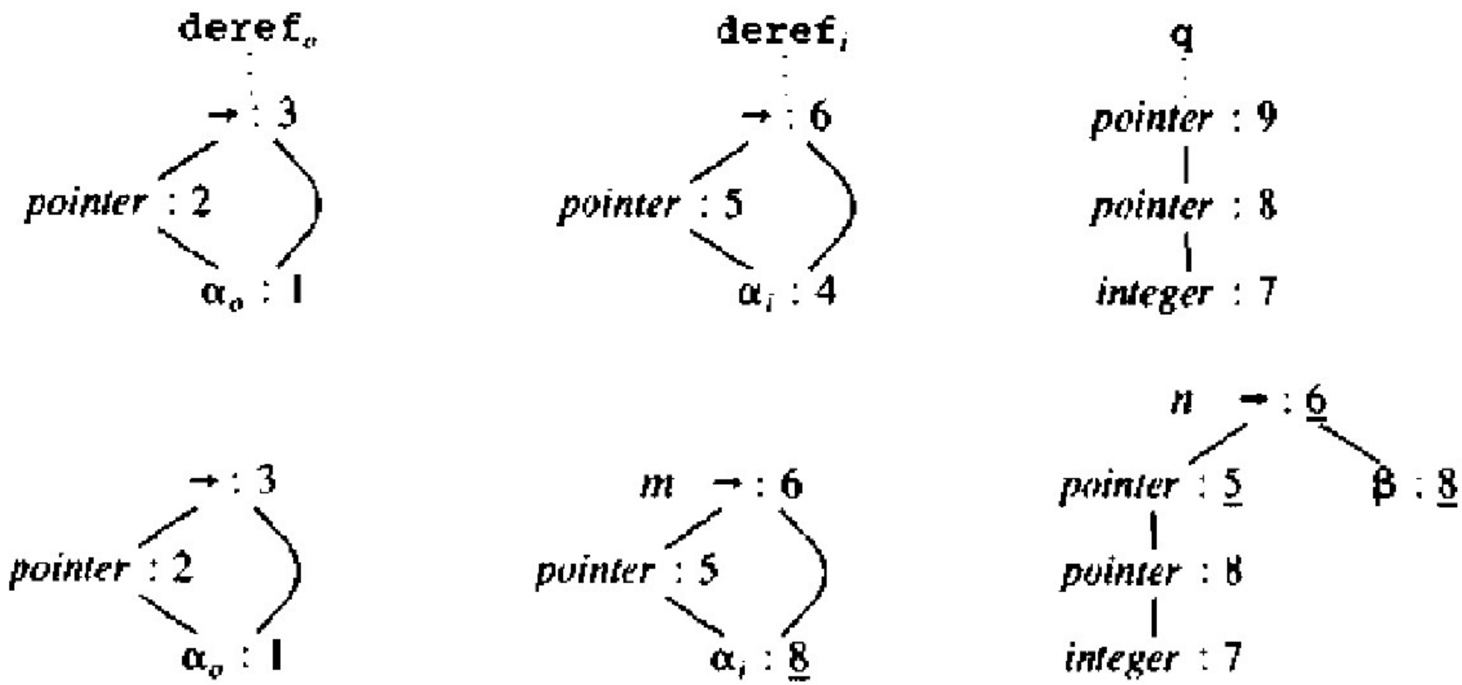
find(n) returns the representative of n

union prefers to make non-variable node a representative node

Checking Polymorphic Functions

- E.g.

```
deref: ∀α.pointer(α) -> α;
q    : pointer(pointer(integer));
deref(deref(q))
```



Unification

- Quiz
  - Draw a dag for the following type expressions
  - Unify the following two type expressions
    - `((α1->α2)*list(α3))->list(α2)`
    - `((α3->α4)*list(α3))->α5`
  - Check the structural equivalence of the following two type expressions
    - `e: real->e`
    - `f: real->(real->f)`