

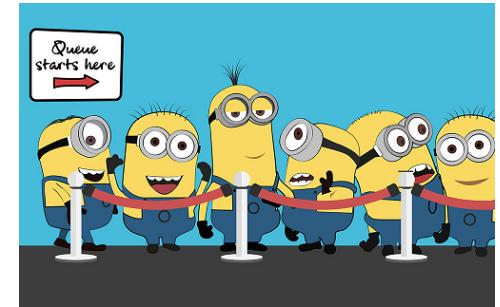
# CSE214 Data Structures

## Queues

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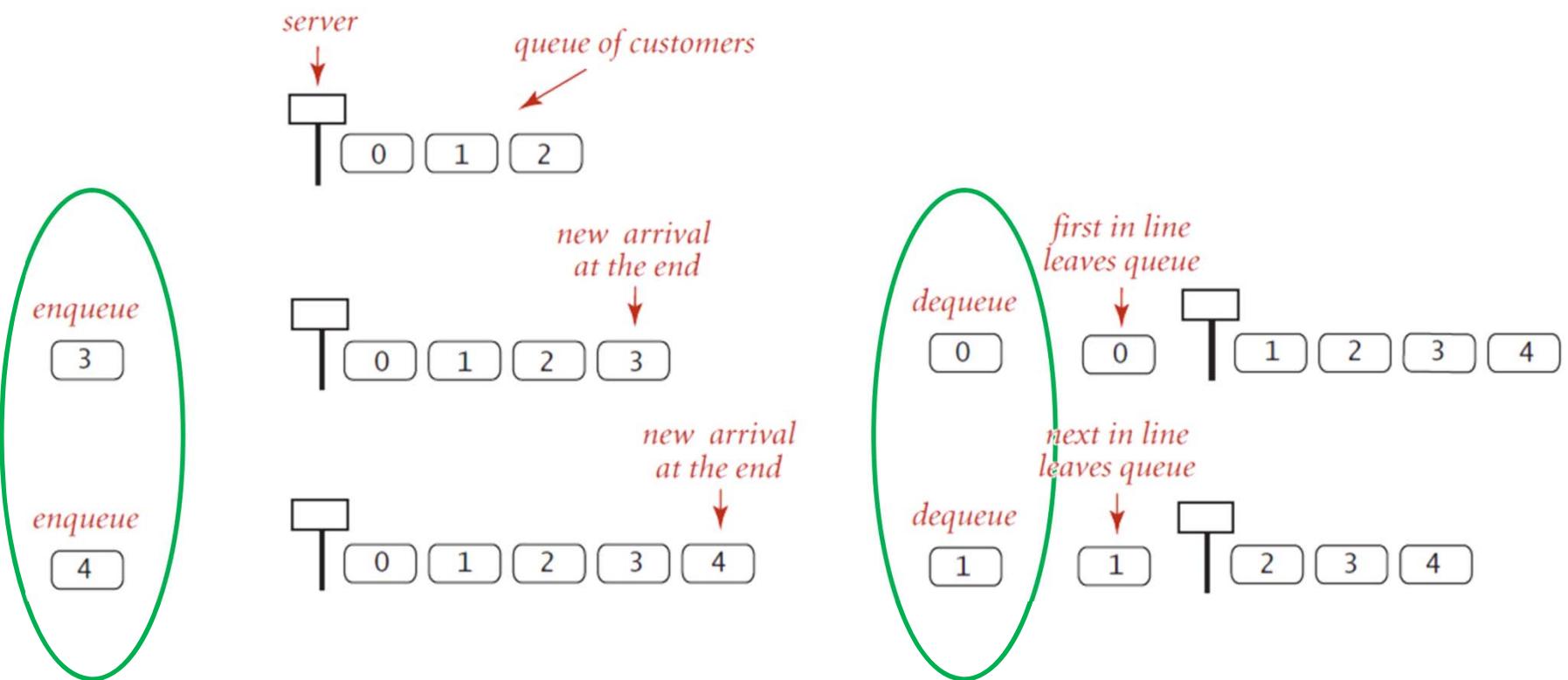


# Queues



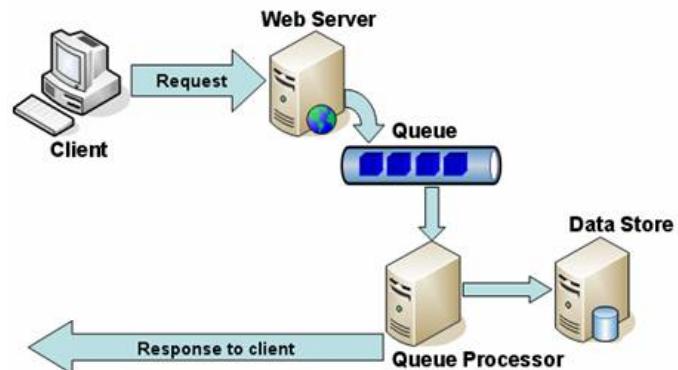
- Queue
  - Queue is a collection of objects that are inserted and removed according to the **First-In, First-Out (FIFO)** principle
- Main operations on queues
  - **enqueue**: adding an element to a queue
  - **dequeue**: removing the element that has been in the queue the longest

# Queue Operations



# Examples of Queues

- A line of people waiting to get in to a store
- A call service system at a customer service center
- Web server
- Networked printer ...



# Queue Abstract Data Type

`enqueue(e)`: Adds element *e* to the back of queue.

`dequeue()`: Removes and returns the first element from the queue  
(or null if the queue is empty).

`first()`: Returns the first element of the queue, without removing it  
(or null if the queue is empty).

`size()`: Returns the number of elements in the queue.

`isEmpty()`: Returns a boolean indicating whether the queue is empty.

# Queue Operation Example

Method	Return Value	$\text{first} \leftarrow Q \leftarrow \text{last}$
enqueue(5)	–	(5)
enqueue(3)	–	(5, 3)
size()	2	(5, 3)
dequeue()	5	(3)
isEmpty()	false	(3)
dequeue()	3	()
isEmpty()	true	()
dequeue()	null	()
enqueue(7)	–	(7)
enqueue(9)	–	(7, 9)
first()	7	(7, 9)
enqueue(4)	–	(7, 9, 4)

# Interface Queue

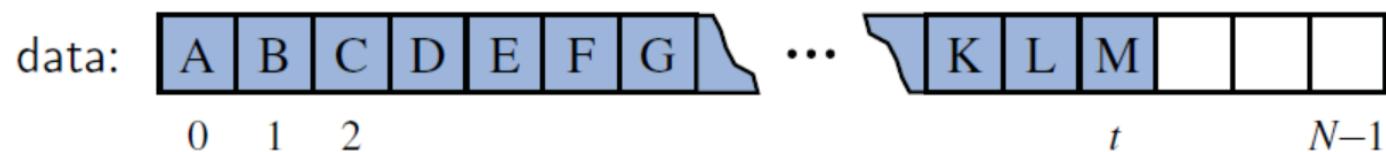
```
public interface Queue<E> {  
    public int size();  
    public boolean isEmpty();  
    public void enqueue(E e);  
    public E dequeue();  
    public E first();  
}
```

Our Queue ADT	Interface <code>java.util.Queue</code>	
	throws exceptions	returns special value
<code>enqueue(e)</code>	<code>add(e)</code>	<code>offer(e)</code>
<code>dequeue()</code>	<code>remove()</code>	<code>poll()</code>
<code>first()</code>	<code>element()</code>	<code>peek()</code>
<code>size()</code>		<code>size()</code>
<code>isEmpty()</code>		<code>isEmpty()</code>

# *Array-Based Queue Implementation*

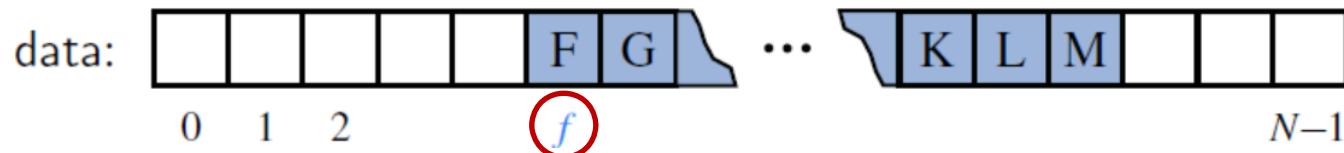
## ■ Implementation 1

- Enqueue: store elements at the end of existing elements (  $O(1)$  )
  - Dequeue: remove the first element and shift the rest (  $O(n)$  )



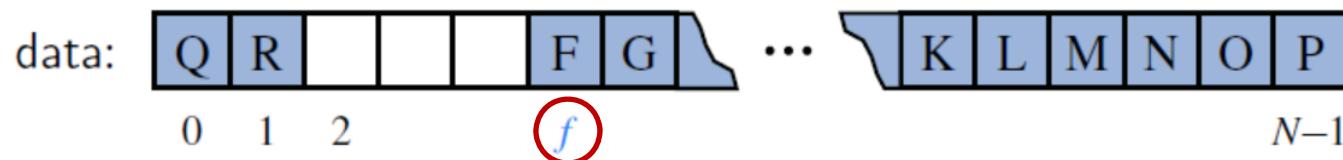
# *Array-Based Queue Implementation*

- Implementation 2
  - Enqueue: store elements at the end of existing elements (  $O(1)$  )
  - Dequeue: remove the element at index  $f$  and **advance  $f$**  by 1 (  $O(1)$  )
  - $f$  will **drift** backward and the back of the queue would eventually reach the end of the array



# *Array-Based Queue Implementation*

- Implementation 3
  - Using an array **circularly**
  - The contents of the queue **wraps around** the end of the array as necessary
    - New elements to be placed at index  $N, N+1, \dots$  are placed at  $0, 1, \dots$
    - Easy to implement:  $f = (f + 1) \% N$



# *Array-Based Queue Implementation*

- Dequeue
  - Remove the element at index  $f$
  - $f = (f + 1) \% N$
  - Decrease size
- Enqueue
  - Add element to  $(f + size) \% N$
  - Increase size

```
@SuppressWarnings("unchecked")
public class ArrayQueue<E> implements Queue<E> {
    protected static final int CAPACITY = 16;
    protected E[] data;
    protected int size;
    protected int f;

    public ArrayQueue() { this(CAPACITY); }
    public ArrayQueue(int capacity) { data = (E[])new Object[capacity]; }

    //
    // Interface Queue
    //
    public int size() { return size; }
    public boolean isEmpty() { return size == 0; }
    public E first() {
        if(isEmpty())
            return null;
        return data[f];
    }
}
```

```
public void enqueue(E e) {
    if(size == data.length)
        throw new IllegalStateException("Queue is full");
    int avail = (f + size) % data.length;
    data[avail] = e;
    size++;
}

public E dequeue() {
    if(isEmpty())
        throw new IllegalStateException("Queue is empty");
    E ret = data[f];
    data[f] = null;
    f = (f + 1) % data.length;
    size--;
    return ret;
}
```

```
protected static void onFalseThrow(boolean b) {  
    if(!b)  
        throw new RuntimeException("Error: unexpected");  
  
}  
  
public static void main(String[] args) {  
    ArrayQueue<Integer> queue = new ArrayQueue<Integer>();  
  
    queue.enqueue(1);  
    queue.enqueue(2);  
    queue.enqueue(3);  
    queue.enqueue(4);  
  
    onFalseThrow(queue.dequeue() == 1);  
    onFalseThrow(queue.dequeue() == 2);  
    onFalseThrow(queue.dequeue() == 3);  
    onFalseThrow(queue.dequeue() == 4);  
  
    System.out.println("Success!");  
}
```

# Big-Oh of Array-Based Queue

Method	Running Time
size	$O(1)$
isEmpty	$O(1)$
first	$O(1)$
enqueue	$O(1)$
dequeue	$O(1)$

# *List-Based* Queue Implementation

- Use a singly linked list
  - No limit in the queue capacity
  - $O(1)$  enqueue and dequeue operations

```

public class ListQueue<E> implements Queue<E> {
    //Node for singly linked list
    private static class Node<E> {
        public Node<E> next;
        public E e;
        public Node() {}
        public Node(E e, Node<E>next) { this.e = e; this.next = next; }
    }

    //sentinel
    private Node<E> head,      //head is a sentinel
                  tail;      //tail points to the last node or
                  //head when empty
    private int size;

    public ListQueue() {
        head = tail = new Node<E>();
        size = 0;
    }

    // Interface Queue
    //
    public int size()          { return size; }
    public boolean isEmpty() { return size == 0; }
}

```

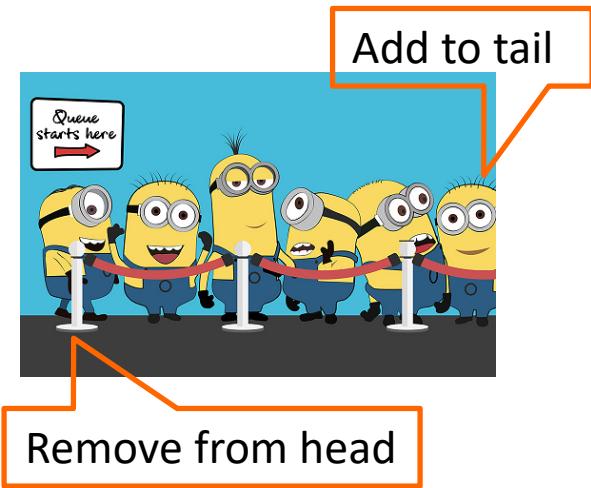
```

//add e after tail
public void enqueue(E e) {
    tail.next = new Node<E>(e, null);
    tail = tail.next;
    size++;
}

//remove the node after head
public E dequeue() {
    if(isEmpty())
        return null;
    Node<E> f = head.next;
    head.next = f.next;
    size--;
    //if tail is removed, reset tail to head
    if(size == 0)
        tail = head;
    return f.e;
}

public E first() {
    if(isEmpty())
        return null;
    return head.next.e;
}

```



# Josephus Problem (Hot Potato Game)



- Given a **number k** and **n players**
  - Each player is passing a potato to the next player **k** times
  - The person received the potato at the **k<sup>th</sup> step** is out of the game
  - The play repeats until only 1 play is left
  - The last player left is the winner
- We can simulate the Josephus problem using a queue

```
public class Josephus {  
  
    public static <E> E Play(Queue<E> queue, int k) {  
        if(queue.isEmpty())  
            return null;  
  
        //find victims until only 1 player is left  
        while(queue.size() > 1) {  
  
            //rotate the potato k-1 times  
            for(int i = 0; i < k-1; i++)  
                queue.enqueue(queue.dequeue());  
  
            //evict the victim (k-th player)  
            E e = queue.dequeue();  
            System.out.println("      " + e + " is out");  
        }  
  
        //the last player is the winner  
        return queue.dequeue();  
    }  
}
```

```

public static <E> Queue<E> buildQueue(E a[]) {
    Queue<E> queue = new ListQueue<E>();

    //add all elements in a to queue
    for(E e : a)
        queue.enqueue(e);

    //return the queue
    return queue;
}

public static void main(String[] args) {
    String[] a1 = {"Alice", "Bob", "Cindy", "Doug", "Ed", "Fred"};
    String[] a2 = {"Gene", "Hope", "Irene", "Jack", "Kim", "Lance"};
    String[] a3 = {"Mike", "Roberto"};

    System.out.println("First winnder is " + Play(buildQueue(a1), 3));
    System.out.println("Second winnder is " + Play(buildQueue(a2), 10));
    System.out.println("Third winnder is " + Play(buildQueue(a3), 7));
}
}

```

# Josephus Problem

Expected output

```
> java Josephus
```

```
Cindy is out  
Fred is out  
Doug is out  
Bob is out  
Ed is out
```

```
First winner is Alice
```

```
Jack is out  
Irene is out  
Lance is out  
Gene is out  
Kim is out
```

```
Second winner is Hope
```

```
Mike is out
```

```
Third winner is Roberto
```

# Double-Ended Queues

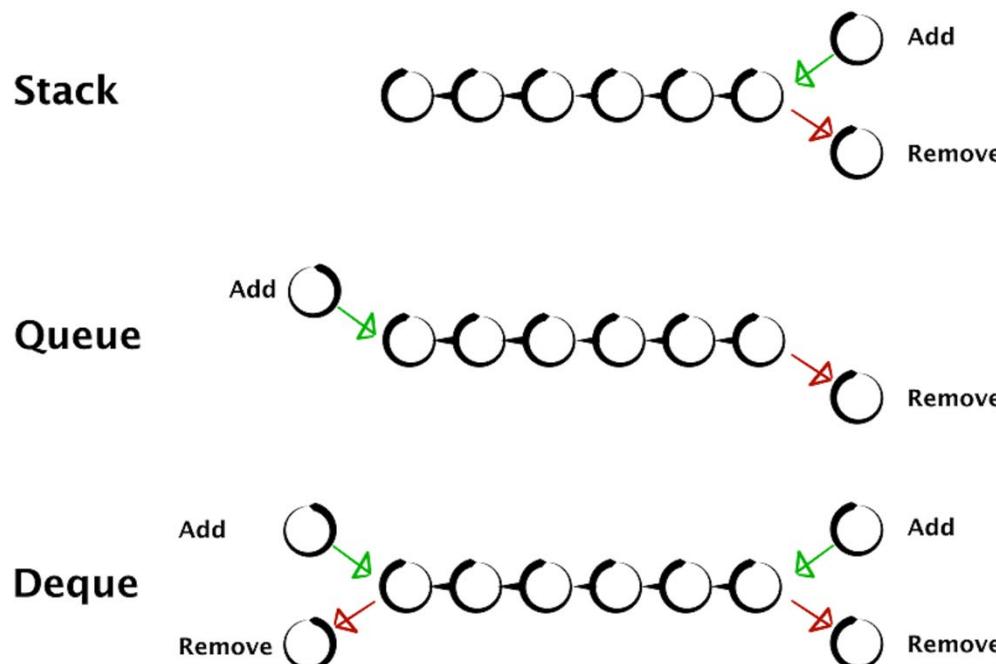
- Deque (Double-Ended Queue)
  - Supports insertion and deletion at both the front and the back of the queue



- Example
  - Guests removed from the head of the waitlist
    - They don't like the table and want to be relisted back to the head of the queue
  - Guests at the tail of the waitlist queue grow impatient and leave

# Deque

- Stack vs Queue vs Deque



# Deque Abstract Data Type

`addFirst(e)`: Insert a new element *e* at the front of the deque.

`addLast(e)`: Insert a new element *e* at the back of the deque.

`removeFirst()`: Remove and return the first element of the deque  
(or null if the deque is empty).

`removeLast()`: Remove and return the last element of the deque  
(or null if the deque is empty).

`first()`: Returns the first element of the deque, without removing it  
(or null if the deque is empty).

`last()`: Returns the last element of the deque, without removing it  
(or null if the deque is empty).

`size()`: Returns the number of elements in the deque.

`isEmpty()`: Returns a boolean indicating whether the deque is empty.

# Deque Operation Example

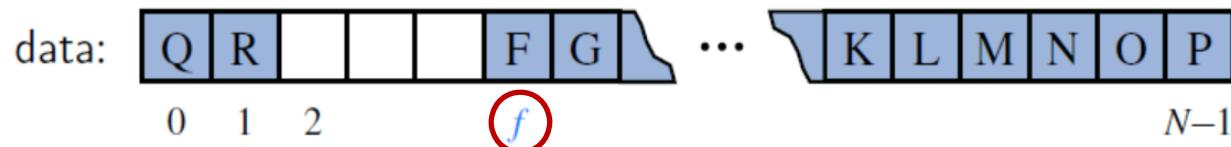
Method	Return Value	D
addLast(5)	–	(5)
addFirst(3)	–	(3, 5)
addFirst(7)	–	(7, 3, 5)
first()	7	(7, 3, 5)
removeLast()	5	(7, 3)
size()	2	(7, 3)
removeLast()	3	(7)
removeFirst()	7	()
addFirst(6)	–	(6)
last()	6	(6)
addFirst(8)	–	(8, 6)
isEmpty()	false	(8, 6)
last()	6	(8, 6)

# Interface Deque

```
public interface Deque<E> {
    public int size();
    public boolean isEmpty();
    public E first();
    public E last();
    public void addFirst(E e);
    public void addLast(E e);
    public E removeFirst();
    public E removeLast();
}
```

# Implementing a Deque

- First element
  - Index:  $f$
  - After add:  $f = (f - 1 + N) \% N$ ,  $size++$
  - After remove:  $f = (f + 1) \% N$ ,  $size--$
- Last element
  - Index:  $(f + size - 1 + N) \% N$
  - After add:  $(f + size) \% N$ ,  $size++$
  - After remove:  $(f + size - 2 + N) \% N$ ,  $size--$



```
@SuppressWarnings("unchecked")
public class ArrayDeque<E> implements Deque<E> {
    protected static final int CAPACITY = 16;
    protected E[] data;
    protected int size;
    protected int f;

    public ArrayDeque() { this(CAPACITY); }
    public ArrayDeque(int capacity) { data = (E[])new Object[capacity]; }

    //Interface Deque
    public int size() { return size; }
    public boolean isEmpty() { return size == 0; }

    private int index(int i) { //take care of the wrapping
        return (i + data.length) % data.length;
    }

    private int lastIndex() {
        return index(f + size - 1);
    }
```

```

public E first() {
    if(isEmpty())
        throw new IllegalStateException("Deque is empty");
    return data[f];
}
public E last() {
    if(isEmpty())
        throw new IllegalStateException("Deque is empty");
    return data[lastIndex()];
}

public void addFirst(E e) {
    if(size == data.length)
        throw new IllegalStateException("Deque is full");
    f = index(f - 1);
    data[f] = e;
    size++;
}
public void addLast(E e) {
    if(size == data.length)
        throw new IllegalStateException("Deque is full");
    int i = index(lastIndex() + 1);
    data[i] = e;
    size++;
}

```

```
public E removeFirst() {
    if(isEmpty())
        throw new IllegalStateException("Deque is empty");
    E ret = data[f];
    data[f] = null;
    f = index(f + 1);
    size--;
    return ret;
}

public E removeLast() {
    if(isEmpty())
        throw new IllegalStateException("Deque is empty");
    int i = lastIndex();
    E ret = data[i];
    data[i] = null;
    size--;
    return ret;
}
```

```
protected static void onFalseThrow(boolean b) {  
    if(!b)  
        throw new RuntimeException("Error: unexpected");  
}  
  
public static void main(String[] args) {  
    Deque<Integer> deque = new ArrayDeque<Integer>();  
  
    deque.addFirst(1);  
    deque.addLast(2);  
    deque.addFirst(3);  
    deque.addLast(4);  
  
    onFalseThrow(deque.size() == 4);  
    onFalseThrow(deque.first() == 3);  
    onFalseThrow(deque.last() == 4);  
    onFalseThrow(deque.removeFirst() == 3);  
    onFalseThrow(deque.removeLast() == 4);  
    onFalseThrow(deque.removeLast() == 2);  
    onFalseThrow(deque.removeLast() == 1);  
    onFalseThrow(deque.size() == 0);  
  
    System.out.println("Success!");  
}  
}
```

# Deque in Java

- java.util.ArrayDeque, java.util.LinkedList

Our Deque ADT	Interface <code>java.util.Deque</code>	
	throws exceptions	returns special value
<code>first()</code>	<code>getFirst()</code>	<code>peekFirst()</code>
<code>last()</code>	<code>getLast()</code>	<code>peekLast()</code>
<code>addFirst(<i>e</i>)</code>	<code>addFirst(<i>e</i>)</code>	<code>offerFirst(<i>e</i>)</code>
<code>addLast(<i>e</i>)</code>	<code>addLast(<i>e</i>)</code>	<code>offerLast(<i>e</i>)</code>
<code>removeFirst()</code>	<code>removeFirst()</code>	<code>pollFirst()</code>
<code>removeLast()</code>	<code>removeLast()</code>	<code>pollLast()</code>
<code>size()</code>	<code>size()</code>	
<code>isEmpty()</code>	<code>isEmpty()</code>	

# Priority Queues

- Priority queue
  - A collection of prioritized elements
  - Arbitrary element insertion
  - Removal of the **first priority** element
- Key
  - Elements in priority queues are associated with a key
  - Elements with the minimal key will be the next to be removed
  - Keys are comparable (**total order**)

# The Priority Queue ADT

- insert( $k, v$ ):** Creates an entry with key  $k$  and value  $v$  in the priority queue.
- min( ):** Returns (but does not remove) a priority queue entry  $(k,v)$  having minimal key; returns null if the priority queue is empty.
- removeMin( ):** Removes and returns an entry  $(k,v)$  having minimal key from the priority queue; returns null if the priority queue is empty.
- size( ):** Returns the number of entries in the priority queue.
- isEmpty( ):** Returns a boolean indicating whether the priority queue is empty.

# Priority Queue Operation Example

Method	Return Value	Priority Queue Contents
insert(5,A)		{ (5,A) }
insert(9,C)		{ (5,A), (9,C) }
insert(3,B)		{ (3,B), (5,A), (9,C) }
min()	(3,B)	{ (3,B), (5,A), (9,C) }
removeMin()	(3,B)	{ (5,A), (9,C) }
insert(7,D)		{ (5,A), (7,D), (9,C) }
removeMin()	(5,A)	{ (7,D), (9,C) }
removeMin()	(7,D)	{ (9,C) }
removeMin()	(9,C)	{ }
removeMin()	null	{ }
isEmpty()	true	{ }

# Implementing a Priority Queue

- The entry **composite**

```
/** Interface for a key-value pair. */
public interface Entry<K,V> {
    K getKey(); // returns the key stored in this entry
    V getValue(); // returns the value stored in this entry
}
```

# Priority Queue Interface

```
public interface PriorityQueue<K, V> {  
    int size();  
    boolean isEmpty();  
    Entry<K, V> insert(K key, V value)  
        throws IllegalArgumentException;  
    Entry<K, V> min();  
    Entry<K, V> removeMin();  
}
```

# Comparing Keys (Total Order)

- Partial order  $\leq$ 
  - **Reflexive property:**  $k \leq k$ .
  - **Antisymmetric property:** if  $k_1 \leq k_2$  and  $k_2 \leq k_1$ , then  $k_1 = k_2$ .
  - **Transitive property:** if  $k_1 \leq k_2$  and  $k_2 \leq k_3$ , then  $k_1 \leq k_3$ .
- Total order
  - A partial order  $\leq$  is a total order if for any keys  $k_1$  and  $k_2$ 
    - **Comparability property:**  $k_1 \leq k_2$  or  $k_2 \leq k_1$ .

# Interface `java.lang.Comparable`

- Define **the natural ordering** of instances
  - E.g. orders among numbers
  - E.g. the lexicographic ordering of strings
- `a.compareTo(b)` must return an integer  $i$  such that
  - $i < 0$  designates that  $a < b$ .
  - $i = 0$  designates that  $a = b$ .
  - $i > 0$  designates that  $a > b$ .

# Interface `java.util.Comparator`

- Provides an order **other than the natural ordering**
  - E.g. order strings by their length
- External to the class
- `compare(a, b)`
  - Returns an integer with a similar meaning to `compareTo`

# Interface java.util.Comparator

- Example

```
public class StringLengthComparator implements Comparator<String> {  
    /** Compares two strings according to their lengths. */  
    public int compare(String a, String b) {  
        if (a.length() < b.length()) return -1;  
        else if (a.length() == b.length()) return 0;  
        else return 1;  
    }  
}
```

# Comparators and Priority Queue ADT

- Priority Queue ADT
  - Allows users to choose key types
  - Allows users to use **custom key Comparators**
  - Makes the priority queue general and reusable

# Comparators and Priority Queue ADT

- **Default comparator** for the natural ordering

```
import java.util.Comparator;

public class DefaultComparator<E extends Comparable<E>>
    implements Comparator<E> {

    public int compare(E a, E b) throws ClassCastException {
        return ((Comparable<E>)a).compareTo(b);
    }

}
```

# AbstractPriorityQueue Base Class

```
import java.util.Comparator;

public abstract class AbstractPriorityQueue<K extends Comparable<K>, V>
    implements PriorityQueue<K, V> {

    protected static class PQEntry<K extends Comparable<K>, V>
        implements Entry<K, V> {
        private K k;
        private V v;

        public PQEntry(K key, V value) { k = key; v = value; }
        public K getKey() { return k; }
        public V getValue() { return v; }

        protected void setKey(K key) { k = key; }
        protected void setValue(V value) { v = value; }
    }
}
```

```
private Comparator<K> comp;

protected AbstractPriorityQueue() {
    this(new DefaultComparator<K>());
}
protected AbstractPriorityQueue(Comparator<K> c) {
    comp = c;
}

protected int compare(Entry<K,V> a, Entry<K,V> b) {
    return comp.compare(a.getKey(), b.getKey());
}

protected boolean checkKey(K key) throws IllegalArgumentException {
    try {
        return comp.compare(key, key) == 0;
    } catch(ClassCastException e) {
        throw new IllegalArgumentException("Incompatible key");
    }
}

public boolean isEmpty() { return size() == 0; }
}
```

# Priority Queue with an Unsorted List

```
import java.util.Comparator;

public class UnsortedPriorityQueue<K extends Comparable<K>, V>
    extends AbstractPriorityQueue<K, V>{
    private PositionalList<Entry<K,V>> list =
        new PositionalDbiLinkedList<>();

    public UnsortedPriorityQueue() { super(); }
    public UnsortedPriorityQueue(Comparator<K> comp) {
        super(comp);
    }

    private Position<Entry<K,V>> findMin() {
        Position<Entry<K,V>> small = list.first();
        for(Position<Entry<K,V>> walk : list) {
            if(compare(walk.getElement(), small.getElement()) < 0)
                small = walk;
        }
        return small;
    }
}
```

```
public int size() {
    return list.size();
}

public Entry<K,V> insert(K key, V value)
        throws IllegalArgumentException {
    checkKey(key);
    Entry<K,V> newest = new PQEntry<K,V>(key, value);
    list.addLast(newest);
    return newest;
}

public Entry<K,V> min() {
    if(list.isEmpty())
        return null;
    return findMin().getElement();
}

public Entry<K,V> removeMin() {
    if(list.isEmpty())
        return null;
    return list.remove(findMin());
}
```

```
private static void onFalseThrow(boolean b) {
    if(!b)
        throw new RuntimeException("Error: unexpected");
}

public static void main(String[] args) {
    PriorityQueue<Integer, Integer> pq =
        new UnsortedPriorityQueue<Integer, Integer>();
    pq.insert(3, 3);
    pq.insert(2, 2);
    pq.insert(1, 1);
    pq.insert(4, 4);
    pq.insert(5, 5);

    for(int i = 1; i <= 5; i++) {
        onFalseThrow(pq.min().getValue() == i);
        onFalseThrow(pq.removeMin().getValue() == i);
    }
    System.out.println("Success!");
}
```

# Priority Queue with an Unsorted List

- Running time of `UnsortedPriorityQueue`

Method	Running Time
<code>size</code>	$O(1)$
<code>isEmpty</code>	$O(1)$
<code>insert</code>	$O(1)$
<code>min</code>	$O(n)$
<code>removeMin</code>	$O(n)$

# Priority Queue with a Sorted List

```
import java.util.Comparator;

public class SortedPriorityQueue<K extends Comparable<K>, V>
    extends AbstractPriorityQueue<K, V>{
    private PositionalList<Entry<K,V>> list =
        new PositionalDblLinkedList<>();

    public SortedPriorityQueue() { super(); }
    public SortedPriorityQueue(Comparator<K> comp) { super(comp); }
    public int size() { return list.size(); }

    public Entry<K,V> min() {
        if(list.isEmpty())
            return null;
        return list.first().getElement();
    }
    public Entry<K,V> removeMin() {
        if(list.isEmpty())
            return null;
        return list.remove(list.first());
    }
}
```

```

public Entry<K,V> insert(K key, V value)
    throws IllegalArgumentException {
    checkKey(key);
    //new Entry
    Entry<K,V> newest = new PQEntry<K,V>(key, value);

    //find where newest should be inserted
    Position<Entry<K,V>> pos = null;
    for(Position<Entry<K,V>> walk : list) {
        if(compare(newest, walk.getElement()) < 0) {
            pos = walk;
            break;
        }
    }

    //add it to the last position or insert it before pos
    if(pos == null)
        list.addLast(newest);          //new key is the largest
    else
        list.addBefore(pos, newest); //new key goes before pos

    return newest;
}

```

```
private static void onFalseThrow(boolean b) {
    if(!b)
        throw new RuntimeException("Error: unexpected");
}

public static void main(String[] args) {
    PriorityQueue<Integer, Integer> pq =
        new SortedPriorityQueue<Integer, Integer>();
    pq.insert(3, 3);
    pq.insert(2, 2);
    pq.insert(1, 1);
    pq.insert(4, 4);
    pq.insert(5, 5);

    for(int i = 1; i <= 5; i++) {
        onFalseThrow(pq.min().getValue() == i);
        onFalseThrow(pq.removeMin().getValue() == i);
    }
    System.out.println("Success!");
}
}
```

# Priority Queue with a Sorted List

- Running time comparison

Method	Unsorted List	Sorted List
size	$O(1)$	$O(1)$
isEmpty	$O(1)$	$O(1)$
insert	$O(1)$	$O(n)$
min	$O(n)$	$O(1)$
removeMin	$O(n)$	$O(1)$

# Assignment 5

- We will implement a “**Snake Bite**” game in this assignment
  - Implement the code marked with TODO tags
  - Submit modified source files in a single zip file
- Due date: 4/21/2022

# Assignment 5

- Implement `DynamicArrayDeque`
  - Finish implementing the TODOs
  - You should pass all unit tests in its main
- Implement `SnakeWorld`
  - Finish implementing the TODOs

- When all TODOs are implemented correctly, you can play the “Snake Bite” game

