Issues Related to Using Finger-Fitts law to Model One-Dimensional Touch Pointing Tasks

Yu-Jung Ko
yujko@cs.stonybrook.edu
Department of Computer Science,
Stony Brook University
New York, USA

Hang Zhao
zhao8@cs.stonybrook.edu
Department of Computer Science,
Stony Brook University
New York, USA

IV Ramakrishnan
ram@cs.stonybrook.edu
Department of Computer Science,
Stony Brook University
New York, USA

Shumin Zhai
zhai@acm.org
Google, Mountain View
California, USA

Xiaojun Bi
xiaojun@cs.stonybrook.edu
Department of Computer Science,
Stony Brook University
New York, USA

ABSTRACT

Finger-Fitts law [6] is a variant of Fitts' law which accounts for the finger ambiguity in touch pointing. In this paper we investigated two research questions related to Finger-Fitts law: (1) Should Finger-Fitts law use nominal target width W or effect target width W_e to model MT? and (2) should Finger-Fitts law use a pre-defined value (denoted by σ_a) or a free parameter (denoted by c) to represent the absolute ambiguity caused by finger touch? Our investigation on two touch pointing datasets showed that there are cases where using nominal width has stronger model fitness, and also cases where using effective width is better. Regarding the representation of finger ambiguity, using a free parameter c to represent the ambiguity of finger touch always leads to stronger model fitness than using the pre-defined σ_a, after controlling for overfitting. It indicates that viewing the finger ambiguity as an empirically determined parameter has more flexibility to capture the ambiguity of finger touch involved in the study. Overall, our research advances the understanding on how to model Finger touch input with Finger-Fitts law.

CCS CONCEPTS

• **Human-centered computing** → **HCI theory, concepts and models**; **Pointing**.

KEYWORDS

empirical study, Fitts' Law, finger input, pointing models

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1 INTRODUCTION

Among a number of finger-touch based interactions, pointing has been a dominant input modality on mobile devices such as smartphones and tablets. Due to its prevalence, modeling touch pointing is crucial in designing touch interfaces. Fitts' law [13, 23] (Equation 1), which relates the pointing movement time (MT) to the relative precision of the tasks ($\frac{A}{W}$), is the most widely known pointing model. However, despite its success in modeling pointing actions with a mouse or stylus, Fitts' law does not address the ambiguity caused by finger touch, which is the widely recognized "fat finger" problem. Hence, it cannot accurately model touch-based pointing.

$$MT = a + b\log_2(\frac{A}{W} + 1). \tag{1}$$

Finger-Fitts law (a.k.a FFitts law, Equation 2) [6] is a refinement of Fitts' law for modeling touch pointing:

$$\begin{aligned} MT &= a + b\log_2\big(\frac{A}{\sqrt{2\pi e(\sigma^2 - \sigma_a^2)}} + 1\big) \\ &= a + b\log_2\big(\frac{A}{\sqrt{W_e^2 - 2\pi e\sigma_a^2}} + 1\big). \end{aligned} \tag{2}$$

Previous research [6, 34] has shown that Finger-Fitts law (Equation 2) can more accurately model finger-touch pointing than Fitts' law, and has been used for modeling typing speed on soft keyboard [4], for developing a keyboard decoding algorithm [5], and for modeling other touch interaction such as crossing [22].

The recent work of Ko et al. [20] indicates that the nominal target width W (the width defined by the task parameter) can be used in lieu of the effective target width W_e in Finger-Fitts law to model touch pointing. Although the model proposed by Ko et al. [20] is for 2D pointing (Equation 8 in [20]), for 1D pointing, their model becomes the following form:

$$MT = a + b\log_2\Big(\frac{A}{\sqrt{W^2 - c^2}} + 1\Big), \tag{3}$$

where a, b, and c are all empirically determined parameters.

Compared to the original Finger-Fitts law (Equation 2), equation 3 has two changes: (1) the effective width W_e is replaced with the nominal target width W, and (2) it uses a free parameter c rather than a pre-defined σ_a to represent the ambiguity caused by finger touch.

How would these changes influence the performance of models? The understanding of this question would advance our knowledge of touch modeling.

In this paper, we conducted a user study, and analyzed the previously reported data in [6], to answer two research questions regarding the determination of target width, and the representation of finger ambiguity in Finger-Fitts law. The first question is which target width should we use in Finger-Fitts law, nominal width W or effective width W_e? The second question is how should the model represent the absolute finger ambiguity, using the predefined σ_a from separate calibration tasks (e.g., the task described in [6]) or treating it as a free parameter estimated from data [20]?

Our investigation showed mixed results for using W vs. W_e: there are cases where using nominal width is better than effective width while there are cases that showed opposite results. Our study also showed that using the free parameter c results in stronger model fitness than using the pre-defined σ_a to represent the finger ambiguity, after controlling for overfitting. Compared to pre-defined ambiguity from the calibration task, the free parameter c offers more flexibility to capture the uncertainty induced by the finger touch during the study. Overall, our investigation advances the understanding of using Finger-Fitts law to model touch pointing.

2 RELATED WORK

We review related work on (1) using Fitts' law and its variants to model pointing, and (2) modeling finger touch pointing with Finger-Fitts law.

2.1 Modeling 1D pointing

As one of the best known theoretical foundations of HCI, Fitts' law (Equation 1) [13, 23] has served as a cornerstone for interface and input device evaluation [9, 23], interface optimization [21], and interaction behavior modeling [11].

The beauty of the original Fitts' law lies in its simplicity. It is a pure task model of human pointing performance, in which all of the model's independent variables are *a priori* task parameters A and W. For a given graphical object's distance and size, for example, designers can predict or estimate the average time it takes a user to complete a pointing task at it.

One challenge of applying Fitts' law is that a user might or might not comply with the task precision defined by A/W when performing the tasks, causing over- or under-utilization of target width [35]. This is partly because a user may adopt different speed-accuracy trade-off policies [3, 4, 16, 17, 24, 25, 33]. The way researchers have addressed the varied degree of task compliance is to bend Fitts' law away from a pure task model towards a behavioral one by changing an independent variable in the model from a task parameter W (target width) to "effective width", an *a posterior* quantity depending on user's behavior. First proposed by Crossman [12] and explored further [23, 26, 32], the effective width adjustment method has shown a stronger model fit if the observed error rates deviate from 4%. It replaces the nominal target width W with the so-called effective width W_e (i.e., $\sqrt{2\pi e}\sigma$), as shown in Equation 4.

$$MT = a + b \cdot \log_2\left(\frac{A}{\sqrt{2\pi e}\sigma} + 1\right) \quad (4)$$

$$= a + b \cdot \log_2\left(\frac{A}{W_e} + 1\right), \quad (5)$$

Controlled studies [35] showed that using W_e could partially but not fully account for the subjective layer of a speed-accuracy tradeoff. Involving the posterior variable σ complicates Fitts' law as a predictive tool for design. Later in the next section, we explain in detail that because the Fitts' law with effective width adjustment (Equations 4 and 5) is the basis of Finger-Fitts law [6]), the limitation of involving a posterior variable also limits the predictive power of Finger-Fitts law.

Another line of Fitts' law research closely related to the current work is about modeling small-sized target acquisition tasks. Previous researchers [32] have proposed using $W - c$ instead of $W_e = \sqrt{2\pi e}\sigma$ to adjust the target width in Fitts' law, where c was an experimentally determined constant attributed to hand tremor. The modified version gave a good fit for both pencil-based [32] and mouse-based [10] pointing tasks. Our research later shows that c-constant model could serve as a simplification of the refined Finger-Fitts model, with similar model fitness.

2.2 Modeling finger touch pointing

As finger touch has become the dominant input modality in mobile computing, a sizable amount of research has been carried out to understand and model the uncertainty in touch interaction. On a capacitive touchscreen, a touch point is converted from the contact region of the finger. This is an ambiguous and "noisy" procedure, which inevitably introduces errors. Factors such as finger angle [18, 19] and pressure [15] may affect the size and shape of the contact region, unintentionally altering the touch position. The lack of visual feedback on where the finger lands due to occlusion (the "fat finger" problem) further exacerbates the issue [18, 19, 27–29]. As a result, it is hard to precisely control the touch position even with fine motor control ability.

This "fat finger" problem, or the lack of absolute precision in finger touch, presented a challenge to use Fitts' law as a model for finger touch-based pointing, because the only variable in Fitts' law, namely Fitts' index of difficulty, $log_2(A/W + 1)$, is solely determined by the relative movement precision, or the distance to target size ratio.

Bi, Li and Zhai [6–8] identified this challenge, and proposed the Finger Fitts law [6] to address it. They derived their model by separating two sources of end point variance - those due to the absolute imprecision of finger touch (denoted by σ_a) and those due to the speed-accuracy trade-off demonstrated in a pointing process (denoted by ${\sigma_r}^2$). The end point variance caused by the imprecision of finger touch (σ_a) is independent to the speed-accuracy trade-off so it should be accounted for. They accounted for it by subtracting ${\sigma_a}^2$ from the observed variance σ^2, which led to Finger-Fitts law (Equation 2). Following the notation of effective width $W_e = \sqrt{2\pi e}\sigma$

(or 4.133σ) [12, 26, 32], Finger-Fitts law (Equation 2) can be re-expressed as Equation 6:

$$MT = a + b\log_2\left(\frac{A}{\sqrt{W_e{}^2 - 2\pi e\sigma_a{}^2}} + 1\right). \tag{6}$$

Later research [4, 6, 22, 34] showed that Finger-Fitts law was successful in modeling touch interaction. For example, research [4] showed it was more accurate than the typical Fitts' law in estimating the upper bound of typing speed on a virtual keyboard. Researchers [22] extended the Finger-Fitts law to the crossing action with finger touch, which improved the model fitness (R^2) from 0.75 to 0.84 over the original Fitts' law. The recent work [20] extends Finger-Fitts law from 1D to 2D, which shows using nominal target width and height is valid for modeling 2-dimensional touch pointing. Complementary to the previous work [20], this work investigates modeling 1-dimensional target selection with nominal target widths. We also compare effective width vs. nominal width while the previous work [20] did not draw such a comparison.

As alluded to earlier, previous research on Finger-Fitts law is mostly based on using the effective width W_e. Next, we describe how we use the nominal width W in Finger-Fitts law (a.k.a the Finger-Fitts-W model), and present a study comparing it with using effective width and the typical Fitts' law.

3 MODEL CANDIDATES FOR FINGER-FITTS LAW

We have two options to represent the target width in Finger-Fitts law: using the effective width or the nominal target width.

Additionally, there are two approaches of representing the finger ambiguity. The first approach relies on the calibration task which results in a pre-defined σ_a [6]. Another approach is to view the finger ambiguity as a free parameter (denoted by c^2) estimated from the empirical data [20].

With two options of representing target width and two approaches of representing finger ambiguity, we have four versions of Finger-Fitts law:

- **Finger-Fitts-W_e-σ_a model**:

$$MT = a + b\log_2\left(\frac{A}{\sqrt{W_e^2 - 2\pi e\sigma_a^2}} + 1\right). \tag{7}$$

  where a, b are empirically determined parameters, and σ_a is a pre-defined value. We adopted the value proposed by Bi et al. [6]: σ_a = 0.94mm for horizontal bar target, and σ_a = 1.5mm for circular targets.
- **Finger-Fitts-W-σ_a model**:

$$MT = a + b\log_2\left(\frac{A}{\sqrt{W^2 - 2\pi e\sigma_a^2}} + 1\right) \tag{8}$$

  where a, b and σ_a are defined the same as the previous model.
- **Finger-Fitts-W_e-c model**:

$$MT = a + b\log_2\left(\frac{A}{\sqrt{W_e^2 - c^2}} + 1\right). \tag{9}$$

  where a, b, and c are all empirically determined parameters.
- **Finger-Fitts-W-c model**:

$$MT = a + b\log_2\left(\frac{A}{\sqrt{W^2 - c^2}} + 1\right) \tag{10}$$

  where a, b, and c are defined in the same way as in the previous model.

Additionally, we also include the typical Fitts' law (Equation 1), and the Fitts' law with effective width (Equation 4) as another two model candidates. Therefore, we have six models in total.

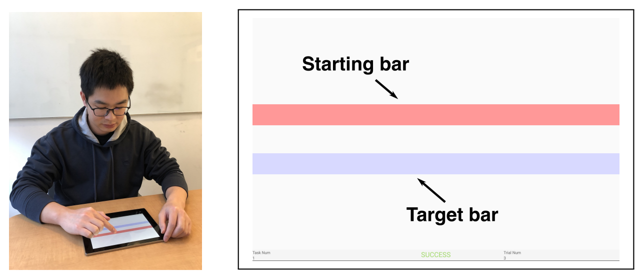
We carried out two studies to evaluate the finesses of these six models on the horizontal bar and circular target pointing tasks, respectively. Additionally, we also evaluated these models on the previously reported data from Bi, Li, and Zhai [6].

4 EXPERIMENT 1: EVALUATION IN 1D POINTING TASKS WITH HORIZONTAL BARS

We first carried out a study to evaluate the proposed models in horizontal bar selection tasks.

4.1 Participants and Apparatus

We recruited 23 subjects for an IRB approved study (7 females; aged from 21 - 36). All of them were right-handed and daily smartphone users. A Google Pixel C tablet with 2560x1800 resolution and 308 PPI were used throughout the experiment. Each participant was instructed to perform the tasks on the tablet. They were instructed to select the target with the index finger as fast and accurately as possible.



(a) Experiment Setup **(b) Targets (horizontal bars)**

Figure 1: (a) A participant was doing the task. (b) A screenshot of the task.

4.2 Design and Data Processing

4.2.1 Target Acquisition Task. We designed a within-subject reciprocal target acquisition task with horizontal bars with different widths and distances between the bars. In this experiment, we applied a similar 1-D shape in [6], horizontal bars, as our targets.

The study included 15 conditions with 5 levels (2, 4, 8, 12, 20 mm) of target height (W) and 3 levels (24, 48, 80 mm) of distance (A). The wide range of target height from 2 to 20 mm comprises the most practical design of UI elements on mobile devices and tablets. Each condition included 20 touches (19 trials, where the first touches in each condition are considered the starting action) and the condition would show up in random order. We have 23 (participants) $\times$ 15 (conditions) $\times$ 10 (successful trials in one condition) = 3,450 successful trials in total.

At the beginning of each trial, two horizontal bars were displayed on the touch screen. One starting bar colored in red and one in blue. The blue horizontal bar indicates the destination bar after successfully touching the starting bar. The participant was instructed to select the start bar to start the trial. Upon successfully selecting the start bar, the colors of start and destination bars got swapped and the participant was instructed to select the destination bar as fast and accurately as possible. A successful sound would be played if the target was successfully selected. Otherwise, a failure sound was played. The elapsed time between the moment the user successfully selected the start bar and the moment the user subsequently landed down the touch point to select the destination bar was recorded as the movement time of the current trial; the touch point for selecting the destination bar was the location of the endpoint, regardless of whether the touch point was within or outside the target boundary. If the participant succeeded in selecting the destination bar, the colors of the two circles were swapped again. This would be recorded as a successful trial and move on to the next successful trial requirement immediately. If the participant failed in selecting the destination bar, she had to successfully select it again to start the next trial. This setting ensured that in each trial the finger always starts from somewhere within the starting bar, reducing the noise in measuring A.

Heights (mm)	MT Mean [SD] (s)	Error rate
2	0.63 [0.16]	39.3%
4	0.54 [0.11]	18.1%
8	0.46 [0.11]	4.8%
12	0.42 [0.11]	2.1%
20	0.38 [0.11]	0.0%

Table 1: Movement time and error rates over different target widths

Distances (mm)	MT Mean [SD] (s)	Error rate
24	0.44 [0.13]	14.1%
48	0.51 [0.15]	15.5%
80	0.56 [0.16]	17.8%

Table 2: Movement time and error rates over different distances

4.2.2 Data processing. We pre-processed the data by removing touch points that fell beyond 3 standard deviations to the target center. In horizontal bar acquisition tasks, 100 out of 4,109 touch points (2.4%) were removed as outliers. This results in 3,450 successful trials out of 4,099 trials in total.

4.3 Results

4.3.1 MT and error rates across the condition. We observed movement time and the error rates across different target widths and distances (Table 1 and 2).

For movement times, a repeated measure ANOVA test showed that both width W ($F_{4,88} = 158.4$, $p < 0.001$) and distance A ($F_{2,44} = 69.38$, $p < 0.001$) had a statistically significant effect. The interaction effect of width and distance was also significant ($F_{8,176} = 2.564$, $p = 0.011$). For error rates, a repeated measure ANOVA test showed that width W had a significant effect ($F_{4,88} = 122.5$, $p < 0.001$), but not distance A ($F_{2,44} = 2.311$, $p = 0.111$). The interaction effect of width and distance was not significant ($F_{8,176} = 1.569$, $p = 0.137$).

We also evaluated if the participants were error-prone with smaller targets. A pairwise t-test with Bonferroni correction showed that W = 2, 4mm had a significant effect on error rates against the cases that target width W = 8, 12, and 20mm, with p values significantly lower than 0.05. These results concurred with the conclusion from other research [6, 8]

4.3.2 Regression for MT vs. ID. Figure 2 shows the regression results of MT vs. ID. As shown, the Finger-Fitts-W-c has the highest R^2 value (0.984) among all the test models, indicating its high model fitness.

4.3.3 RMSE of MT Prediction. To increase the external validity of the evaluation, we also examined the Root Mean Square Error (RMSE) of MT prediction with cross validation. We conducted leave-one-(A, W)-out cross validation and obtained the mean and standard deviation [SD] of RMSE (Unit: Second) for Finger-Fitts-W, Finger-Fitts-W_e, Fitts' law - W_e and Fitts' law - W (Table 2).

4.3.4 Information Criteria. Information criteria [1, 2, 30, 31] have been widely used to compare the quality of models because they take into account the complexity of the model (i.e., the number of free parameters). Commonly used information criteria include AIC (Akaike Information Criterion), $WAIC$ (Widely Applicable Information Criterion) and BIC (Bayesian information criterion) [14], all of which penalize the complexity of a model. In general, the smaller the information criterion, the better the model is. We have calculated multiple information criteria including AIC, $WAIC$, and BIC (Table 3). As shown, the Finger-Fitts-W-c outperforms Finger-Fitts-W-σ_a, Finger-Fitts-W_e-σ_a and Finger-Fitts-W_e-c, Fitts' law - W and Fitts' law - W_e in these metrics.

4.3.5 Model Fitness. The result of all six models in the touch input data we collected is shown in Table 3. Compared to the models with effective width W_e (models #4 - #6), models using a free parameter c to represent the finger touch ambiguity (model #1 - #3) lead to better performance.

4.4 Model Fitness on Bi, Li, and Zhai's Horizontal Bar Pointing Data Set [6]

We also evaluated the fitness of all six models with the horizontal-bar target acquisition data reported in Bi, Li, and Zhai's paper [6]. As shown in Table 4, in both nominal width W and effective width W_e conditions, models using a free parameter c to represent the finger touch ambiguity (models #3 and #6) lead to the best performance. Comparing the model using W with its counterpart using W_e, the results are mixed: there are situations where models with W are better (e.g., models #1 vs. #4), and also situations where models with W_e are better (e.g., models #2 vs. #5, and models #3 vs. #6).

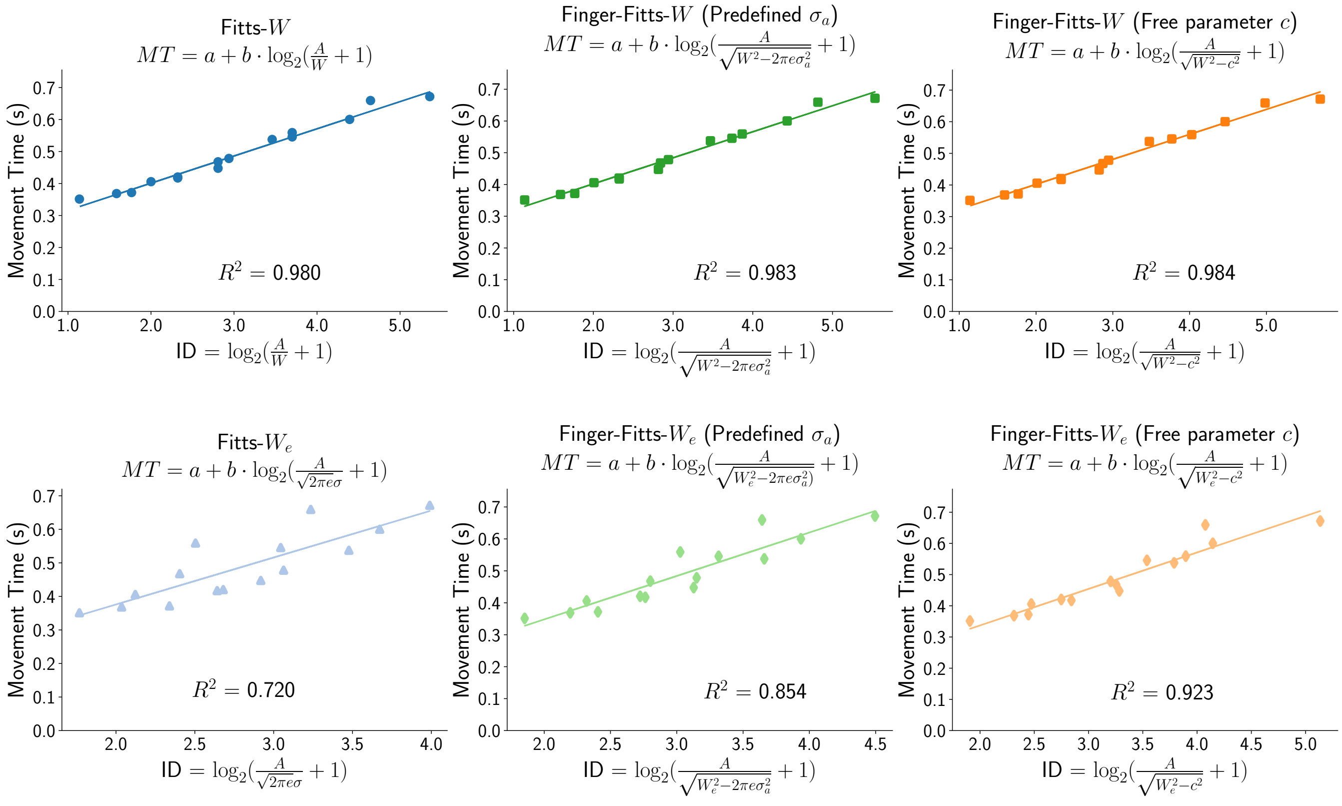


Figure 2: Horizontal Bar: ***MT vs. ID*** **regressions for Fitts' law, Fitts' law with effective width, Finger-Fitts-W, and Finger-Fitts-W_e models. As shown, Finger-Fitts-W with free parameter c model shows the best model fitness.**

Model				*ID*	R^2	RMSE [SD]	AIC	WAIC	BIC	Parameters
Nominal Width W	#1		Fitts-W Eq. (1)	$\log_2(\frac{A}{W}+1)$	0.98	0.014 [0.003]	-75.20	-77.58	-73.08	$a=0.231, b=0.085$
	#2		Finger-Fitts-W-σ_a Eq. (8)	$\log_2(\frac{A}{\sqrt{W^2-2\pi e\sigma_a^2}}+1)$	0.983	0.013 [0.003]	-78.11	-80.45	-75.98	$a=0.238, b=0.082$, $\sigma_a^2=0.884$
	#3		**Finger-Fitts-W-c Eq. (10)**	$\log_2(\frac{A}{\sqrt{W^2-c^2}}+1)$	**0.984**	**0.013 [0.002]**	**-79.31**	**-81.97**	**-77.18**	a=0.244,b=0.079, c^2=1.54
Effective Width W_e	#4		Fitts-W_e Eq. (4)	$\log_2(\frac{A}{\sqrt{2\pi e}\sigma}+1)$	0.72	0.054 [0.011]	-35.65	-39.00	-33.53	$a=0.096, b=0.14$
	#5		Finger-Fitts-W_e-σ_a Eq. (7)	$\log_2(\frac{A}{\sqrt{W_e^2-2\pi e\sigma_a^2}}+1)$	0.854	0.038 [0.01]	-45.36	-48.59	-43.23	$a=0.076, b=0.136$, $\sigma_a^2=0.884$
	#6		Finger-Fitts-W_e-c Eq. (9)	$\log_2(\frac{A}{\sqrt{W_e^2-c^2}}+1)$	0.923	0.027 [0.01]	-55.07	-56.72	-52.94	$a=0.103, b=0.117$, $c^2=23.38$

Table 3: Horizontal Bar: The parameters, R^2, *RMSE* of leave-one-(A, W)-out cross validation, and the information criteria AIC, WAIC and BIC of the models. For the information criteria, the smaller the values, the more accurate the model prediction.

5 EXPERIMENT 2: EVALUATION IN 1D POINTING TASKS WITH CIRCULAR TARGETS

In addition to the 1D horizontal bar experiment, we carried out a study with circular targets which has the same reciprocal target acquisition setting as the horizontal bar experiment.

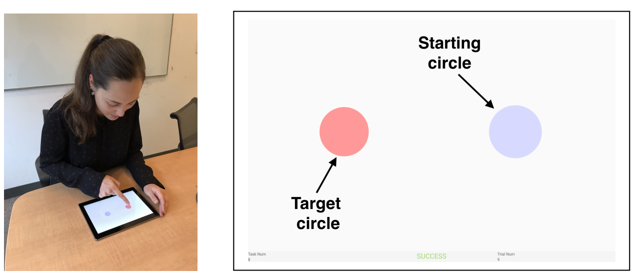
For this experiment, we recruited 14 subjects for an IRB approved study (3 females; aged from 24 - 35). All of them were right-handed and daily smartphone users. The participants practice the experiment on the same apparatus and perform the tasks with the same instruction as in the horizontal bar experiment.

In this experiment with circular targets, 15 conditions with 5 levels (4, 6, 8, 10, 12 mm) of diameters (W) and 3 levels (16, 28, 60 mm) of distance (A) were considered. It had two different movement directions, which are vertical and horizontal movements. Each condition included 20 touches (19 trials, where the first touches in each condition are considered the starting action) and the condition would show up in random order. Unlike the horizontal bar experiment, once 19 valid trials are operated, the experiment will move on to the next non-repeated, randomly ordered condition. Except for this selection, The acquisition task setting follows the one in the horizontal target experiment. In total, We have 14 (participants) × 15 (conditions) × 2 (directions) × 19 (trials) = 7,980 trials.

Model		*ID*	R^2	RMSE	AIC	WAIC	BIC	Parameters
Nominal Width W	#1	$\log_2(\frac{A}{W}+1)$	0.956	0.009 [0.001]	-29.37	-33.17	-29.99	$a=0.25, b=0.06$
	#2	$\log_2(\frac{A}{\sqrt{W^2-2\pi e\sigma_a^2}}+1)$	0.946	0.010 [0.001]	-28.37	-32.04	-28.99	$a=0.25, b=0.06, \sigma_a^2=0.884$
	#3	$\log_2(\frac{A}{\sqrt{W^2-c^2}}+1)$	0.956	0.009 [0.001]	-29.37	-33.17	-29.99	$a=0.24, b=0.06, c^2=0$
Effective Width W_e	#4	$\log_2(\frac{A}{\sqrt{2\pi e}\sigma}+1)$	0.864	0.016 [0.003]	-22.43	-26.28	-23.06	$a=0.08, b=0.14$
	#5	$\log_2(\frac{A}{\sqrt{W_e^2-2\pi e\sigma_a^2}}+1)$	0.958	0.009 [0.002]	-29.17	-32.06	-29.79	$a=0.11, b=0.12, \sigma_a^2=0.884$
	#6	$\log_2(\frac{A}{\sqrt{W_e^2-c^2}}+1)$	0.961	0.009 [0.002]	-29.36	-32.89	-29.99	$a=0.10, b=0.12, c^2=13.56$

Table 4: Data of the horizontal bar experiment in FFitts law [6]: Prameters, R^2, $RMSE$ of leave-one-(A,W)-out cross validation, and the information criteria AIC, WAIC and BIC of the models.

We pre-processed the data by removing touch points that fell beyond 3 standard deviations to the target center. In circular acquisition tasks, 50 out of 7,980 touch points (0.63%) were removed as outliers.



(a) Experiment Setup **(b) Targets (Circular)**

Figure 3: (a) A participant was doing the task. (b) A screenshot of the task.

Diameters (mm)	MT Mean [SD] (s)	Error rate
4	0.50 [0.13]	24.9%
6	0.37 [0.13]	10.9%
8	0.31 [0.11]	6.4%
10	0.28 [0.10]	2.8%
12	0.25 [0.09]	1.1%

Table 5: Movement time and error rates over different target widths

Distances (mm)	MT Mean [SD] (s)	Error rate
16	0.26 [0.11]	8.2%
28	0.31 [0.12]	9.3%
60	0.45 [0.13]	10.0%

Table 6: Movement time and error rates over different distances

5.1 Results

5.1.1 MT and error rates across the condition. We observed movement time and the error rates across different target widths and distances (Table 5 and 6).

For movement times, a repeated measure ANOVA test showed that both width W ($F_{4,52} = 175.3$, $p < 0.0001$) and distance A ($F_{2,26} = 320.7$, $p < 0.0001$) had a statistically significant effect. The interaction effect of width and distance was also significant ($F_{8,104} = 2.077$, $p < 0.05$). For error rates, a repeated measure ANOVA test showed that width W had a significant effect ($F_{4,52} = 56.19$, $p < .0001$), but not distance A ($F_{2,26} = 1.443$, $p = 0.255$). The interaction effect of width and distance was not significant ($F_{8,104} = 1.965$, $p = 0.058$).

A pairwise t-test with Bonferroni correction was used to evaluate if the participants were error-prone with smaller targets. It showed that the size of the targets with W = 4, 6mm had a significant effect on error rates against the cases with target width W = 8, 10, and 12mm, where p values were significantly lower than 0.05.

5.1.2 Regression for MT vs. ID. Figure 4 shows the regression results of MT vs. ID. As shown, the Finger-Fitts-W law with free parameter c has the highest R^2 value (0.986) among all the test models, indicating its high model fitness. The results also showed that Finger-Fitts-W_e model free parameter c and pre-defined σ_a were better than the typical Fitts' law - W and the Fitts' law - W_e, consistent with findings from previous work [6].

5.1.3 RMSE of MT Prediction. To increase the external validity of the evaluation, we also examined the Root Mean Square Error (RMSE) of MT prediction with cross-validation. We conducted leave-one-(A, W)-out cross-validation and obtained the RMSE for Finger-Fitts-W, Finger-Fitts-W_e, Fitts' law - W_e and Fitts' law - W.

5.1.4 Information Criteria. Similar to Experiment 1, we calculated information criteria including *AIC*, *WAIC*, and *BIC*. As shown, the Finger-Fitts-W law with free parameter c outperforms all other test models in these metrics.

5.1.5 Model Fitness. The result of all six models in the touch input data we collected is shown in Table 7. Finger-Fitts-W-c (model #3) results in a better performance compared with its counterpart using W_e (model #6). On the contrary, Finger-Fitts-W_e-σ_a (model

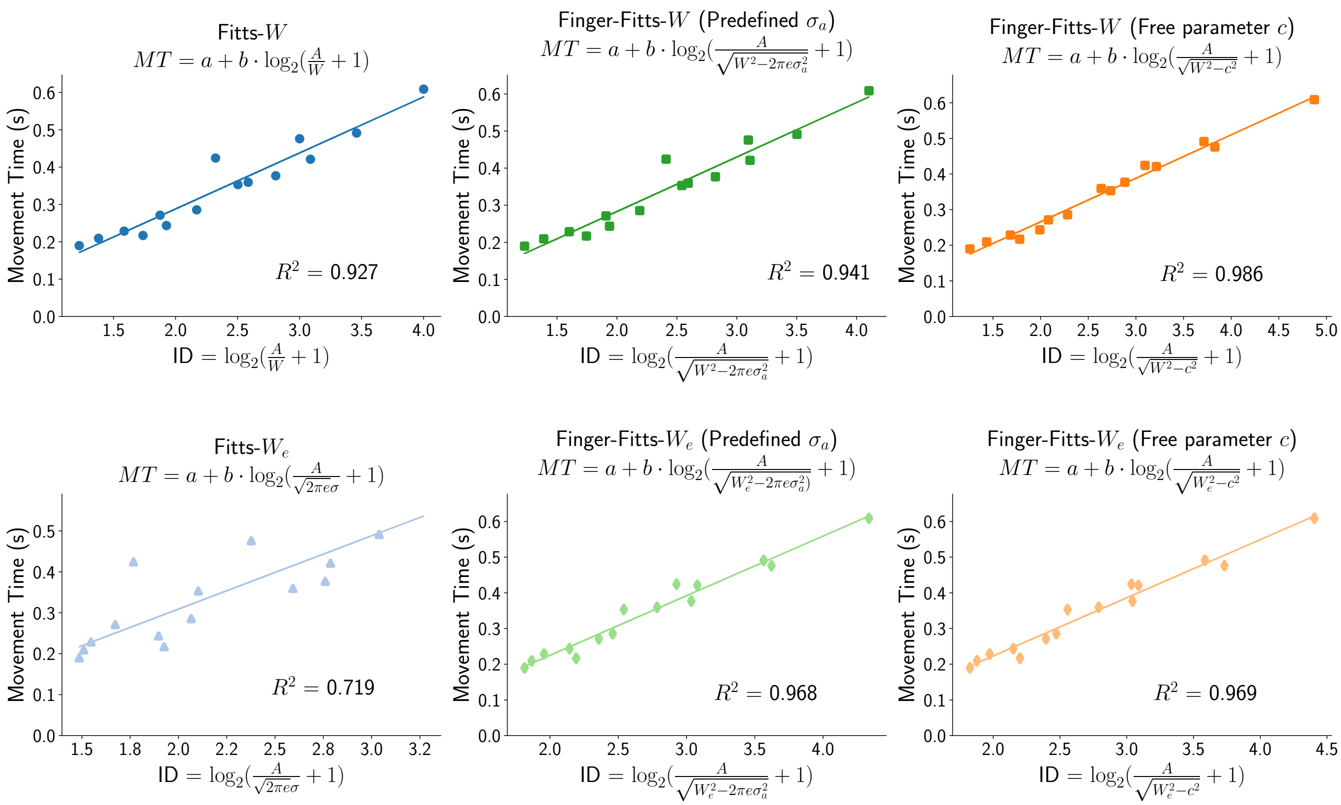


Figure 4: Circular Target: $MT vs. ID$ regressions for Fitts' law, Fitts' law with effective width, Finger-Fitts-W, and Finger-Fitts-W_e models. As shown, Finger-Fitts-W with free parameter c model shows the best model fitness.

Model				ID	R^2	RMSE [SD]	AIC	WAIC	BIC	Parameters
Nominal Width W	#1	●	Fitts-W Eq. (1)	$\log_2(\frac{A}{W}+1)$	0.927	0.033 [0.008]	-50.49	-52.98	-48.37	$a=-0.012, b=0.150$
	#2	■	Finger-Fitts-W-σ_a Eq. (8)	$\log_2(\frac{A}{\sqrt{W^2-2\pi e \sigma_a^2}}+1)$	0.941	0.029 [0.007]	-54.18	-57.66	-52.06	$a=-0.011, b=0.147$, $\sigma_a^2=2.25$
	#3	■	**Finger-Fitts-W-c Eq. (10)**	$\log_2(\frac{A}{\sqrt{W^2-c^2}}+1)$	**0.986**	**0.014 [0.002]**	**-75.68**	**-79.24**	**-73.55**	a=0.022,b=0.122, c^2=11.506
Effective Width W_e	#4	▲	Fitts-W_e Eq. (4)	$\log_2(\frac{A}{\sqrt{2\pi e}\sigma}+1)$	0.719	0.064 [0.012]	-30.84	-33.94	-28.72	$a=-0.049, b=0.179$
	#5	◆	Finger-Fitts-W_e-σ_a Eq. (7)	$\log_2(\frac{A}{\sqrt{W_e-2\pi e \sigma_a^2}}+1)$	0.968	0.021 [0.004]	-63.47	-67.11	-61.35	$a=-0.109, b=0.167$, $\sigma_a^2=2.25$
	#6	◆	Finger-Fitts-W_e-c Eq. (9)	$\log_2(\frac{A}{\sqrt{W_e^2-c^2}}+1)$	0.969	0.02 [0.005]	-64.18	-67.80	-62.06	$a=-0.103, b=0.163$, $c^2=39.36$

Table 7: Circular Target: The parameters, R^2, $RMSE$ of leave-one-(A,W)-out cross validation, and the information criteria AIC, WAIC and BIC of the models. For the information criteria, the smaller the values, the more accurate the model prediction.

#5) performs better than Finger-Fitts-W-σ_a (model #3). Compared to Fitts' law - W_e (model #4), Fitts' law - W (model #1) leads to better model fitness.

5.2 Model Fitness on Bi, Li, and Zhai's Circular Pointing Data Set [6]

We also evaluated the fitness of all six models in the touch input data reported in Bi, Li, and Zhai's paper [6] as in tasks with horizontal bars. As shown in Table 8, models #3 and #6, which use a free parameter c to represent the finger touch ambiguity result in the best performance. Similar to the tasks with horizontal bars, the comparison between using W_e vs. using W generates mixed results: there are situations where models with W are better (e.g., models #1 vs. #4), and also situations where models with W_e are better (e.g., models #2 vs. #5, and models #3 vs. #6).

6 GENERAL DISCUSSION

Representing finger ambiguity with a free parameter c leads to stronger model fitness than using a pre-defined σ_a. The results from our user studies and investigation on Bi, Li, and Zhai's [6] both show that representing finger ambiguity with a free parameter c leads to stronger model fitness than using a pre-defined σ_a. The models with parameter c all have stronger model fitness than their

Model		*ID*	R^2	RMSE	AIC	WAIC	BIC	Parameters
Nominal Width W	#1	$\log_2(\frac{A}{W}+1)$	0.849	0.015 [0.000]	-23.23	-27.02	-23.86	$a=0.29, b=0.05$
	#2	$\log_2(\frac{A}{\sqrt{W^2-2\pi e\sigma_a^2}}+1)$	0.789	0.018 [0.001]	-21.52	-25.38	-22.15	$a=0.32, b=0.04, \sigma_a^2=2.25$
	#3	$\log_2(\frac{A}{\sqrt{W^2-c^2}}+1)$	0.849	0.015 [0.000]	-23.23	-27.02	-23.86	$a=0.29, b=0.05, c^2=0$
Effective Width W_e	#4	$\log_2(\frac{A}{\sqrt{2\pi e}\sigma}+1)$	0.791	0.017 [0.003]	-21.28	-25.02	-21.91	$a=0.15, b=0.14$
	#5	$\log_2(\frac{A}{\sqrt{W_e^2-2\pi e\sigma_a^2}}+1)$	0.949	0.008 [0.001]	-29.64	-33.51	-30.27	$a=0.16, b=0.10, \sigma_a^2=2.25$
	#6	$\log_2(\frac{A}{\sqrt{W_e^2-c^2}}+1)$	0.968	0.006 [0.002]	-32.56	-36.39	-33.19	$a=0.13, b=0.12, c^2=34.39$

Table 8: Data of the circular target experiment in FFitts law [6]: Parameters, R^2, $RMSE$ of leave-one-(A,W)-out cross validation, and the information criteria AIC, WAIC and BIC of the models.

counterparts with predefined σ_a, after controlling for overfitting (e.g., cross-one-$(A,/w)$-out cross-validation.)

The implication of representing finger ambiguity with free parameter c is that σ_a may differ across task contexts, and treating it as a free parameter would provide more flexibility in modeling. It also addresses a potential problem which is that it leaves the equation undefined if $W < \sqrt{2\pi e}\sigma_a$. However, this formulation induces the drawback that it introduces an extra free parameter c to the model.

Nominal width vs. effective width. Our evaluation shows mixed results of nominal width vs. effective width. In the circular target selection task in our user study, model #5 (with W_e) outperformed its counterpart of using W (model #2). However, for other model candidates, using nominal target widths outperformed their counterparts of using effective target width. The evaluation on Bi, Li, and Zhai's [6] shows models #5, and #6 which used effective width outperformed their counterparts of using nominal target width. However, model #1 which used nominal target width outperformed model #4 which used effective target width.

These mixed results show that both nominal and effective target widths are valid representations of target widths in Finger-Fitts law. There is no clear winner of these two approaches. Although the original Finger-Fitts law [6] used the effective target width, it is still valid to use nominal target width to model touch pointing behaviors. Replacing W_e with W also has a physical meaning. The W_e represents the observed variance in the endpoint distribution, which is the actual endpoint variability a user exhibits. In contrast, W^2 represents the endpoint variability allowance specified by the task parameter, which is the variability allowance a user is supposed to consume.

7 CONCLUSIONS

We investigate two issues related to modeling touch pointing tasks with Finger-Fitts law: (1) Should nominal or effective target width be used?, and (2) should the ambiguity of finger touch be represented by a pre-defined σ_a or by a free parameter c estimated from user data?

Our investigation shows that both nominal and effective widths could be used to model touch pointing. There is no clear winner between them: there are cases where using nominal width is better than effective width while there are cases that showed the opposite results.

Regarding the representation of the finger ambiguity, models using free parameter c lead to stronger model fitness than using the predefined σ_a to interpret the finger ambiguity after controlling for the overfitting (e.g., performing leave-one-(A,W)-out cross-validation). With the free parameter c, the model can more accurately reflect the uncertainty introduced by the finger touch in the study. In sum, our investigation deepens the understanding of how to use Finger-Fitts law to model touch pointing.

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