

# Brief Announcement: Lower and Upper Bounds for Attacks on Authentication Protocols

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Many authentication protocols are intended to work correctly in the presence of an adversary that can intercept messages, perform an unbounded number of encryptions and other operations while fabricating messages, and prompt honest principals to engage in an unbounded number of concurrent (*i.e.*, interleaved) runs of the protocol. The amount of local state maintained by a single run of an authentication protocol is bounded. This suggests the existence of upper bounds on the resources needed to attack a protocol. Such bounds provide a rigorous basis for automated verification. We sketch a Language for Authentication Protocols (LAP), based on [WL93], and establish an exponential lower bound on the worst-case number of concurrent runs needed in a successful attack on a LAP protocol. Details appear in [Sto98a]. An exponential upper bound would be too large to enable automated verification. This shows the need to impose additional restrictions on the class of protocols, as done in [Sto98b], which gives a polynomial upper bound.

The relevant kinds of statements (slightly simplified) in LAP are: `NewValue( $v$ )`, which generates a unique value (*e.g.*, a nonce or session key) and binds variable  $v$  to it; `Send( $x, t$ )`, which sends a message  $t$  to  $x$ ; and `Receive( $pat$ )`, which receives a message  $m$  and binds the unbound variables in pattern  $pat$  to the corresponding subterms of  $m$ . The `Receive` statement attempts pattern-matching between a candidate message  $m$  and the pattern. A pattern can express that the message should be a ciphertext produced with a given key. If  $m$  is encrypted with the given key (if any) and there exist bindings for the unbound variables of  $pat$  such that  $pat$  with those bindings equals  $m$ , then the `Receive` statement executes and establishes those bindings. The `Receive` statement blocks until this condition is satisfied.

A *local protocol* is a finite sequence of statements satisfying some well-formedness requirements. A *protocol* is, roughly, a set of local protocols, one for each role (or participant) in the protocol. A *secrecy* requirement asserts that certain values are not revealed to the adversary.

**Theorem 1.** There exists a family of LAP protocols  $\Pi^\ell$  and a secrecy property  $\phi$  such that the minimum number of concurrent runs in an execution of  $\Pi^\ell$  that violates  $\phi$  is  $\Omega((\ell/2-4)^{(\ell/2-4)})$ , where  $\ell$  is the maximum number of `Send` statements in a local protocol of  $\Pi^\ell$ .

**Proof sketch:** Protocol  $\Pi^\ell$  involves three local protocols:  $P_I$ ,  $P_R$ , and  $P_S$ . Intuitively, an execution of  $\Pi^\ell$  performs two depth-first traversals of a conceptual  $\ell$ -ary tree of height  $\ell$  before violating  $\phi$ . Each non-leaf node of the tree corresponds to a run of a local protocol. A run of  $P_I$  corresponds to the root. Runs of  $P_R$  correspond to non-root non-leaf nodes. Runs of  $P_S$  correspond to leaves. The protocol involves *two* depth-first traversals in order to force all the runs of  $P_R$  to be concurrent. Values generated by `NewValue` are used to ensure that each node in the tree corresponds to a distinct run. By design, the secrecy requirement  $\phi$  is violated iff  $P_I$  runs to completion, and  $P_I$  can do this only in executions containing  $\Omega((\ell/2-4)^{(\ell/2-4)})$  concurrent runs of  $P_R$ . ■

## References

- [Sto98a] Scott D. Stoller. Justifying finite resources for adversaries in automated analysis of authentication protocols. Tech Report 506, C. S. Dept., Indiana U., March 1998 (revised Feb. 1999).
- [Sto98b] Scott D. Stoller. Reductions for automated analysis of authentication protocols. Tech Report 520, C.S. Dept., Indiana U., Dec. 1998.
- [WL93] Thomas Y. C. Woo and Simon S. Lam. A semantic model for authentication protocols. In *Proc. 14th IEEE Symposium on Research in Security and Privacy*, pages 178–194, 1993.