

Table of Contents

Preface	ix
■ About <i>Combinatorica</i> ■ What's Between the Covers ■ Why <i>Mathematica</i> ? ■ Acknowledgments ■ Caveat ■ Dedication	
Chapter 1. <i>Combinatorica</i>: An Explorer's Guide	
1.1 Combinatorial Objects: Permutations, Subsets, Partitions	3
■ Permutations and Subsets ■ Partitions, Compositions, and Young Tableaux	
1.2 Graph Theory and Algorithms	10
■ Representing Graphs ■ Drawing Graphs ■ Generating Graphs ■ Properties of Graphs ■ Algorithmic Graph Theory	
1.3 <i>Combinatorica</i> Conversion Guide	32
■ The Main Differences ■ Functions Whose Usage Has Changed	
1.4 An Overview of <i>Mathematica</i>	41
■ The Structure of Functions ■ Mathematical Operations ■ List Manipulation ■ Iteration ■ Ten Little n -Sums ■ Conditionals ■ Compiling <i>Mathematica</i> Code	
Chapter 2. Permutations and Combinations	
2.1 Generating Permutations	55
■ Lexicographically Ordered Permutations ■ Ranking and Unranking Permutations ■ Random Permutations ■ Minimum Change Permutations	
2.2 Inversions and Inversion Vectors	69
■ Inversion Vectors ■ Counting Inversions ■ The Index of a Permutation ■ Runs and Eulerian Numbers	
2.3 Combinations	76
■ Subsets via Binary Representation ■ Gray Codes ■ Lexicographically Ordered Subsets ■ Generating k -Subsets ■ Strings	
2.4 Exercises	89
■ Thought Exercises ■ Programming Exercises ■ Experimental Exercises	
Chapter 3. Algebraic Combinatorics	
3.1 The Cycle Structure of Permutations	93
■ Odd and Even Permutations ■ Types of Permutations ■ Hiding Cycles ■ Counting Cycles	
3.2 Special Classes of Permutations	104
■ Involutions ■ Derangements	
3.3 Pólya Theory	109
■ Permutation Groups ■ Group Action ■ Equivalence Classes and Orbits ■ Cycle Index of Permutation Groups ■ Applying Pólya's Theorem	
3.4 Exercises	131
■ Thought Exercises ■ Programming Exercises ■ Experimental Exercises	

Chapter 4. Partitions, Compositions, and Young Tableaux

4.1 Integer Partitions	135
▪ Generating Partitions ▪ Generating Functions and Partitions ▪ Ferrers Diagrams ▪ Random Partitions	
4.2 Compositions	146
▪ Random Compositions ▪ Generating Compositions	
4.3 Set Partitions	149
▪ Generating Set Partitions ▪ Stirling and Bell Numbers ▪ Ranking, Unranking, and Random Set Partitions ▪ Set Partitions and Restricted Growth Functions	
4.4 Young Tableaux	162
▪ Insertion and Deletion ▪ Permutations and Pairs of Tableaux ▪ Generating Young Tableaux ▪ Counting Tableaux by Shape ▪ Random Tableaux ▪ Longest Increasing Subsequences	
4.5 Exercises	173
▪ Thought Exercises ▪ Programming Exercises ▪ Experimental Exercises	

Chapter 5. Graph Representation

5.1 Data Structures for Graphs	179
▪ The Internal Representation ▪ Edge Lists ▪ Adjacency Lists ▪ Adjacency Matrices ▪ Incidence Matrices	
5.2 Modifying Graphs	192
▪ Additions, Deletions, and Changes ▪ Setting Graph Options	
5.3 Classifying Graphs	198
5.4 Displaying Graphs	200
▪ The Vertex and Edge Options ▪ Inherited Options ▪ A Hierarchy of Options ▪ Highlighting and Animation	
5.5 Basic Graph Embeddings	213
▪ Circular Embeddings ▪ Ranked Embeddings ▪ Radial Embeddings ▪ Rooted Embeddings	
5.6 Improving Embeddings	219
▪ Translating, Dilating, and Rotating Graphs ▪ Shaking Graphs ▪ Spring Embeddings	
5.7 Storing and Editing Graphs	224
5.8 Exercises	226
▪ Thought Exercises ▪ Programming Exercises ▪ Experimental Exercises	

Chapter 6. Generating Graphs

6.1 Building Graphs from Other Graphs	231
▪ Contracting Vertices ▪ Inducing and Permuting Subgraphs ▪ Unions and Intersections ▪ Sums and Differences ▪ Joins of Graphs ▪ Products of Graphs ▪ Line Graphs	
6.2 Regular Structures	244
▪ Complete Graphs ▪ Circulant Graphs ▪ Complete k -Partite Graphs ▪ Cycles, Stars, and Wheels ▪ Grid Graphs ▪ Interconnection Networks	

6.3	Trees	258
	■ Labeled Trees ■ Complete Trees	
6.4	Random Graphs	262
	■ Constructing Random Graphs ■ Realizing Degree Sequences	
6.5	Relations and Functional Graphs	269
	■ Graphs from Relations ■ Functional Graphs	
6.6	Exercises	273
	■ Thought Exercises ■ Programming Exercises ■ Experimental Exercises	
Chapter 7. Properties of Graphs		
7.1	Graph Traversals	277
	■ Breadth-First Search ■ Depth-First Search	
7.2	Connectivity	283
	■ Connected Components ■ Strong and Weak Connectivity ■ Orienting Graphs ■ Biconnected Components ■ k -Connectivity ■ Harary Graphs	
7.3	Cycles in Graphs	294
	■ Acyclic Graphs ■ Girth ■ Eulerian Cycles ■ Hamiltonian Cycles and Paths ■ Traveling Salesman Tours	
7.4	Graph Coloring	306
	■ Bipartite Graphs ■ Chromatic Polynomials ■ Finding a Vertex Coloring ■ Edge Colorings	
7.5	Cliques, Vertex Covers, and Independent Sets	316
	■ Maximum Clique ■ Minimum Vertex Cover ■ Maximum Independent Set	
7.6	Exercises	319
	■ Thought Exercises ■ Programming Exercises ■ Experimental Exercises	
Chapter 8. Algorithmic Graph Theory		
8.1	Shortest Paths	323
	■ Single-Source Shortest Paths ■ All-Pairs Shortest Paths ■ Applications of All-Pairs Shortest Paths ■ Number of Paths	
8.2	Minimum Spanning Trees	335
	■ Union-Find ■ Kruskal's Algorithm ■ Counting Spanning Trees	
8.3	Network Flow	340
8.4	Matching	343
	■ Maximal Matching ■ Bipartite Matching ■ Weighted Bipartite Matching and Vertex Cover ■ Stable Marriages	
8.5	Partial Orders	352
	■ Topological Sorting ■ Transitive Closure and Reduction ■ Hasse Diagrams ■ Dilworth's Theorem	
8.6	Graph Isomorphism	363
	■ Finding Isomorphisms ■ Tree Isomorphism ■ Self-Complementary Graphs	

8.7 Planar Graphs	370
▪ Testing Planarity	
8.8 Exercises	372
▪ Thought Exercises ▪ Programming Exercises ▪ Experimental Exercises	
Appendix	375
Reference Guide	376
Bibliography	447
Index	459