Lecture 7: Bounds on Options Prices

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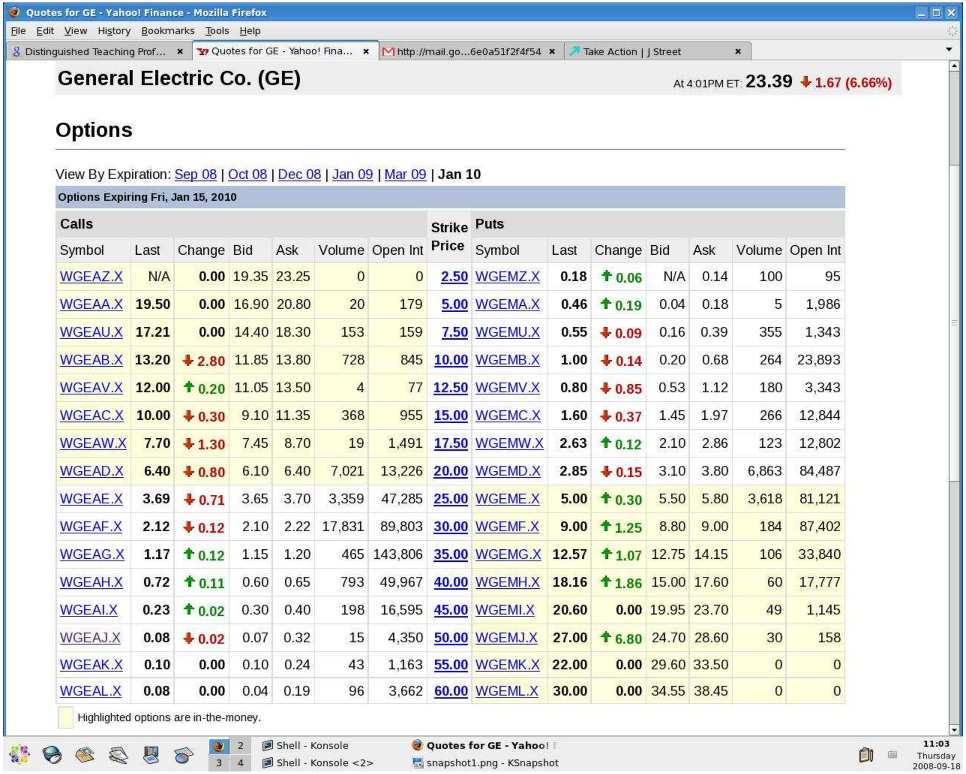
Option Price Quotes



General Electric Co. (GE)

At 4:01PM ET: **23.39** ↓ 1.67 (6.66%)

Options

View By Expiration: Sep 08 | Oct 08 | Dec 08 | Jan 09 | Mar 09 | **Jan 10**

Options Expiring Fri, Jan 15, 2010

Calls							Strike Price	Puts						
Symbol	Last	Change	Bid	Ask	Volume	Open Int		Symbol	Last	Change	Bid	Ask	Volume	Open Int
WGEAZ.X	N/A	0.00	19.35	23.25	0	0	2.50	WGEMZ.X	0.18	↑ 0.06	N/A	0.14	100	95
WGEAA.X	19.50	0.00	16.90	20.80	20	179	5.00	WGEMA.X	0.46	↑ 0.19	0.04	0.18	5	1,986
WGEAU.X	17.21	0.00	14.40	18.30	153	159	7.50	WGEMU.X	0.55	↓ 0.09	0.16	0.39	355	1,343
WGEAB.X	13.20	↓ 2.80	11.85	13.80	728	845	10.00	WGEMB.X	1.00	↓ 0.14	0.20	0.68	264	23,893
WGEAV.X	12.00	↑ 0.20	11.05	13.50	4	77	12.50	WGEMV.X	0.80	↓ 0.85	0.53	1.12	180	3,343
WGEAC.X	10.00	↓ 0.30	9.10	11.35	368	955	15.00	WGEMC.X	1.60	↓ 0.37	1.45	1.97	266	12,844
WGEAW.X	7.70	↓ 1.30	7.45	8.70	19	1,491	17.50	WGEMW.X	2.63	↑ 0.12	2.10	2.86	123	12,802
WGEAD.X	6.40	↓ 0.80	6.10	6.40	7,021	13,226	20.00	WGEMD.X	2.85	↓ 0.15	3.10	3.80	6,863	84,487
WGEAE.X	3.69	↓ 0.71	3.65	3.70	3,359	47,285	25.00	WGEME.X	5.00	↑ 0.30	5.50	5.80	3,618	81,121
WGEAF.X	2.12	↓ 0.12	2.10	2.22	17,831	89,803	30.00	WGEMF.X	9.00	↑ 1.25	8.80	9.00	184	87,402
WGEAG.X	1.17	↑ 0.12	1.15	1.20	465	143,806	35.00	WGEMG.X	12.57	↑ 1.07	12.75	14.15	106	33,840
WGEAH.X	0.72	↑ 0.11	0.60	0.65	793	49,967	40.00	WGEMH.X	18.16	↑ 1.86	15.00	17.60	60	17,777
WGEAI.X	0.23	↑ 0.02	0.30	0.40	198	16,595	45.00	WGEMI.X	20.60	0.00	19.95	23.70	49	1,145
WGEAJ.X	0.08	↓ 0.02	0.07	0.32	15	4,350	50.00	WGEMJ.X	27.00	↑ 6.80	24.70	28.60	30	158
WGEAK.X	0.10	0.00	0.10	0.24	43	1,163	55.00	WGEMK.X	22.00	0.00	29.60	33.50	0	0
WGEAL.X	0.08	0.00	0.04	0.19	96	3,662	60.00	WGEML.X	30.00	0.00	34.55	38.45	0	0

Highlighted options are in-the-money.

Reading the Quotes

Bid and Ask are what the market maker is willing to buy and sell at now. The Last sales price is usually between them.
Volume is the number of transactions in the option that day.
Open interest counts the number of contracts that have been issued to date which have not yet been offset/closed out.
Option prices decrease (increase) with respect to strike price for calls (puts).

Factors Affecting Option Prices

1. The current stock price S_0.
2. The option strike price K.
3. The time to expiration T.
4. The volatility of the stock price σ.
5. The risk-free interest rate r.
6. The value of dividends expected during the life of the option.

Impact on Option Prices

Suppose each relevant factor increased in isolation. How does that impact the value of an *American* option?

variable	call	put
stock price	+	–
strike price	–	+
expiration date	+	+
volatility	+	+
risk-free rate	+	–
dividends	–	+

Impact of Rate Changes?

The confusing one is the impact of increasing the risk-free rate. If interest increases *independently* of stock price, the present cost (value) of my call purchase (put sale) in the future decreases – good for call, bad for put.

In practice, when interest rates go up usually stock prices down, so a rate increase is bad for a call and good for a put.

The only change with European option prices are that there is no certain relationship with expiration date – for example, increasing T might cover an extra dividend payment.

Upper/Lower Bounds on European Option Prices

Let p and c be the value of European put and call options. We assume no stocks pay dividends.

Since an option to buy a share of stock at any positive price cannot be worth more than the value of the share, so $c \leq S_0$.

Since an option to sell a share of stock at the strike price cannot be worth more than the strike price, so $p \leq K$.

Since the European option is worth at most K at maturity, it cannot be worth more than the discounted value of this amount, so $p \leq Ke^{-rT}$.

Better Bounds by Arbitrage Arguments

Portfolio A consists of a European call option for one share at K plus Ke^{-rT} worth of cash to execute it at time T.

At time T, this is worth S_T if $S_T > K$, or K if $K > S_T$ and we don't buy the option, or in other words $\max(S_T, K)$.

Portfolio B consists of one share, and will be worth S_T at time T.

That A is worth more than B at time T, implies A is worth more than B now since otherwise an arbitrage possibility exists.

Thus $c + Ke^{-rT} \geq S_0$. Since no option can have negative value,

$$c \geq \max(S_0 - Ke^{-rT}, 0)$$

Lower Bound on Puts

A similar argument for puts shows that

$$p \geq \max(Ke^{-rT} - S_0, 0)$$

Portfolio A consists of a European put option for one share at K at time T plus one share.

At time T, this is worth S_T if $S_T > K$, or K if $K > S_T$ and we don't buy the option, or in other words $\max(S_T, K)$.

Portfolio B consists of Ke^{-rT} worth of cash

That A is worth more than B at time T, implies A is worth more than B now since otherwise an arbitrage possibility exists.

Thus $p + S_0 \geq Ke^{-rT}$.

Put-Call Parity

In fact, there is a tight *put-call parity* relationship between the value of a European put and call. Consider the following portfolios:

Portfolio A consists of a European call option for one share plus Ke^{-rT} worth of cash to execute it at time T, worth $\max(S_T, K)$ at time T by our previous arguments.

Portfolio C consists of a European put option plus one share of stock, also worth $\max(S_T, K)$ at time T.

Since they have equal value then they must have equal value now, so

$$c + Ke^{-rT} = p + S_0$$

What About American Options?

Similar but weaker bounds hold for American options

$$S_0 - K \leq C - P \leq S_0 - Ke^{rT}$$

The gap here is associated with the observation that one can execute an American option immediately, but the holder of a European option cannot cash in until time T, when it will be discounted.

Empirical results basically support the bound on over reasonable time scales to the resolution of transaction costs.

Another View of Put-Call Parity

Suppose I buy a call option and sell a put option, both at strike price K and expiration time T.

The sum of these two payoff functions is linear, just like owning a share! The curve is shifted so profit is zero at K, however.

The payoff at T is $S_T - K$. Discounting this to the present gives

$$c - p = S_0 - Ke^{-rT}$$

or

$$c + Ke^{-rT} = p + S_0$$