Lecture 25:
Money Management

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Money Management Techniques

The trading strategies we have studied point towards possible investment opportunities, but usually do not tell us how much we should invest in each.

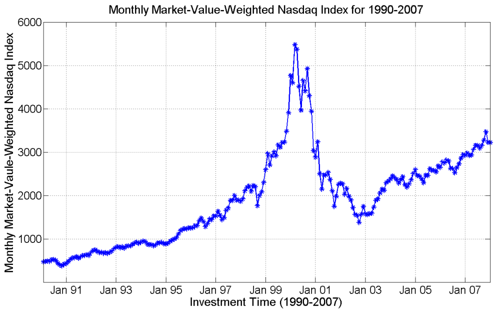
Money management issues are implicit in discussions of (1) risk vs. return, (2) portfolio optimization, and (3) market saturation.

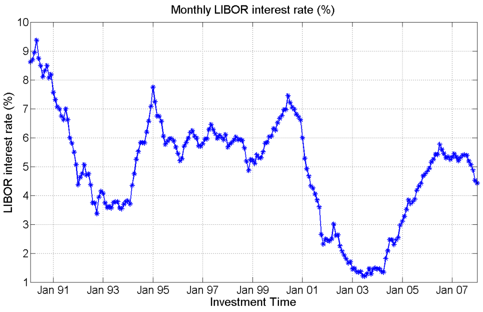
Properly allocating capital to investment opportunities can be as or more important than finding them in the first place.

Leveraged Trading Strategies

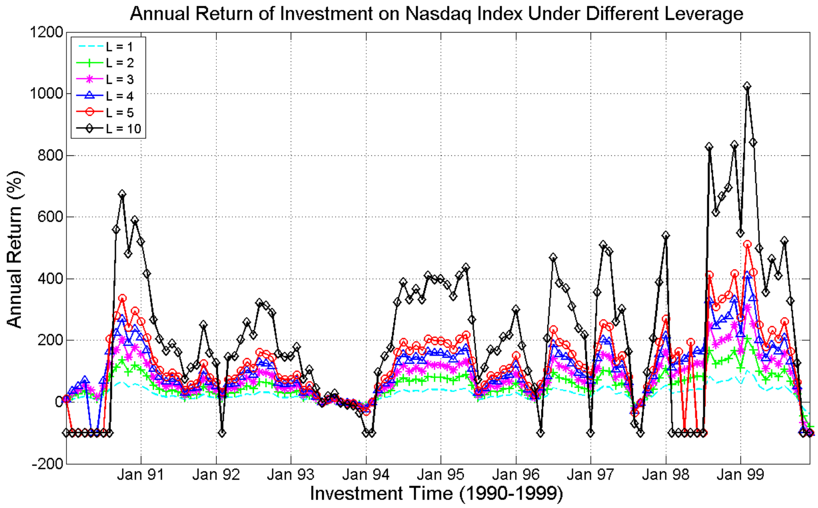
That investment strategies must modulate risk and return in money management is apparent when studying the impact of leverage.

Consider a strategy which borrows money at the LIBOR rate for one year, and invests it in a stock market index (here Nasdaq).

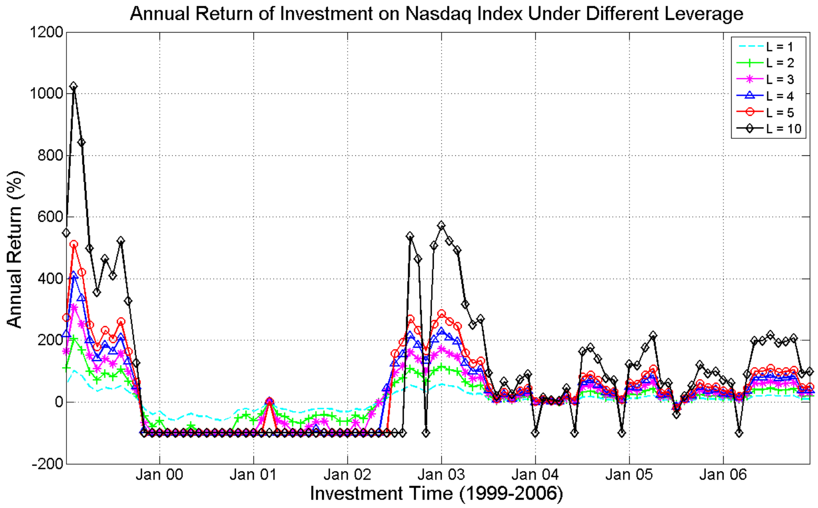




Nasdaq Leveraged Trading 1990-2000



Nasdaq Leveraged Trading 2000-2007



The probability of going bust is as meaningful notion of risk as volatility…

Managing Money When You Have an Edge

You play a sequence of games, where:

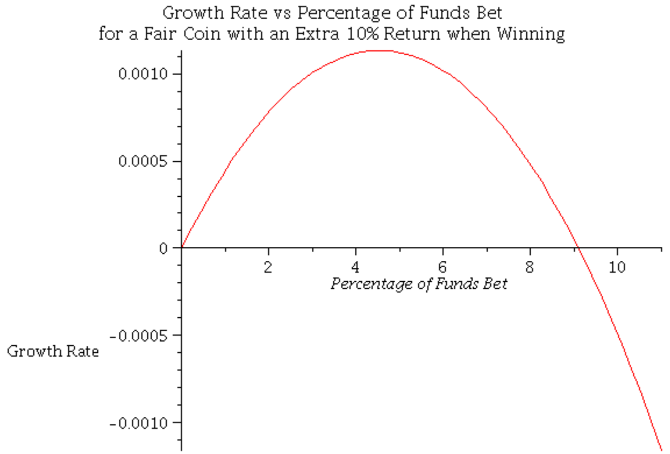
- If you win, you get W dollars for each dollar bet
- If you lose, you lose your bet
- For each game, the probability of winning is p and losing is $q = 1 - p$
- You bet some fixed percentage f of your bankroll B each game, for you have $(1 - f)B$ if you lose and $(W - 1)fB + B$ if you win.

The right value of f is called the Kelly Criterion.

Rigged in our Favor?

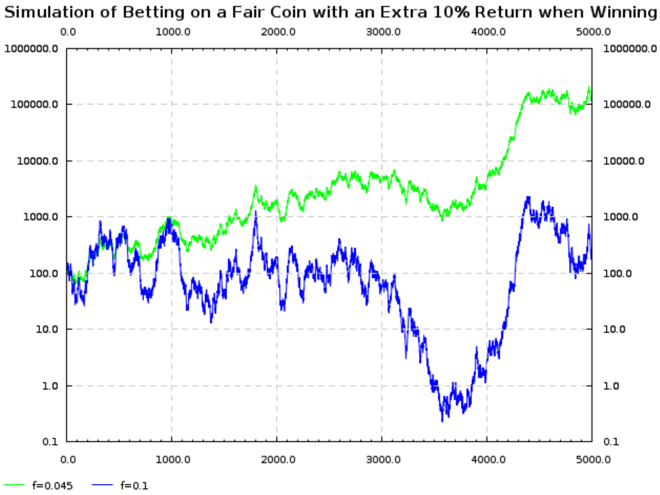
Suppose we bet $1 on a fair coin, but one which pays $2.10 if it comes up heads?

How much of our bankroll should be bet each time?



Bet too much and we lose, even with the odds in our favor!

After 5000 Coin Tosses



Ten straight tails leaves only 1/3 the bankroll at 10%, but almost 2/3 at Kelly (4.5%)

The Kelly Criterion: History

Developed by John Kelly, a physicist at Bell Labs in a 1956 paper "A New Interpretation of Information Rate" published in the Bell System Technical Journal.

He used Information Theory to show how a gambler with inside information should bet.

Thorpe used system to compute optimum bets for blackjack in his 1962 book "Beat the Dealer: A Winning Strategy for the Game of Twenty One", and later as hedge fund manager on Wall Street.

Not So Easy

Suppose you play a (more generous) sequence of games of flipping a fair coin.

On heads, you win two dollars for each dollar bet, or a total of three dollars per dollar bet.

On tails, you lose your one dollar bet.

The odds are in your favor, but if you bet all your money on each game, you will eventually lose a game and be bankrupt

If you bet too little, you will not make as much money as you could have.

Bet Everything?

Suppose your bankroll starts at one dollar and you bet everything (f = 1) each round.

After 10 rounds, there is one chance in 1024 that you will have 59,049 dollars, and 1023 chances in 1024 that you will have 0 dollars

Your expected wealth (arithmetic mean) is 57.67 dollars, but your median wealth is 0 dollars!

Would you bet this way, which maximizes the arithmetic mean of your wealth?

Calculating the Kelly Fraction

You play a sequence of $n = w + l$ games. Each game, you either win W for each dollar bet with probability p or lose your bet with probability $q = 1 - p$.
If after n games, you have won w and lost l games, your total bankroll is

$$B_n = (1 + fW)^w * (1 - f)^l B$$

Dividing by B gives the gain in your bankroll:

$$Gain_n = (1 + fW)^w * (1 - f)^l$$

Computing Gain

We want to find the value of f that maximizes the geometric mean of the gain or (equivallently) the arithmetic mean of the log of the gain

The geometric mean G, is the limit as $n \rightarrow \infty$ of the $n$throot of the gain

$$G = \lim_{n \rightarrow \infty} ((1 + fW)^{w/n} * (1 - f)^{l/n})$$

which is

$$G = (1 + fW)^p * (1 - f)^q$$

The value of your bankroll after n games is

$$B_n = G^n * B$$

so G is the gain per game.

Maximizing the Arithmetic Mean of the Log of the Gain

To find the f that maximizes G, we take the derivative of

$$G = (1 + fW)^p * (1 - f)^q$$

with respect to f and set it equal to 0:

$$(1 + fW)^p * (-q(1 - f)^{q-1}) + Wp(1 + fW)^{p-1} * (1 - f)^q = 0$$

Solving for f gives

$$f = (pW - q)/W = p - q/W$$

This is the Kelly Criterion for this problem

Theorem: The log of the geometric mean of a random variable equals the arithmetic mean of the log of that variable

Edge over Odds

The Kelly criterion $f = (pW - q)/W$ is sometimes written as

$$f = edge/odds$$

The *odds* are how much you will win if you win, e.g. the tote-board odds at a racetrack.

Edge is how much you expect to win, e.g. p is your inside knowledge of which horse will win.

If $pW - q = 0$, you have no advantage and shouldn't bet anything, so $f = 0$.

If $q = 0$, then $f = 1$ and you should borrow to bet all you possibly could.

Biased Coin Flipping

For the generous coin flipping example ($W = 2$, $p = 1/2$), $f = 0.5 - .5/2 = 0.25$, where $G = 1.0607$.
After 10 rounds starting from $B = 1$, this expected (mean) final wealth = 3.25 and median final wealth = 1.80.
By comparison, if we bet all the money (f = 1) we would have expected (mean) final wealth = 57.67 and Median wealth = 0.

Generalizations

The analysis can be generalized to the case we win W or lose L each round, which is more like investing.
Using the same math, the value of f that maximizes G is

$$f = (pW - qL)/WL = p/L - q/W$$

As an example, consider $p = 1/2$, $W = 1$, and $L = 0.5$. Then $f = 0.5$ and $G = 1.0607$.
It can also be generalized to n possible outcomes x_i, each with probability p_i.

Shannon's Example

Claude Shannon (of Information Theory fame) proposed an approach to profiting from random variations in stock prices based on the preceding example.

Consider the "game" as the value of the stock at the end of a single step/day.

If you "win" the stock doubles in value, since $W = 1$.

If you "lose" the stock halves in value since $L = 1/2$.

If the stock just oscillates around its initial value, Shannon would be making $(1.0607)^n$ gain in n days!

Properties of the Kelly Criterion

It maximizes (1) the geometric mean of wealth, and (2) the arithmetic mean of the log of wealth.

In the long term (an infinite sequence), with probability 1 it maximizes the final value of the wealth compared with any other strategy, as well as the median of the wealth.

Half the distribution of the “final” wealth is above the median and half below it.

Finally, it minimizes the expected time required to reach a specified goal for the wealth

Fluctuations using the Kelly Criterion

The Kelly Criterion f leads to a large amount of volatility in the bankroll.
For example, the probability of the bankroll dropping to $1/n$ of its initial value at some point in an infinite sequence is $1/n$.
Thus there is a 50% chance the bankroll will drop to 1/2 of its value at some time in an infinite sequence.

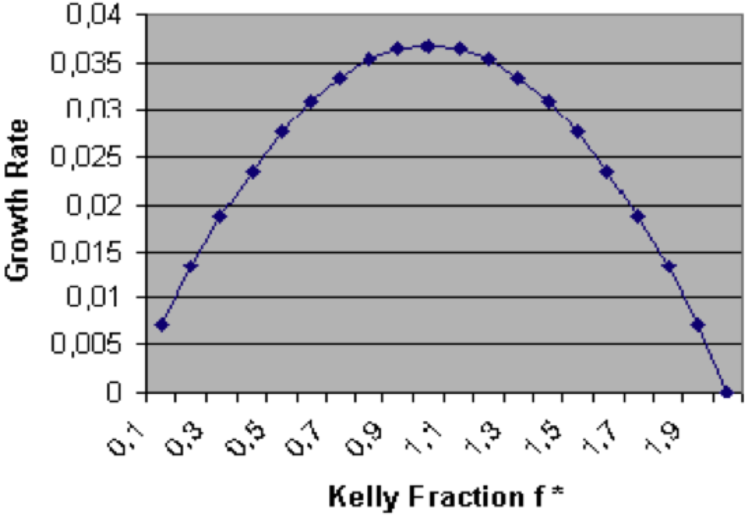
Varying the Kelly Criterion

Many people propose using a value of f equal to half Kelly or some other fraction of the Kelly Criterion to obtain less volatility with somewhat slower growth

Half Kelly produces about 75% of the growth rate of the full Kelly fraction.

An important reason to be conservative (e.g. half Kelly) is that people often overestimate their edge.

Growth Rate for Different Kelly Fractions



All the Possibilities in Four Games

Consider again the example where $p = 1/2$, $W = 1$, $L = 0.5$, and $B = 100$, limited to four games.

		Final Bankroll			
		½ Kelly	**Kelly**	**1½ Kelly**	**2 Kelly**
		f = .25	**f = .5**	**f = .75**	**f = 1.0**
4 wins 0 losses	(.06)	244	506	937	1600
3 wins 1 loss	(.25)	171	235	335	400
2 wins 2 losses	(.38)	120	127	120	100
1 win 3 losses	(.25)	84	63	43	25
0 wins 4 losses	(.06)	59	32	15	6
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Arithmetic Mean		128	160	194	281
Geometric Mean		105	106	105	100

Should You Use the Kelly Criterion?

- You are going to make only a relatively short sequence of bets compared to the infinite sequence used in the math.
- The properties of infinite sequences might not be an appropriate guide for a finite sequence of bets
- You might not be comfortable with the volatility. Do you really want to maximize the arithmetic mean of the log of your wealth (or the geometric mean of your wealth)?

What is the marginal utility of money?

References

- Poundstone, William, "Fortunes Formula: The Untold Story of the Scientific Betting System that Beat the Casinos and Wall Street," Hill and Wang, New York, NY, 2005
- Kelly, John L, Jr., A New Interpretation of Information Rate, Bell Systems Technical Journal, Vol. 35, pp. 917-926, 1956

Final Thoughts on Investing

There are several coherent theories of how prices are formed in the financial markets. None of them lead to clear strategies for you to beat the market.

However an understanding of these theories should help you avoid stupid investment strategies:

- the importance of diversification
- the relation of return to risk
- the apparently inherent unpredictability of efficient markets
- the strength of buy-and-hold vs. active strategies.

- the importance of long time horizons to minimize variance/risk.

None of this is grounds for discouragement. The stock market has been called “a casino rigged in favor of the investor”.

Skiena’s advice: – Stick your money in a broad index fund with low expenses, and forget about it.