The Capital Assets Pricing Model

The *Capital Assets Pricing Model* or CAPM relates the rate of return on an asset to the level of risk associated with the asset.

Risk is a notion associated with the *variance* or *volatility* of the investment.

The returns from a *risk-free* investment such as U.S. Treasury bills are guaranteed, but fairly low.

The expected returns from a more volatile investment such as stocks is higher, but the variance in actual returns is also higher.

The *Capital Assets Pricing Model* assumes investors are *risk-adverse*, that they will take riskier investments only if the expected returns are higher.

Types of Risk

The risk of a given portfolio can be partitioned into *systematic risk* and *specific risk*.

The *specific risk* or *non-systematic risk* is the risk associated with a given asset. For example, will the company's management perform worse than it should?

The *systematic risk* or *market risk* is the risk common to all securities. For example, will the Fed raise interest rates?

Note that the specific risk can be diversified away by buying a variety of different assets, since the specific risks will cancel each other out.

If several stocks have the same expected return, we are better off owning all of them than of them, because we remove specific risk.

The expected return is the same, but the variance is lower.

The volatility of an asset effects the value of an option for it, since more volatile assets are more likely to significantly increase or decrease value in a given time window.

Diversification and Portfolio Theory

The CAPM assumes that the risk-return profile of a portfolio can be optimized, where an optimal portfolio displays the lowest possible level of risk for its level of return.

Since adding a new asset to a portfolio further diversifies it, the optimal portfolio in theory contains all possible assets appropriately value-weighted.

The ratio of risky assets to risk-free assets determines the overall return of a portfolio.

There is an optimal portfolio for *any* desired rate of return (up to the highest of any security) because we can always invest some fraction of capital at the risk-free rate and the rest in an optimal portfolio at some higher rate.

Diversification in Practice

Such *diversification* explains why stock index funds perform better than holding individual stocks.

Much of the difference in the performance of different investment managers is due to variable outcomes in their specific risk more than investment skill.

In practice, about 15 well-selected stocks can provide much the same level of diversification as a market index.

To maximize diversification, we selecting securities whose performance is *uncorrelated* or (even better) *negatively correlated*.

Suppose stock A returns 20% in summer/fall and 0% in winter/spring and stock B returns 0% in summer/fall and 20% in winter/spring.

Each stock has a 10% return with high variance, but because they are perfectly negatively correlated buying half of each gives us 10% return with zero variance.

Asset Pricing Under the CAPM

The required rate of return for a particular asset in a market depends on its sensitivity to the movement of the market portfolio (i.e. the broader market).

This sensitivity is known as the asset β and reflects asset specific risk. For the market portfolio $\beta = 1$ by definition.

More sensitive (risky) stocks will have a higher beta, less sensitive stocks will have lower betas.

According to the CAPM the required rate of return for a stock is

$$r_s = \beta(r_m - r_f) + r_f$$

where r_s is the required rate of return on a stock, r_m is the return rate of the market portfolio, r_f is the risk-free interest rate, and β is the beta of the stock.

This says that the relative return of a given stock is a completely a function of its volatility!

There is some tension between CAPM and arbitrage theory in how assets are priced, but we will accept both.

This model is consistent with *random walk models* that predict prices by repeatedly taking random steps of a given size/variance.

Does CAPM adequately explain the variation in stock returns? As with all simple models of complex phenomena, the answer is not completely.

However, that it remains the most widely used method for calculating the cost of capital means it is worth our respect.

The Efficient Market Hypothesis

The *Efficient Market Hypothesis* states that the price of a financial asset reflects all available public information available, and responds only to unexpected news.

If so, prices are optimal estimates of investment value at all times.

If so, it is impossible for investors to predict whether the price will move up or down.

The distinction between public and non-public information explains why there are rules about *insider trading*.

There are a variety of slightly different formulations of the Efficient Market Hypothesis. For example, suppose that prices are predictable but the function is too hard to compute efficiently. . .

Does the EMH explain the behavior of stock prices? As with all simple models of complex phenomena, the answer is not completely.

However, that it remains debated (although not completely believed) means it is worth our respect.

Expected Future Spot Price

Is the futures price of an asset the expected future spot price?

By Keynes and Hicks, this is a function of the behavior of the speculators on the asset.

Another explanation concerns the relative *risk* or *variance* in the price of the asset.

According to the capital asset pricing model (CAPM) investors demand more return to take on more risk.

Consider a speculator who thinks the spot price will go above the futures price at time t. They should take a long futures position, while simultaneously investing enough money at the risk-free rate to buy the asset at the delivery date.

Long or Short?

Let r be the risk-free rate and k be the expected return rate on the given asset.

The present value of this investment is the discounted expected value of the asset sale at time t minus the amount invested, i.e.

$$E(S_t)e^{-kt} - F_0e^{-rt} = 0$$

since by arbitrage assumptions no investment should have a positive net present value.

Thus

$$E(S_t) = F_0e^{-(r-k)t}$$

If $F_0 = E(S_t)$, then the futures price changes only when the market changes views of $E(S_t)$.

If $F_0 < E(S_t)$ (i.e. $r > k$), then one can make profit by going long on F_0.

If $F_0 > E(S_t)$ (i.e. $r < k$), then one can make profit by going short on F_0.

Empirical evidence suggests there is no systematic bias, or perhaps a weak bias in favor of the long position.