

Convergence of Future and Spot Prices

We expect that the spot price of an asset converges to that of the futures price as the delivery date of the contract approaches – otherwise an arbitrage opportunity exists.

If the futures price stays above the spot price, we can buy the asset now and short a futures contract (i.e. agree to sell the asset later at the future price). Then we deliver and clear a profit.

If the futures price stays below the spot price, anyone who wants the asset should go long on a futures contract and accept delivery instead of paying the spot price.

This convergence means the spot price may go up (down), the futures price may go down (up), or both.

Expected Future Prices

The gap between the spot and futures prices may contain information about the *expected future price* of the asset.

Keynes and Hicks theorized that the expected future price depends upon the behavior of hedgers. Hedgers seek to reduce risk, and are in principle willing to pay for this.

Speculators will only be in the game if they can expect profits.

Thus if hedgers are going short and speculators are going long, the expected future price should be above the future contract price.

Types of Buy/Sell Orders

Brokers can typically perform the following buy/sell orders for exchange traded assets:

- *Market orders* request the trade happen immediately at the best current price.
- *Limit orders* demand a given or better price at which to buy or sell the asset. Nothing happens unless a matching buyer or seller is found.
- *Stop or stop-loss order* becomes a market order when a given price is reached by the market on the downside. This enables an investor to minimize their losses in a market reversal, but does not guarantee them the given price.
- *Market-if-Touched order* (MIT) becomes a market order when a given price is reached by the market on the upside. This enables an investor to take profits when they are available, but does not guarantee them the given price.

The volume and distribution of stop and limit orders in principle contains information about future price movements.

Theory argues against making such orders as giving away an option for no payoff, however, such orders are useful particularly for modest-sized investments.

Arbitrage Assumptions

In our analysis of arbitrage strategies, we will make the following assumptions:

- There are no transaction costs for trading.
- There are no tax and/or accounting and/or storage cost considerations on trading.
- All participants can borrow and lend money at the same risk-free rate of interest.
- Arbitrage opportunities are taken advantage of as they occur.

These assumptions greatly simplify the analysis, but are do not completely reflect the messy truths of the market.

Implementing Short Sales

Many arbitrage arguments assume the ability to short sell an asset, i.e. sell the asset now without owning it.

Unfortunately, such contracts are not available for all assets.

However, if the forward price is too low, anyone who owns the asset should sell the asset, invest the proceeds at the risk-free rate, and buy a forward contract to buy it back later at the fixed price.

Such arbitrage arguments also work if you can rent/borrow the asset for the desired period from someone who already owns it.

Pricing Forward Contracts

By arbitrage arguments, the T year forward price F_0 for an asset S_0 which provides no income is

$$F_0 = S_0 e^{rT}$$

where r is the risk-free interest rate.

If the forward price is higher than this, buy the asset now, short the forward contract, and deliver the asset then.

If the forward price is less than this, short the asset, go long on the forward contract, and deliver it then.

Forward pricing becomes somewhat less when the assets pay interest at an annual rate of q :

$$F_0 = S_0 e^{(r-q)T}$$

because the party who owns the asset gets to keep the interest.

The value of stock index futures (e.g. S&P 500) can be so computed given the expected dividend rate q paid by stocks in the index, otherwise there exists an arbitrage opportunity in buying/selling the component stocks.

If cash with *present* value I is returned to the asset holder during the time period, the value of the forward contract is

$$F_0 = (S_0 - I) e^{rT}$$

Storage costs and combinations of fixed-value payments (coupons) can be priced in this manner.

Foreign Currency Futures

Assume that foreign country f has a risk-free interest rate *in its own currency* of r_f .

Let S_0 be the spot price in dollars of one unit of f , and F_0 be the future price in dollars of one unit of f . Then

$$F_0 = S_0 e^{(r-r_f)T}$$

Note that I can borrow Y units of currency f at rate r_k , (costing me $Y e^{r_k T}$ in currency f then) convert this to dollars and invest (earning me $Y S_0 e^{rT}$ in dollars then).

Thus if $F_0 < S_0 e^{(r-r_f)T}$, I can go long on a forward contract to get the $Y e^{r_k T}$ in currency f to pay them back at less than I earned on my dollars.

Note that I can borrow Y dollars at rate r , (costing me $Y e^{rT}$ in dollars then), convert this to Y/S_0 units of currency f , and invest this at rate r_k (earning me $(Y/S_0) e^{r_k T}$ in currency f then)

Thus if $F_0 > S_0 e^{(r-r_f)T}$ I can go short on a forward contract to sell my currency f , earning more than I paid for my dollars.

Thus future currency prices are purely a function of the interest rates in the two countries!