

# Derivatives

*Derivatives* are financial instruments whose value derives from the values of other, more basic variables.

*Options* give the owner the right, but not the obligation, to buy or sell a security at a specified price on (or perhaps before) a specified date.

The Chicago Board of Exchange ([www.cboe.com](http://www.cboe.com)) trades options on over 1200 stocks and stock indices.

*Futures contracts* gives one the right and obligation to buy or sell a commodity at a given price at a given time.

More *exotic* options can easily be designed and are often available or even traded on exchanges.

# Forward Contracts

*Forward contracts* give the owner the right *and* obligation, to buy or sell a given security stock at a specified price on (or perhaps before) a specified date.

The *long position* on the contract agrees to buy the security on the date. They are betting the the price will go up.

The *short position* on the contract agrees to sell the security on the date. They are betting the the price will go down.

Forward contracts can be worth less than zero. If I have the long position on an expiring forward contract for \$100 and the price is \$90, this will cost me \$10.

The fact that certain options can go negative mandates *margin* requirements, where the owner must put money in escrow to prove they can cover the losses.

# Uses of Forward Contracts

Forward contracts are used in foreign exchange to hedge the risks of currency fluctuations.

If I have to pay \$1 million to someone in 1 year, and I have the equivalent in Euros, I can either (a) take a risk the prices will not change, (b) convert it all now and sit on it, (c) keep the money and buy a forward contract for the equivalent amount of Euros in 1 year.

Forward contracts are signed between two parties, not traded on exchanges.

Closely related *futures contracts* for commodities are traded on exchanges like the Chicago Board of Trade (CBOT).

Futures differ from forward contracts in allowing more flexibility on the timing of when the short position must sell.

# Relating Spot and Forward Prices

What is the correct forward price for a security/asset whose current (spot) price is  $S_T$ ?

If the forward price is too high, I can borrow money to buy the asset, and take a short position to guarantee myself a profit.

Example: Suppose the spot price for Microsoft is \$100 a share, the interest rate is 5% per year, and the forward price is \$115 for a one year contract.

Buying the share will cost me \$5 for borrowing the money, so this strategy will guarantee me  $\$15 - \$5 = \$10$  profit a year from now with no risk.

This simple analysis ignores transaction costs, and the time-value of money.

# Pricing Forward Contracts

If the forward price is too low, and I either own or can borrow the asset, I can take a long position to guarantee myself a profit.

Example: Suppose the spot price for Microsoft is \$100 a share, the interest rate is 5% per year, and the forward price is \$100 for a one year contract.

I can sell my share now, earn interest on the money for a year, and through my forward contract buy back the share a year from now, so this strategy will guarantee me  $\$105 - \$100 = \$5$  profit at no risk.

At any price except \$105, the forward contract gives somebody an arbitrage opportunity.

If we assume that all parties are intelligent and can borrow money at the same interest rate, the forward price is completely determined by this theory.

# Options

*Put options* permit you to *sell* an asset for a certain price by a certain date.

*Call options* permit you to *buy* an asset for a certain price by a certain date.

Suppose you own an unexpired call option to buy a stock for \$100.

What is your option worth if the stock is selling for \$110? At least \$10.

What if the stock is selling for \$90? The value of an option cannot be less than 0, since you can choose not to execute it.

# Types of Options

*American options* can be exercised anytime prior to the expiration date

*European options* can only be executed on the expiration date.

*Asian options* have payoffs dependent on the average price over a specified period.

Most exchange-traded options are American, but the value of European options is easier to determine.

Most futures and options are settled for cash values, instead of delivering the actual goods.

# The Value of European Options

The profit of a *long position* (buying the option) of a *call* (to buy the asset) at strike price  $K$  and current price  $S_t$  is  $\max(S_t - K, 0)$ .

Why? I win if the current price is greater than what I am allowed to buy it for.

The profit for a short position (selling the option) on the call is  $-\max(S_t - K, 0) = \min(K - S_t, 0)$ , by a conservation of money argument.

The seller of an option make their gain from what they were originally paid for the option.

The profit of a *long position* (buying the option) of a *put* (to sell the asset) at strike price  $K$  is  $\max(K - S_t, 0)$ .

The corresponding profit for the seller of this position is  $\min(S_t - k, 0)$ .



# Uses of Options

*Hedgers* can use options to reduce risk.

They can reduce the risk that a stock they own will decline by buying put options on it.

*Speculators* can use options to gamble that prices will change.

They can buy a put option on a stock if they think it will go down.

*Arbitragers* can use options to gain risk-less profits if securities are inconsistently priced.

The volume of open interest on both sides of an option may tell something about market sentiment on the future value of an asset.