CSE 613: Parallel Programming

Lecture 4
( Greedy Scheduling )
( inspiration for some slides comes from lectures given by Charles Leiserson )

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A *runtime/online scheduler* maps tasks to processing elements dynamically at runtime.

The map is called a *schedule*.

An *offline scheduler* prepares the schedule prior to the actual execution of the program.
Greedy Scheduling

A strand / task is called *ready* provided all its parents (if any) have already been executed.

- executed task
- ready to be executed
- not yet ready

A *greedy scheduler* tries to perform as much work as possible at every step.
Let $p = \text{number of cores}$

At every step:

- if $\geq p$ tasks are ready:
  execute any $p$ of them
  (complete step)

- if $< p$ tasks are ready:
  execute all of them
  (incomplete step)
Let $p$ = number of cores

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- if $< p$ tasks are ready:
  execute all of them
  (incomplete step)
A Centralized Greedy Scheduler

Let $p = \text{number of cores}$

At every step:

- if $\geq p$ tasks are ready:
  execute any $p$ of them
  \hspace{1cm} ( complete step )

- if $< p$ tasks are ready:
  execute all of them
  \hspace{1cm} ( incomplete step )
A Centralized Greedy Scheduler

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  \hspace{1cm} (complete step)

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Let $p = \text{number of cores}$

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  (incomplete step)
Greed Scheduling Theorem

Theorem [Graham’68, Brent’74]:
For any greedy scheduler,
\[ T_p \leq \frac{T_1}{p} + T_\infty \]

Proof:
\[ T_p = \# \text{complete steps} + \# \text{incomplete steps} \]

- Each complete step performs \( p \) work:
  \[ \# \text{complete steps} \leq \frac{T_1}{p} \]

- Each incomplete step reduces the span by 1:
  \[ \# \text{incomplete steps} \leq T_\infty \]
Optimality of the Greedy Scheduler

Corollary 1: For any greedy scheduler \( T_p \leq 2T_p^* \), where \( T_p^* \) is the running time due to optimal scheduling on \( p \) processing elements.

Proof:

Work law: \( T_p^* \geq \frac{T_1}{p} \)

Span law: \( T_p^* \geq T_\infty \)

\[ \therefore \text{From Graham-Brent Theorem:} \]

\[ T_p \leq \frac{T_1}{p} + T_\infty \leq T_p^* + T_p^* = 2T_p^* \]
Optimality of the Greedy Scheduler

Corollary 2: Any greedy scheduler achieves $S_p \approx p$ (i.e., nearly linear speedup) provided $\frac{T_1}{T_\infty} \gg p$.

Proof:

Given, $\frac{T_1}{T_\infty} \gg p \Rightarrow \frac{T_1}{p} \gg T_\infty$

∴ From Graham-Brent Theorem:

$$T_p \leq \frac{T_1}{p} + T_\infty \approx \frac{T_1}{p}$$

$$\Rightarrow \frac{T_1}{T_p} \approx p \Rightarrow S_p \approx p$$