CSE 373: Analysis of Algorithms

Lecture 14
(Selection)

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Deterministic Select

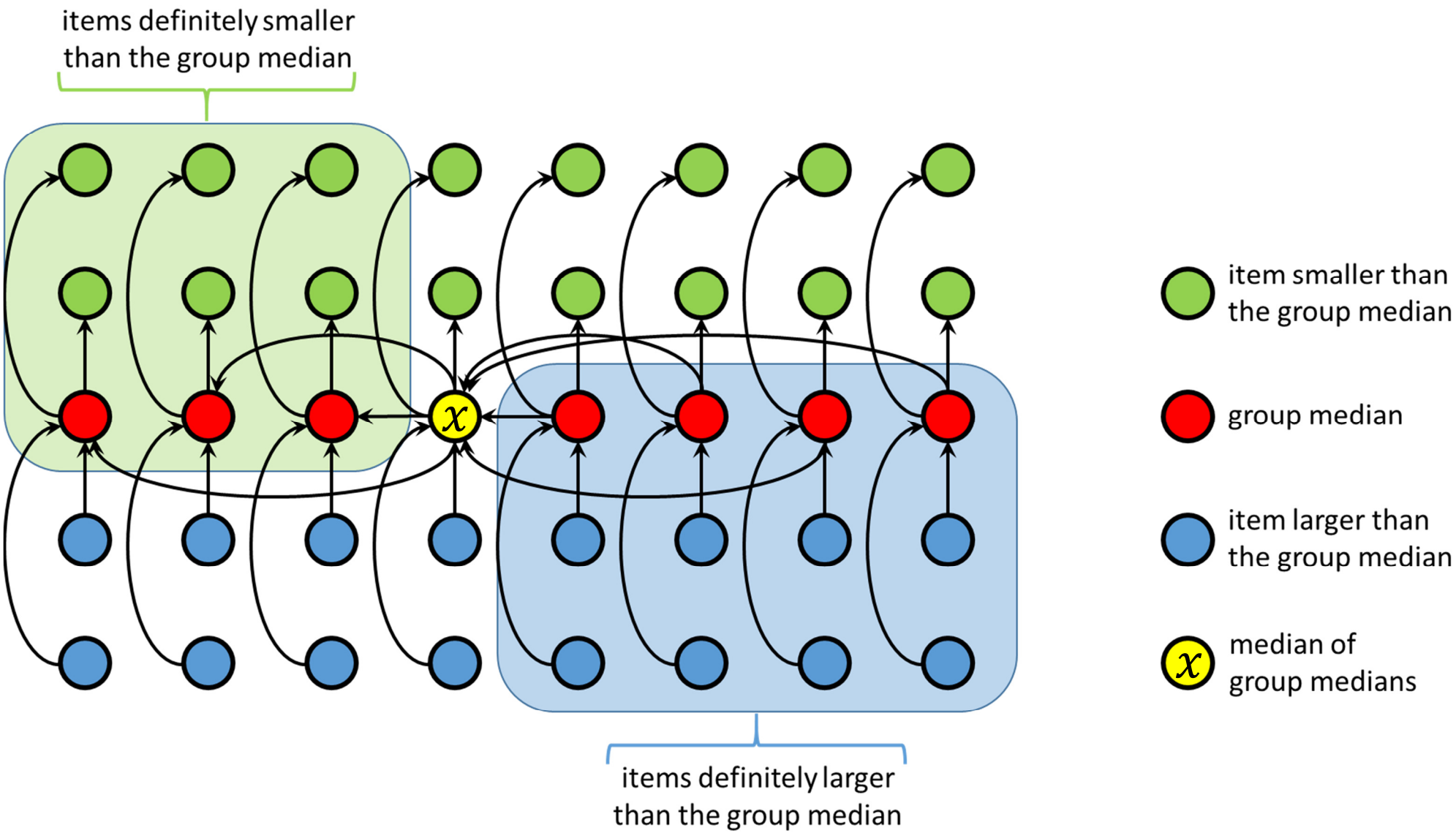
Input: An array $A[\, q : r \,]$ of distinct elements, and integer $k \in [1, r - q + 1]$.

Output: An element x of $A[\, q : r \,]$ such that $rank(x, A[\, q : r \,]) = k$.

```
Select ( A[ q : r ], k )
 1.  n ← r − q + 1
 2.  if n ≤ 140 then
 3.        sort A[ q : r ] and return A[ q + k − 1 ]
 4.  else
 5.        divide A[ q : r ] into blocks B_i's each containing 5 consecutive elements
                ( last block may contain fewer than 5 elements )
 6.        for i ← 1 to ⌈ n / 5 ⌉ do
 7.              M[ i ] ← median of B_i using sorting
 8.        x ← Select ( M[ 1 : ⌈ n / 5 ⌉ ], ⌊( ⌈ n / 5 ⌉ + 1 ) / 2 ⌋ )  { median of medians }
 9.        t ← Partition ( A[ q : r ], x )      { partition around x which ends up at A[ t ] }
10.        if k = t − q + 1 then return A[ t ]
11.        else if k < t − q + 1 then return Select ( A[ q : t − 1 ], k )
12.              else return Select ( A[ t + 1 : r ], k − t + q − 1 )
```

Deterministic Select

SELECT(A, k): Given an unsorted set A of n $(= |A|)$ items, find the k^{th} smallest item in the set



Deterministic Select

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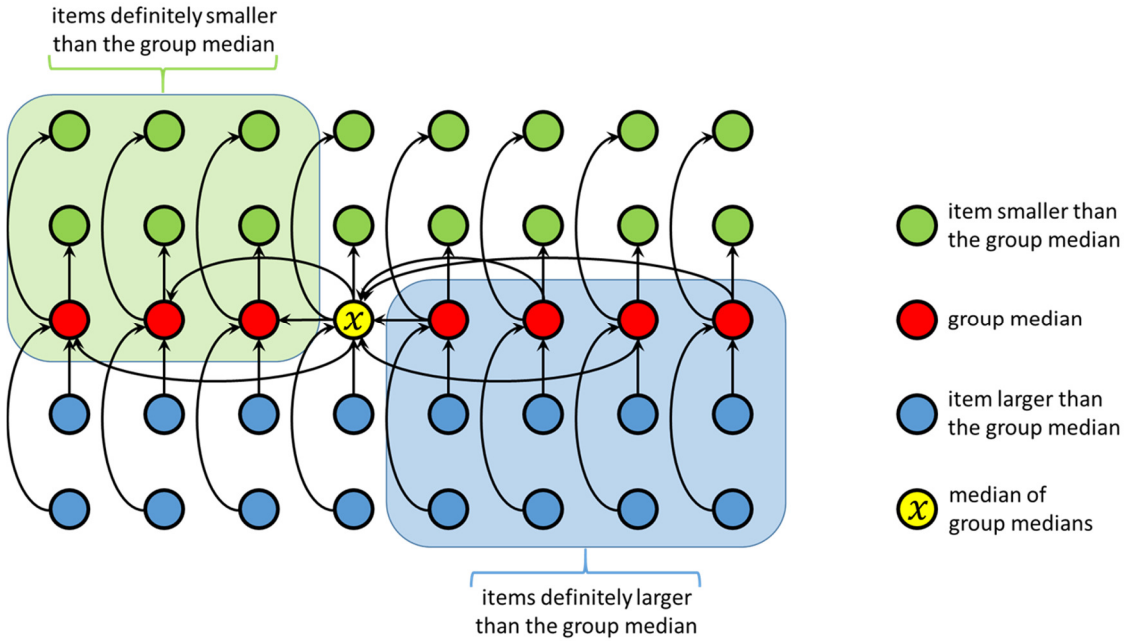

#items definitely smaller than x is $\geq 3\left(\left\lfloor \frac{1}{2}\left\lceil \frac{n}{5}\right\rceil\right\rfloor - 1\right) \geq \frac{3n}{10} - 6$

#items definitely larger than x is $\geq 3\left(\left\lfloor \frac{1}{2}\left\lceil \frac{n}{5}\right\rceil\right\rfloor - 1\right) \geq \frac{3n}{10} - 6$

#items in any recursive call (lines 11/12) $\leq n - \left(\frac{3n}{10} - 6\right) = \frac{7n}{10} + 6$

Deterministic Select

The following recurrence describes the worst-case running time of the deterministic selection algorithm (given in Section 9.3 of CLRS):

$$T(n) \leq \begin{cases} \Theta(1), & if\ n < 140, \\ T\left(\left\lceil \frac{n}{5} \right\rceil\right) + T\left(\frac{7n}{10} + 6\right) + \Theta(n), & if\ n \geq 140. \end{cases}$$

How do you solve for $T(n)$?