First-Order Logic

Chapter 8

Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Pros and cons of propositional logic

- ☺ Propositional logic is declarative
- ☺ Propositional logic allows partial/disjunctive/negated information
  - (unlike most data structures and databases)
  -
- ☺ Propositional logic is compositional:
- ☺
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
  -
- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)
  -
- ☹ Propositional logic has very limited expressive power

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
-
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  -
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus

Syntax of FOL: Basic elements

• Constants	KingJohn, 2, NUS,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf,...
• Variables	x, y, a, b,...
• Connectives	$\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$
• Equality	=
• Quantifiers	$\forall$, $\exists$

Atomic sentences

Atomic sentence = *predicate* ($term_1$,...,$term_n$)
or $term_1 = term_2$

Term = *function* ($term_1$,...,$term_n$)
or *constant* or *variable*

- E.g., *Brother(KingJohn,RichardTheLionheart) > (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives
-

$$\neg S,\ S_1 \wedge S_2,\ S_1 \vee S_2,\ S_1 \Rightarrow S_2,\ S_1 \Leftrightarrow S_2,$$

E.g. *Sibling(KingJohn,Richard)* $\Rightarrow$ *Sibling(Richard,KingJohn)*

$>(1,2) \vee \leq (1,2)$

$>(1,2) \wedge \neg >(1,2)$

Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
-
- Interpretation specifies referents for

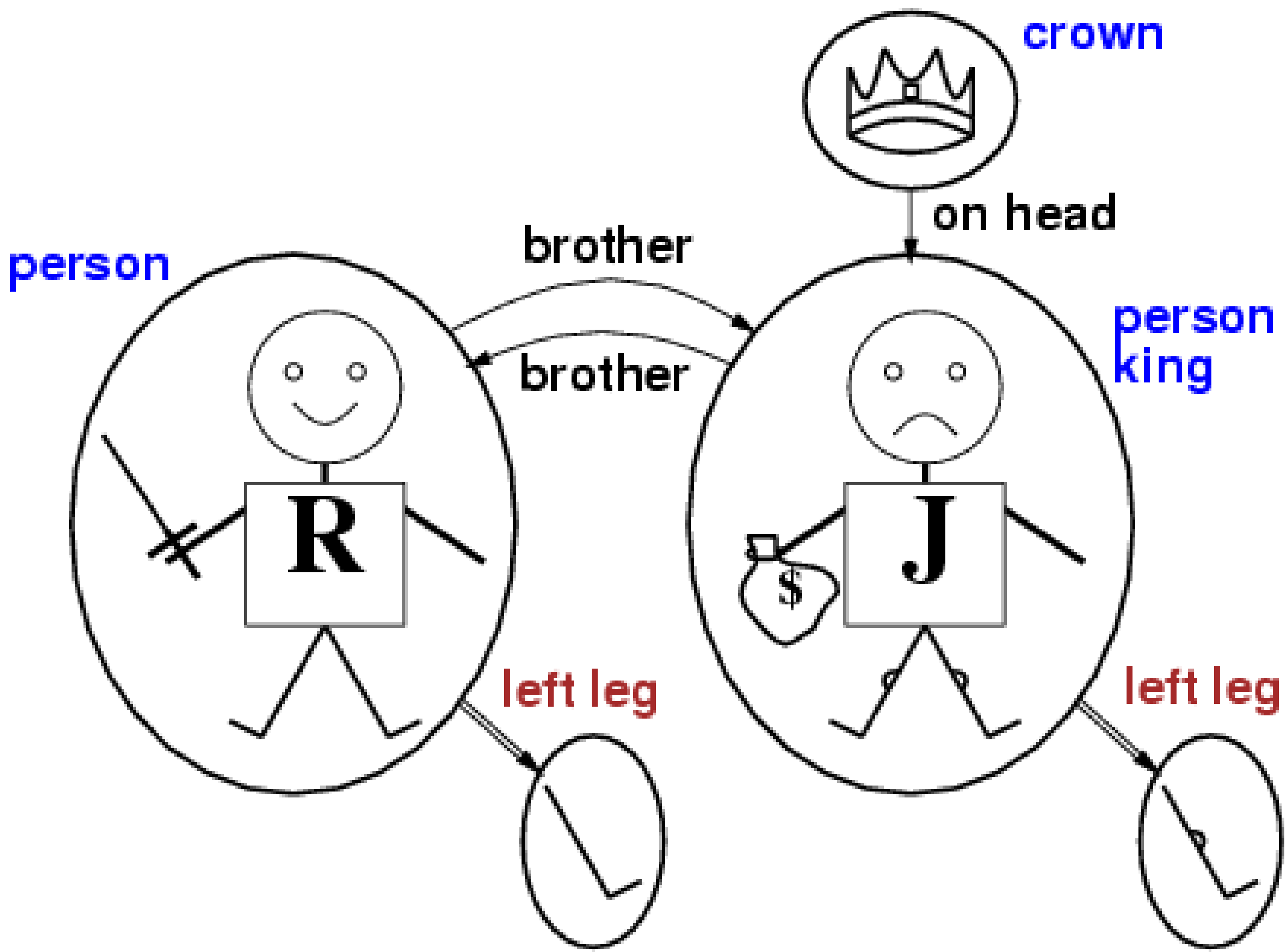
  constant symbols → objects

  predicate symbols → relations

  function symbols → functional relations

- An atomic sentence *predicate*$(term_1,\ldots,term_n)$ is true iff the objects referred to by $term_1,\ldots,term_n$ are in the relation referred to by *predicate*

Models for FOL: Example



Universal quantification

- ∀<*variables*> <*sentence*>

Everyone at NUS is smart:

$\forall x\ At(x,NUS) \Rightarrow Smart(x)$

- $\forall x$ *P* is true in a model *m* iff *P* is true with *x* being each possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of *P*

$$\begin{aligned} & At(KingJohn,NUS) \Rightarrow Smart(KingJohn) \\ \wedge\ & At(Richard,NUS) \Rightarrow Smart(Richard) \\ \wedge\ & At(NUS,NUS) \Rightarrow Smart(NUS) \end{aligned}$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
-
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x$ At(x,NUS) $\wedge$ Smart(x)

  means "Everyone is at NUS and everyone is smart"

Existential quantification

- $\exists$<*variables*> <*sentence*>

- Someone at NUS is smart:
- $\exists x$ At(x,NUS) $\wedge$ Smart(x)\$
-

- $\exists x$ *P* is true in a model *m* iff *P* is true with *x* being some possible object in the model
-

- Roughly speaking, equivalent to the disjunction of instantiations of *P*
-

  At(KingJohn,NUS) $\wedge$ Smart(KingJohn)
  $\vee$ At(Richard,NUS) $\wedge$ Smart(Richard)
  $\vee$ At(NUS,NUS) $\wedge$ Smart(NUS)

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:
-

$$\exists x \; At(x,NUS) \Rightarrow Smart(x)$$

is true if there is anyone who is not at NUS!

Properties of quantifiers

- $\forall x \, \forall y$ is the same as $\forall y \, \forall x$
-
- $\exists x \, \exists y$ is the same as $\exists y \, \exists x$
-

- $\exists x \, \forall y$ is not the same as $\forall y \, \exists x$
-
- $\exists x \, \forall y$ Loves(x,y)
  - "There is a person who loves everyone in the world"
  -
- $\forall y \, \exists x$ Loves(x,y)
  - "Everyone in the world is loved by at least one person"
  -

- Quantifier duality: each can be expressed using the other
-
- $\forall x$ Likes(x,IceCream) $\qquad \neg\exists x \, \neg$Likes(x,IceCream)
-

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
-
- E.g., definition of *Sibling* in terms of *Parent*:
-

$$\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \land \exists m,f\ \neg(m = f) \land Parent(m,x) \land Parent(f,x) \land Parent(m,y) \land Parent(f,y)]$$

Using FOL

The kinship domain:

- Brothers are siblings
- $\forall$x,y *Brother(x,y)* $\Leftrightarrow$ *Sibling(x,y)*
- One's mother is one's female parent
- $\forall$m,c *Mother(c)* = m $\Leftrightarrow$ *(Female(m)* $\wedge$ *Parent(m,c))*
- “Sibling” is symmetric

Using FOL

The set domain:

- $\forall s\ Set(s) \Leftrightarrow (s = \{\}) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
-
- $\neg\exists x,s\ \{x|s\} = \{\}$
-
- $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
-
- $\forall x,s\ x \in s \Leftrightarrow [\ \exists y,s_2\}\ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$
-
- $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
-

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

  `Tell`(KB,Percept([Smell,Breeze,None],5))
  `Ask`(KB,∃a BestAction(a,5))

- I.e., does the KB entail some best action at *t=5*?
-

- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)

- Given a sentence *S* and a substitution σ,
- *S*σ denotes the result of plugging σ into *S*; e.g.,
  S = Smarter(x,y)
  σ = {x/Hillary,y/Bill}
  *S*σ = Smarter(Hillary,Bill)

- `Ask`(KB,S) returns some/all σ such that KB ╞ σ

Knowledge base for the wumpus world

- Perception
  - $\forall t,s,b\ Percept([s,b,Glitter],t) \Rightarrow Glitter(t)$
  -

- Reflex
  - $\forall t\ Glitter(t) \Rightarrow BestAction(Grab,t)$

Deducing hidden properties

- $\forall x,y,a,b$ *Adjacent*([x,y],[a,b]) $\Leftrightarrow$ [a,b] $\in$ {[x+1,y], [x-1,y],[x,y+1],[x,y-1]}

Properties of squares:

- $\forall s,t$ *At*(Agent,s,t) $\wedge$ Breeze(t) $\Rightarrow$ Breezy(s)

Squares are breezy near a pit:

- Diagnostic rule---infer cause from effect

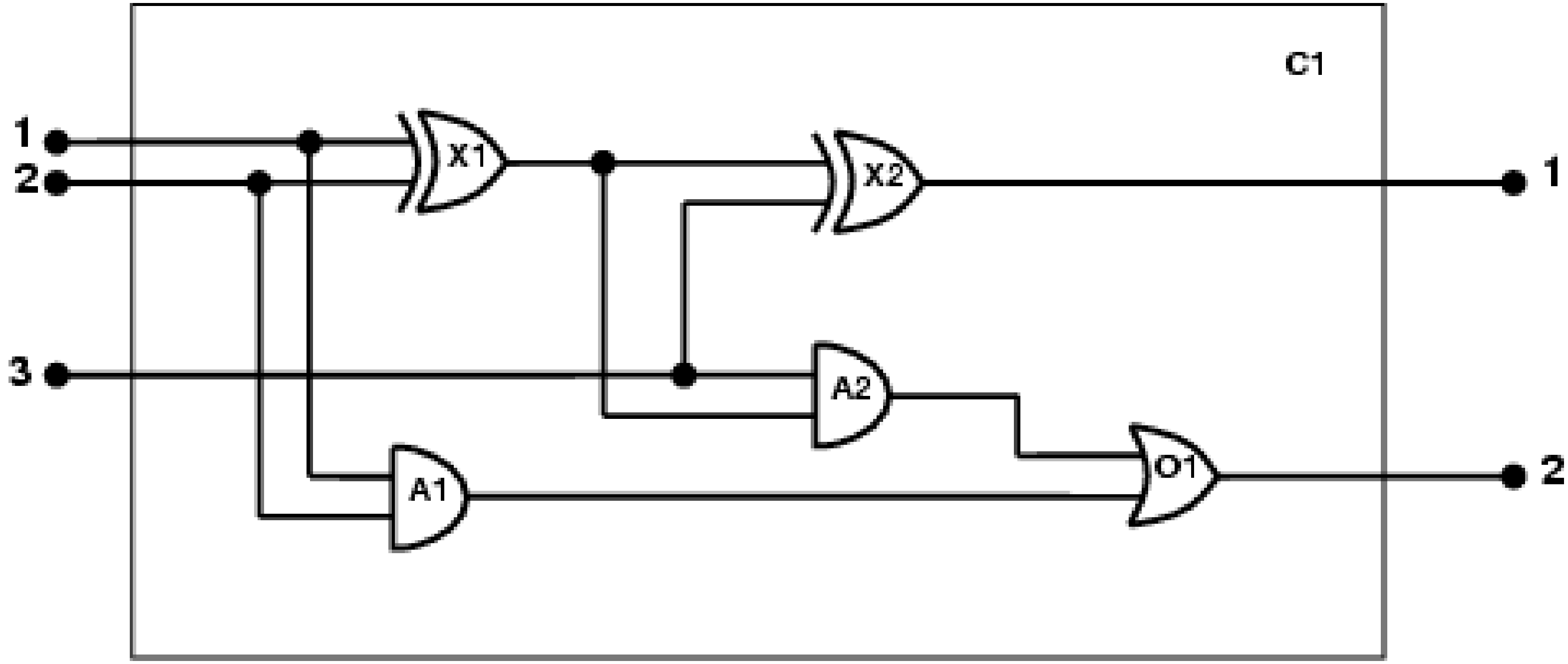
  $\forall$s Breezy(s) $\Rightarrow$ \Exi{r} Adjacent(r,s) $\wedge$ Pit(r)$

Knowledge engineering in FOL

1. Identify the task
2.
2. Assemble the relevant knowledge
3.
3. Decide on a vocabulary of predicates, functions, and constants
4.
4. Encode general knowledge about the domain
5.
5. Encode a description of the specific problem instance
6.

The electronic circuits domain

One-bit full adder



The electronic circuits domain

1. Identify the task

2.

   – Does the circuit actually add properly? (circuit verification)

   –

2. Assemble the relevant knowledge

3.

   – Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)

   –

   – Irrelevant: size, shape, color, cost of gates

   –

3. Decide on a vocabulary

4.

The electronic circuits domain

4. Encode general knowledge of the domain
5.
   - $\forall t_1,t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Signal}(t_1) = \text{Signal}(t_2)$
   - $\forall t \text{ Signal}(t) = 1 \vee \text{Signal}(t) = 0$
   -
   - $1 \neq 0$
   -
   - $\forall t_1,t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Connected}(t_2, t_1)$
   -
   - $\forall g \text{ Type}(g) = \text{OR} \Rightarrow \text{Signal}(\text{Out}(1,g)) = 1 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n,g)) = 1$
   -
   - $\forall g \text{ Type}(g) = \text{AND} \Rightarrow \text{Signal}(\text{Out}(1,g)) = 0 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n,g)) = 0$

The electronic circuits domain

5. Encode the specific problem instance
6.

$Type(X_1) = XOR$
$Type(X_2) = XOR$
$Type(A_1) = AND$
$Type(A_2) = AND$
$Type(O_1) = OR$

$Connected(Out(1,X_1),In(1,X_2))$
$Connected(Out(1,X_1),In(2,A_2))$
$Connected(Out(1,A_2),In(1,O_1))$
$Connected(Out(1,A_1),In(2,O_1))$
$Connected(Out(1,X_2),Out(1,C_1))$
$Connected(Out(1,O_1),Out(2,C_1))$

$Connected(In(1,C_1),In(1,X_1))$
$Connected(In(1,C_1),In(1,A_1))$
$Connected(In(2,C_1),In(2,X_1))$
$Connected(In(2,C_1),In(2,A_1))$
$Connected(In(3,C_1),In(2,X_2))$
$Connected(In(3,C_1),In(1,A_2))$

The electronic circuits domain

6. Pose queries to the inference procedure
7.

What are the possible sets of values of all the terminals for the adder circuit?

$\exists i_1,i_2,i_3,o_1,o_2$ Signal(In(1,C_1)) = $i_1 \wedge$ Signal(In(2,C_1)) = $i_2 \wedge$ Signal(In(3,C_1)) = $i_3 \wedge$ Signal(Out(1,C_1)) = $o_1 \wedge$ Signal(Out(2,C_1)) = o_2

7. Debug the knowledge base

Summary

- First-order logic:
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  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
  -

- Increased expressive power: sufficient to define wumpus world
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