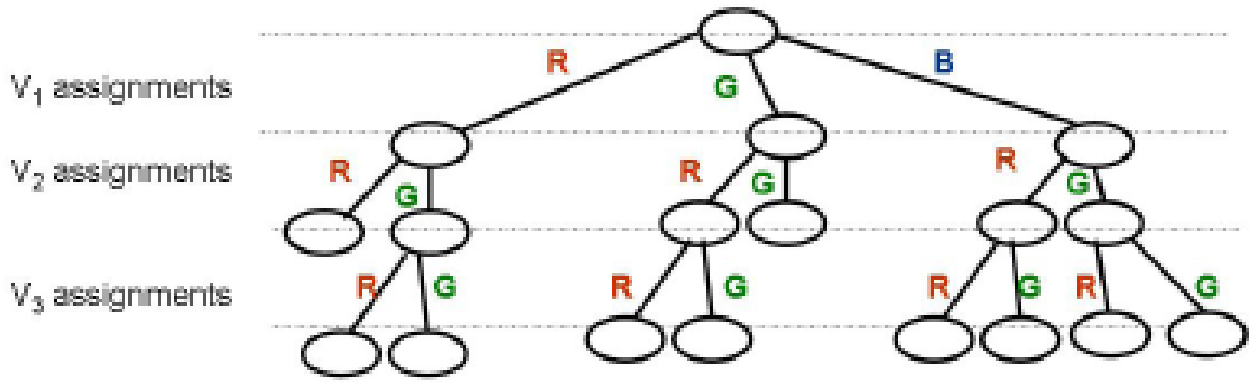
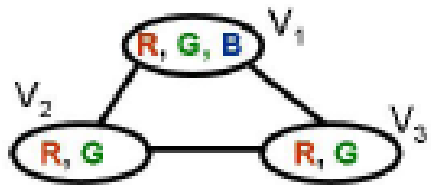
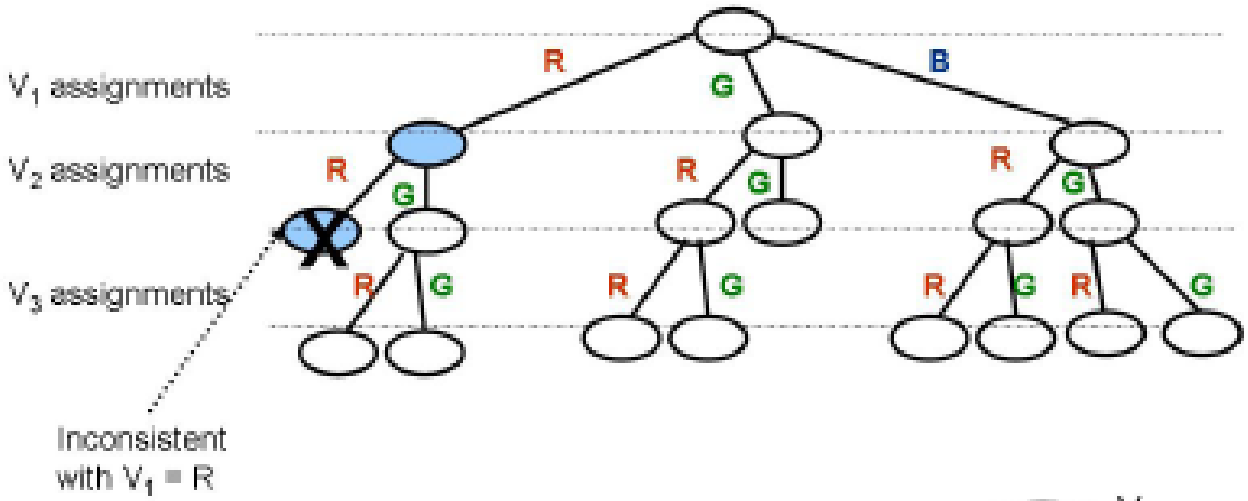
Backtracking

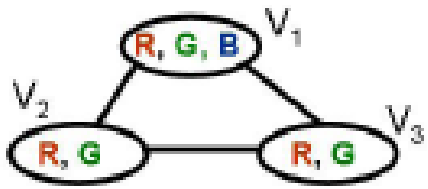




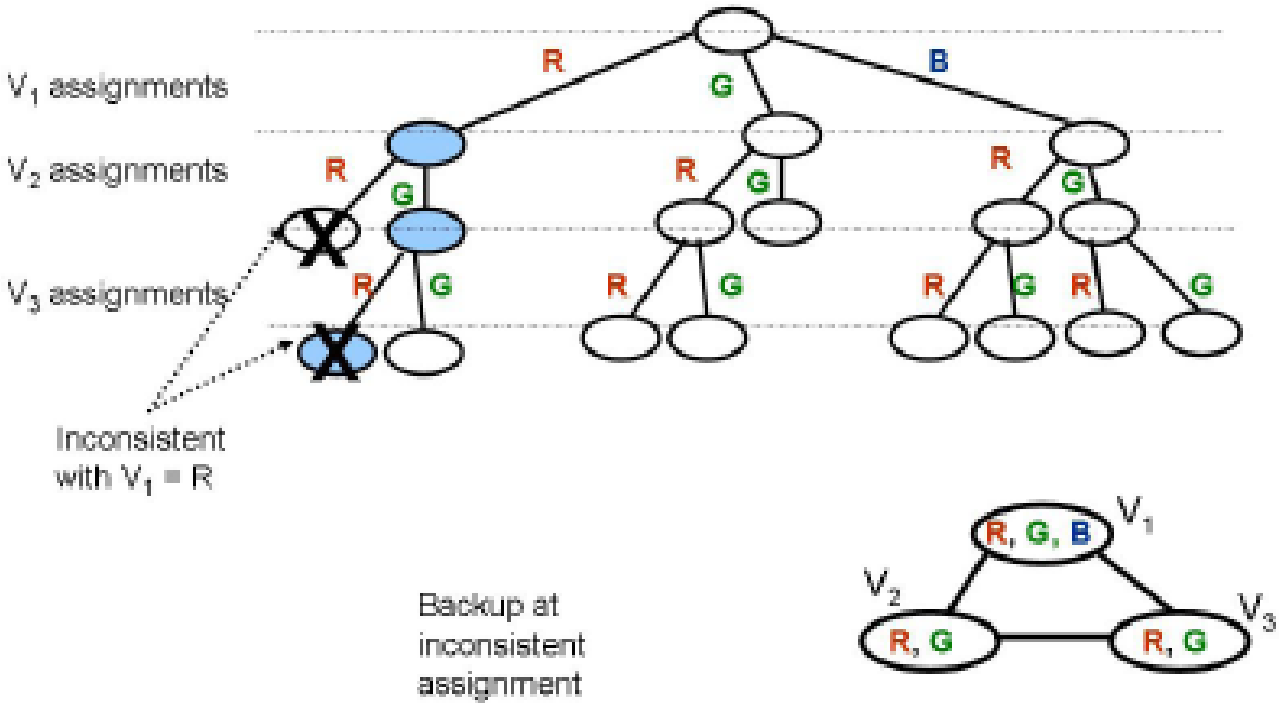
Backtracking



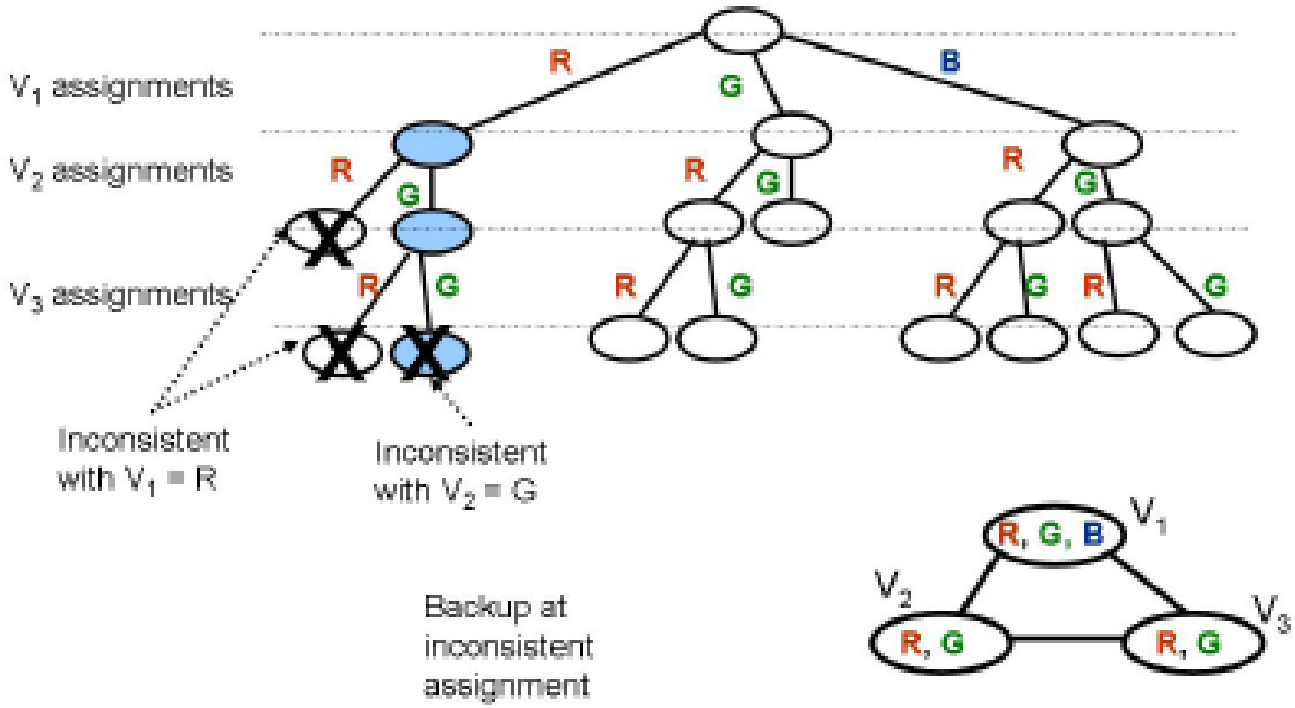
Backup at
inconsistent
assignment



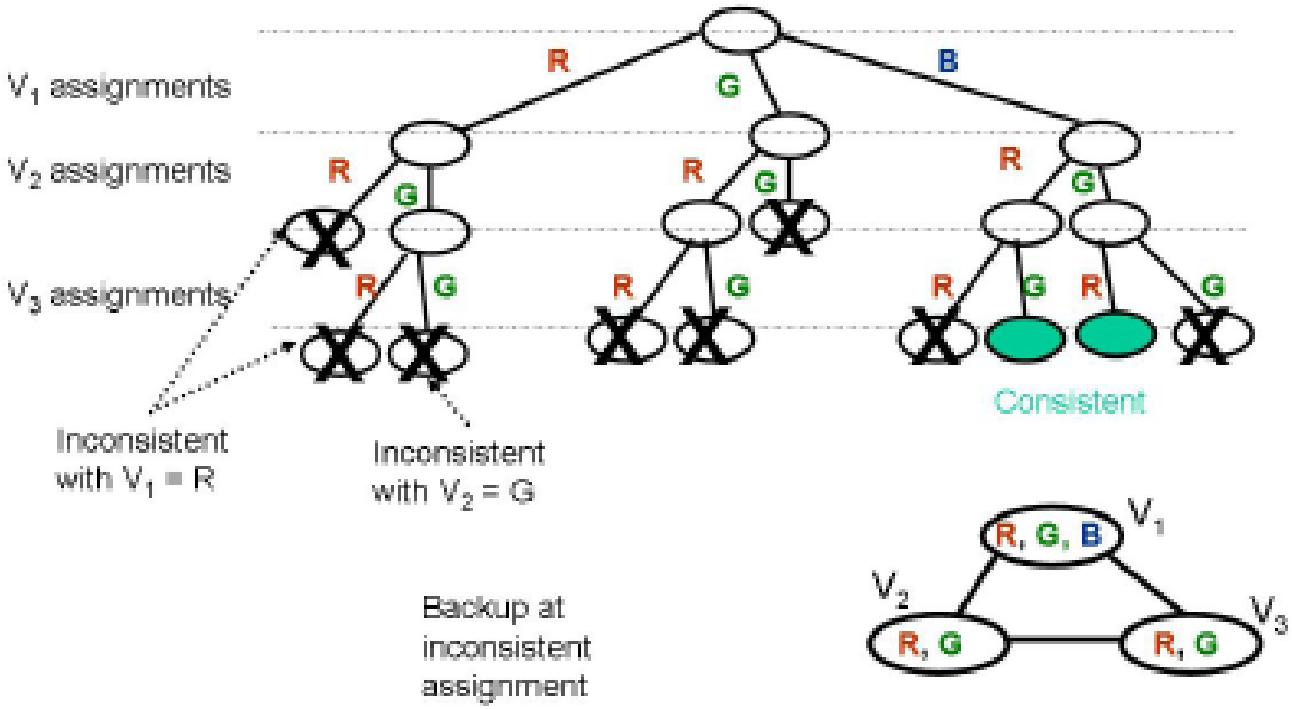
Backtracking



Backtracking



Backtracking



Dynamic Reordering

Traditional backtracking uses fixed ordering of variables & values, e.g., random order or place variables with many constraints first.

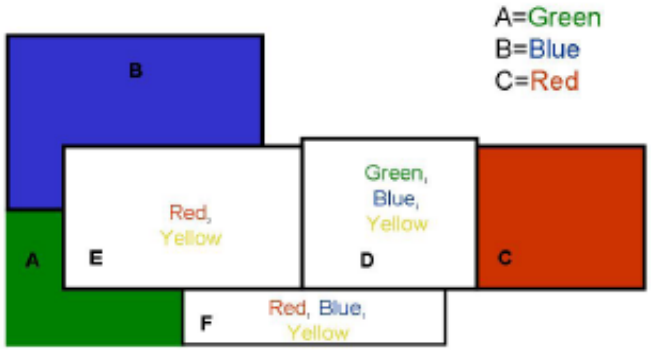
You can usually do better by choosing an order dynamically as the search proceeds.

- Most constrained variable
  when doing forward-checking, pick variable with fewest legal values to assign next (minimizes branching factor)

- Least constraining value
  choose value that rules out the fewest values from neighboring domains

Dynamic Reordering

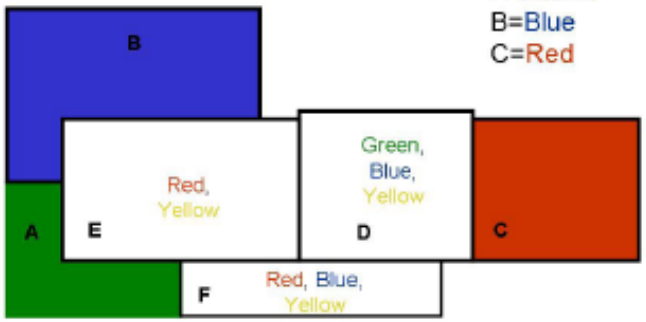
Colors: R, G, B, Y



Which country should we color next →

What color should we pick for it? →

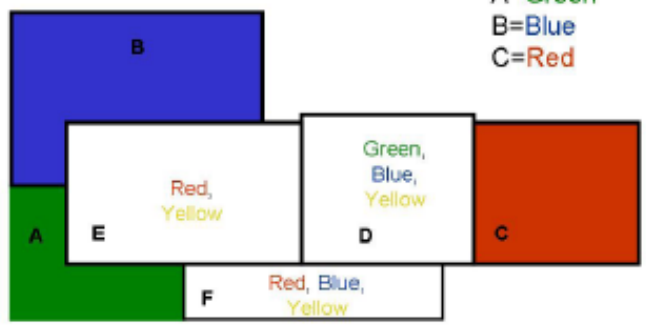
Colors: R, G, B, Y



Which country should we color next → E most-constrained variable (smallest domain)

What color should we pick for it? →

Colors: R, G, B, Y

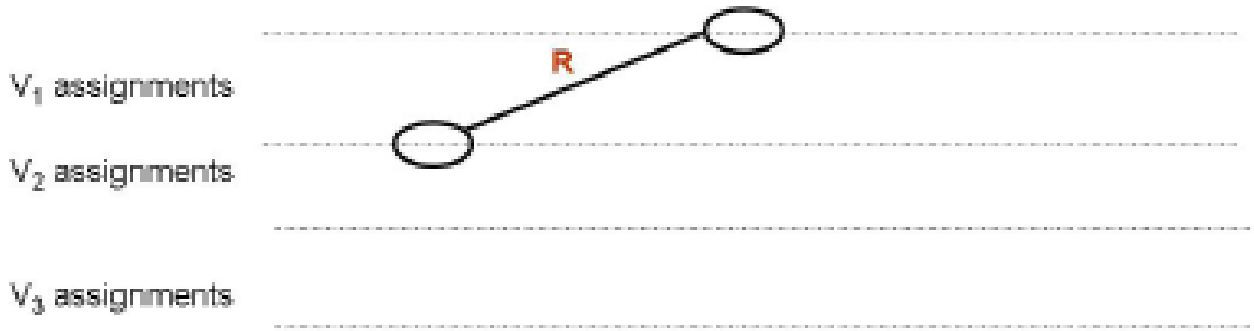


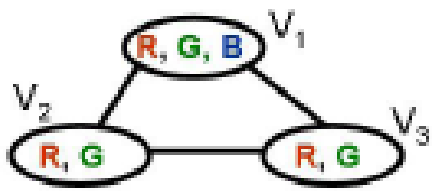
Which country should we color next → E most-constrained variable (smallest domain)

What color should we pick for it? → RED least-constraining value (eliminates fewest values from neighboring domains)

Backtracking with Forward Checking (BT-FC)

When examining assignment $V_i=d_k$, remove any values inconsistent with that assignment from neighboring domains in constraint graph.





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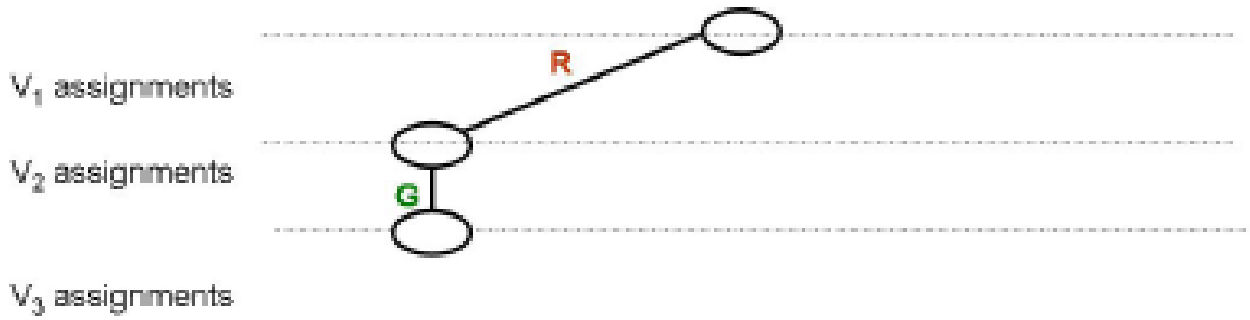

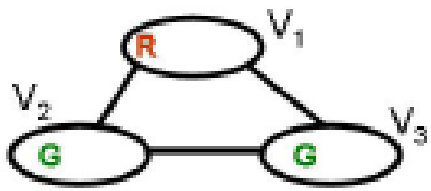

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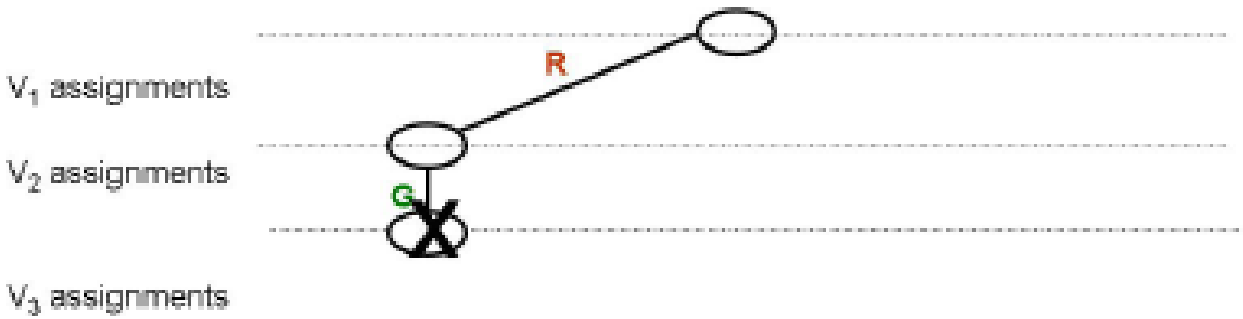
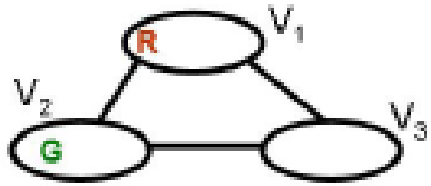
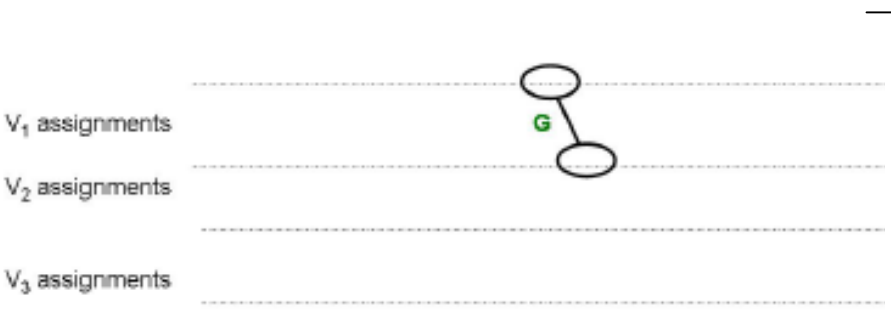
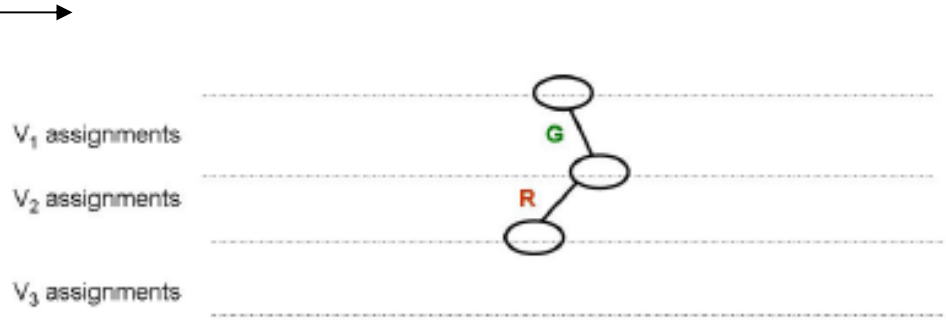


We have a conflict whenever a domain becomes empty.

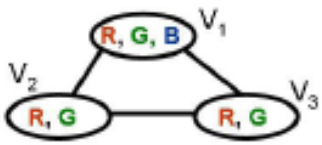


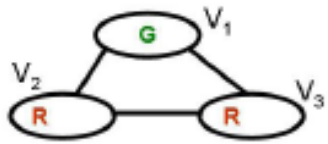
Backtracking with Forward Checking (BT-FC)

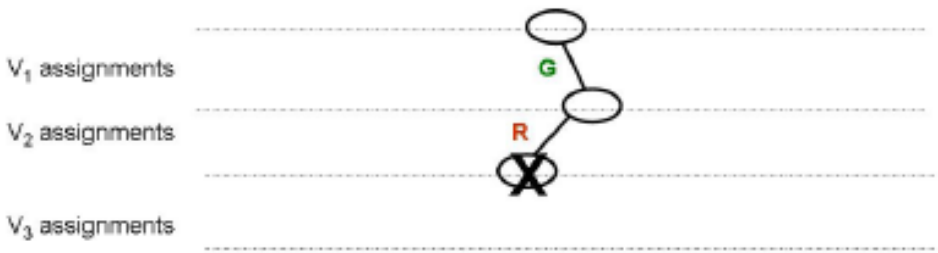


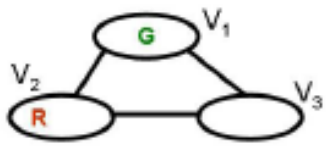


When backing up, need to restore domain values, since deletions were done to reach consistency with tentative assignments considered during search.

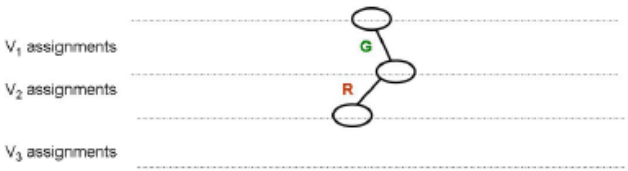


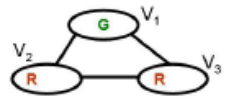


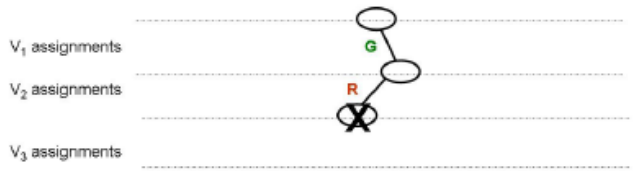


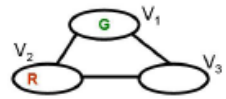


Backtracking with Forward Checking (BT-FC)

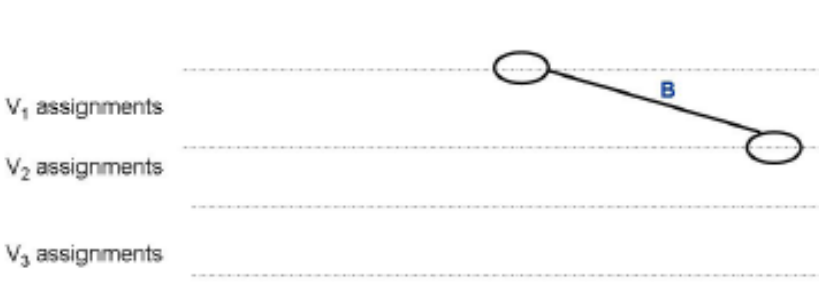


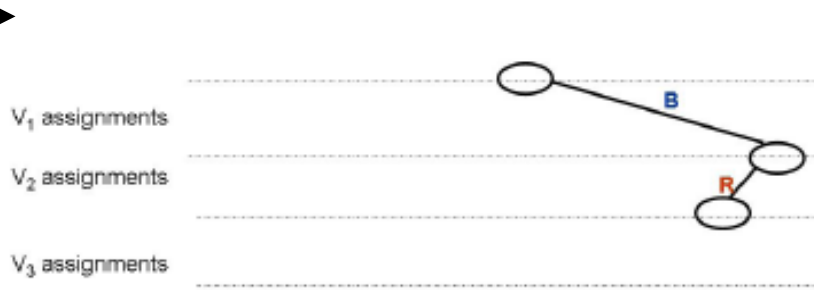


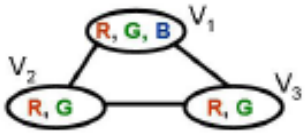


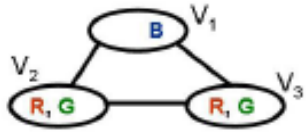


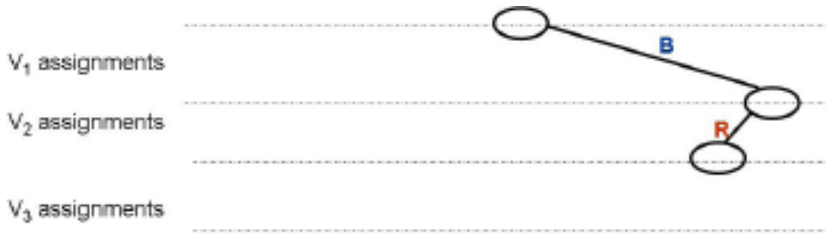
Backtracking with Forward Checking (BT-FC)

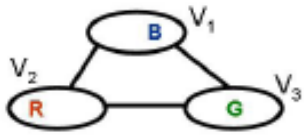


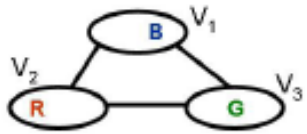












Constraint Propagation (aka Arc Consistency)

Arc consistency eliminates values from domain of variable that can never be part of a consistent solution.

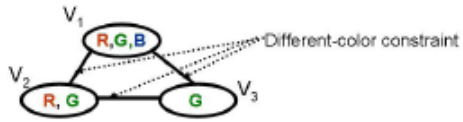
Directed arc (V_i, V_j) is arc consistent if
$\forall x \in D_i \; \exists y \in D_j$ such that (x,y) is allowed by the constraint on the arc

We can achieve consistency on arc by deleting values form D_i (domain of variable at tail of constraint arc) that fail this condition.

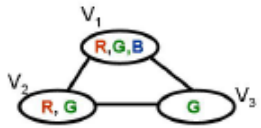
Constraint Propagation Example

Graph Coloring

Initial Domains are indicated



Arc examined	Value deleted



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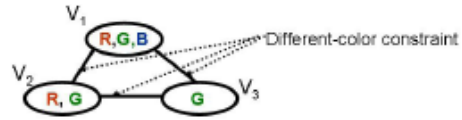
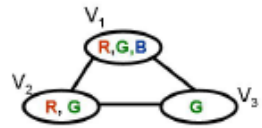


Arc examined	Value deleted
$V_1 - V_2$	none



Graph Coloring

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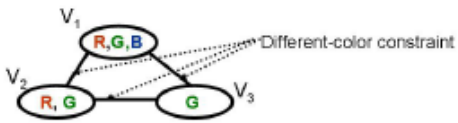
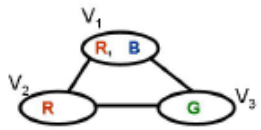


Arc examined	Value deleted
$V_1 - V_2$	none
$V_1 - V_3$	V_1(G)
$V_2 - V_3$	V_2(G)



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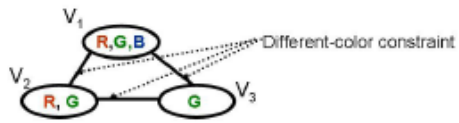
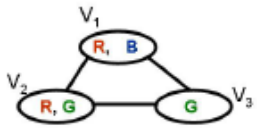
Initial Domains are indicated



Arc examined	Value deleted
$V_1 - V_2$	none
$V_1 - V_3$	V_1(G)



Constraint Propagation Example

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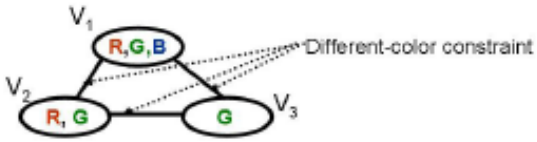
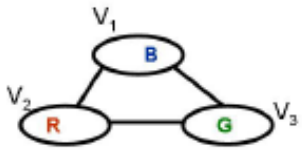
Initial Domains are indicated



Arc examined	Value deleted
$V_1 - V_2$	none
$V_1 - V_3$	V_1(G)
$V_2 - V_3$	V_2(G)
$V_1 - V_2$	V_1(R)



Graph Coloring
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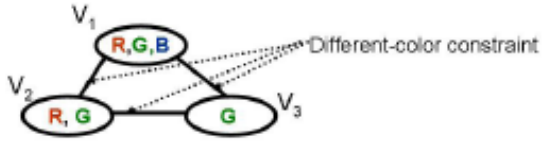
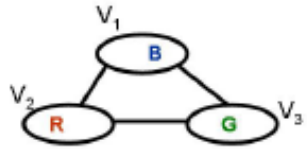
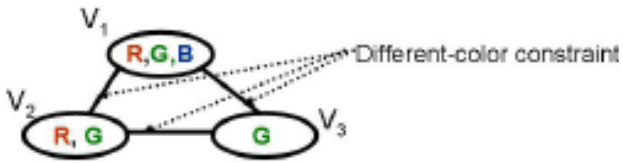


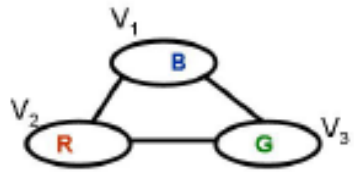
Arc examined	Value deleted
$V_1 - V_2$	none
$V_1 - V_3$	V_1(G)
$V_2 - V_3$	V_2(G)
$V_1 - V_2$	V_1(R)
$V_1 - V_3$	none



Graph Coloring
Initial Domains are indicated



Arc examined	Value deleted
$V_1 - V_2$	none
$V_1 - V_3$	V_1(G)
$V_2 - V_3$	V_2(G)
$V_1 - V_2$	V_1(R)
$V_1 - V_3$	none
$V_2 - V_3$	none



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$V_i \rightarrow V_j$

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We can achieve consistency on arc by deleting values form D_i (domain of variable at tail of constraint arc) that fail this condition.

Assume domains are size at most d and there are e binary constraints.

A simple algorithm for arc consistency is $O(ed^3)$ – note that just verifying arc consistency takes $O(d^2)$ for each arc.