CSE528 Computer Graphics: Theory, Algorithms, and Applications

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Multiple Coordinate Systems

- If ONLY Translation is involved between the two systems

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \rightarrow \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + (-\vec{v})$$

Multiple Coordinate Systems

- What if there is Rotation involved

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \vec{\mathrm{i}}x_1 + \vec{\mathrm{j}}y_1 + \vec{\mathrm{k}}z_1 = \begin{bmatrix} \vec{\mathrm{i}} & \vec{\mathrm{j}} & \vec{\mathrm{k}} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \vec{\mathrm{l}}x_2 + \vec{\mathrm{m}}y_2 + \vec{\mathrm{n}}z_2 = \begin{bmatrix} \vec{\mathrm{l}} & \vec{\mathrm{m}} & \vec{\mathrm{n}} \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

Multiple Coordinate Systems

- If Rotation is involved

$$\begin{bmatrix} i & j & k \end{bmatrix} = \begin{bmatrix} l & m & n \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Multiple Coordinate Systems

$$\begin{bmatrix} i & j & k \end{bmatrix} = \begin{bmatrix} l & m & n \end{bmatrix} \begin{bmatrix} i \bullet l & j \bullet l & k \bullet l \\ i \bullet m & j \bullet m & k \bullet m \\ i \bullet n & j \bullet n & k \bullet n \end{bmatrix}$$

Multiple Coordinate Systems

- Change of bases

$$\begin{bmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \mathrm{l} & \mathrm{m} & \mathrm{n} \end{bmatrix} \begin{bmatrix} \mathrm{i} \bullet \mathrm{l} & \mathrm{j} \bullet \mathrm{l} & \mathrm{k} \bullet \mathrm{l} \\ \mathrm{i} \bullet \mathrm{m} & \mathrm{j} \bullet \mathrm{m} & \mathrm{k} \bullet \mathrm{m} \\ \mathrm{i} \bullet \mathrm{n} & \mathrm{j} \bullet \mathrm{n} & \mathrm{k} \bullet \mathrm{n} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{l} & \mathrm{m} & \mathrm{n} \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

Changes of Bases

$$\mathbf{i} = \mathbf{l}(\mathbf{i} \bullet \mathbf{l}) + \mathbf{m}(\mathbf{i} \bullet \mathbf{m}) + \mathbf{n}(\mathbf{i} \bullet \mathbf{n})$$

$$\mathbf{j} = \mathbf{l}(\mathbf{j} \bullet \mathbf{l}) + \mathbf{m}(\mathbf{j} \bullet \mathbf{m}) + \mathbf{n}(\mathbf{j} \bullet \mathbf{n})$$

$$\mathbf{k} = \mathbf{l}(\mathbf{k} \bullet \mathbf{l}) + \mathbf{m}(\mathbf{k} \bullet \mathbf{m}) + \mathbf{n}(\mathbf{k} \bullet \mathbf{n})$$

Changes of Bases

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

Homogeneous Representations

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{i} \bullet \mathrm{l} & \mathrm{j} \bullet \mathrm{l} & \mathrm{k} \bullet \mathrm{l} & \mathrm{v_x} \\ \mathrm{i} \bullet \mathrm{m} & \mathrm{j} \bullet \mathrm{m} & \mathrm{k} \bullet \mathrm{m} & \mathrm{v_y} \\ \mathrm{i} \bullet \mathrm{n} & \mathrm{j} \bullet \mathrm{n} & \mathrm{k} \bullet \mathrm{n} & \mathrm{v_z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$