CSE528 Computer Graphics: Theory, Algorithms, and Applications

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Non-Uniform Rational B-Splines

NURBS

Pixar Animation Character
‘Woody’ in Toy Story

- Problems: Topological Restrictions Occur!
  - Trimming NURBS is expensive and can have numerical errors
  - When used in animation, very hard to hide seams
What is Subdivision

- Construct a surface from an arbitrary polyhedron
  - Subdivide each face of the polyhedron

- The limit will be a smooth surface

Subdivision Schemes
Subdivision Surfaces

Subdivision surface
(different levels of refinement)
Subdivision Schemes for Interactive Surface Modeling
Subdivision: Key Idea

• Approach limit curve through an iterative refinement process
Subdivision in 3D

Same approach works in 3D

Refinement
Subdivision Surfaces: Motivation

• How do we represent curved surfaces in the computer?
  – Efficiency of representation
  – Continuity
  – Affine invariance
  – Efficiency of rendering

• How do they relate to splines/patches?

• Why use subdivision rather than patches?
Subdivision Type

• **Interpolating schemes**
  – Limit surfaces/curve will pass through original set of data points.

• **Approximating schemes**
  – Limit surface will not necessarily pass through the original set of data points.
Subdivision in Production Environment

• Traditionally spline patches (NURBS) have been used in production for character animation.
• Difficult to control spline patch density in character modeling.

Subdivision in Character Animation
Tony Derose, Michael Kass, Tien Troung
(SIGGRAPH ’98)

(Geri’s Game, Pixar 1998)
Adaptive Subdivision for Rendering

- Not all regions of a model need to be subdivided.
- Idea: Use some criteria and adaptively subdivide mesh where needed.
  - Curvature
  - Screen size (make triangles < size of pixel)
  - View dependence
    - Distance from viewer
    - Silhouettes
    - In view frustum
  - Careful! Must ensure that “cracks” aren’t made

View-dependent refinement of progressive meshes
Hugues Hoppe.
(SIGGRAPH ’87)
Subdivision for Compression

Progressive Geometry Compression
Andrei Khodakovsky, Peter Schröder and Wim Sweldens
(SIGGRAPH 2000)
Subdivision Surfaces
Introduction

• History of subdivision.
• What is subdivision?
• Why subdivision?
History of Subdivision Schemes

Stage I: Create smooth curves from arbitrary mesh
- de Rham, 1947.
- Chaikin, 1974.

Stage II: Generalize splines to arbitrary topology

Stage III: Applied in high end animation industry

Stage IV: Applied in engineering design and CAD
Basic Idea of subdivision

• Start from an initial control polygon.
• Recursively refine it by some rules.
• A smooth surface (curve) in the limit.
Chaikin’s Corner Cutting Scheme
Chaikin’s Corner Cutting Scheme
Chaikin’s Corner Cutting Scheme
Chaikin’s Corner Cutting Scheme
Chaikin’s Algorithm

- A set of control points to define a polygon
  \[ p_0, p_1, p_2, \ldots, p_n \]

- Subdivision process (more control vertices)

- Rules (corner chopping)
  \[
  p_{2i}^{k+1} = \frac{3}{4} p_i^k + \frac{1}{4} p_{i+1}^k \\
  p_{2i+1}^{k+1} = \frac{1}{4} p_i^k + \frac{3}{4} p_{i+1}^k
  \]

- Properties:
  - quadratic B-spline curve, C1 continuous, tangent to each edge at its mid-point
Chaikin’s Algorithm
Chaiken’s Algorithm – Another Example

Apply Iterated Function System

Limit Curve Surface  Think Fractal!

\[ Q_{0} = \frac{1}{4} P_{0} + \frac{3}{4} P_{1} \]
\[ Q_{1} = \frac{3}{4} P_{0} + \frac{1}{4} P_{1} \]
\[ Q_{2} = \frac{1}{4} P_{1} + \frac{3}{4} P_{2} \]
\[ Q_{3} = \frac{3}{4} P_{1} + \frac{1}{4} P_{2} \]
\[ Q_{4} = \frac{1}{4} P_{2} + \frac{3}{4} P_{3} \]
\[ Q_{5} = \frac{3}{4} P_{2} + \frac{1}{4} P_{3} \]
Quadratic Spline
Cubic Spline
Cubic Spline

• **Subdivision rules**

\[ p_{2i}^{k+1} = \frac{1}{2} p_i^k + \frac{1}{2} p_{i+1}^k \]

\[ p_{2i+1}^{k+1} = \frac{1}{4} (\frac{1}{2} p_i^k + \frac{1}{2} p_{i+2}^k) + \frac{3}{4} p_{i+1}^k \]

• **C2 cubic B-spline curve**

• **Corner-chopping**

• **No interpolation**
Curve Interpolation

- Control points
  \[ p^0_{-2}, p^0_{-1}, p^0_0, \ldots, p^0_{n+2} \]

- Rules:
  \[
  \begin{align*}
  p^{k+1}_{2i} &= p^k_i, & -1 \leq i \leq 2^k n + 1 \\
  p^{k+1}_{2i+1} &= \left( \frac{1}{2} + w \right) (p^k_i + p^k_{i+1}) - w (p^k_{i-1} + p^k_{i+2}), \\
  &-1 \leq i \leq 2^k n
  \end{align*}
  \]

- At each stage, we keep all the OLD points and insert NEW points “in between” the OLD ones

- Interpolation!

- The behaviors and properties of the limit curve depend on the parameter \( w \)

- Generalize to SIX-point interpolatory scheme!
Curve Interpolation
Other Modeling Primitives

- Spline patches.
- Polygonal meshes.
Spline Patches

**Advantages:**
- High level control.
- Compact analytical representations.

**Disadvantages:**
- Difficult to maintain and manage inter-patch smoothness constraints.
- Expensive trimming needed to model features.
- Slow rendering for large models.
Polygonal Meshes

**Advantages:**

- Very general.
- Can describe very fine detail accurately.
- Direct hardware implementation.

**Disadvantages:**

- Heavy weight representation.
- A simplification algorithm is always needed.
Subdivision Schemes

Advantages:

• Arbitrary topology.
• Level of detail.
• Unified representation.

Disadvantages:

• Difficult for analysis of properties like smoothness and continuity.
Uniform/Semi-uniform Schemes

- **Catmull-Clark scheme**
  - *Catmull and Clark, CAD 1978*
- **Doo-Sabin scheme**
  - *Doo and Sabin, CAD 1978*
- **Loop scheme**
  - *Loop, Master’s Thesis, 1987*
- **Butterfly scheme**
  - *Dyn, Gregory and Levin, ACM TOG 1990*
- **Mid-edge scheme**
  - *Habib and Warren, SIAM on Geometric Design 1995*
- **Kobbelt scheme**
  - *Kobbelt, Eurographics 1996*
Classification

• **By Mesh type:**
  - Triangular  (Loop, Butterfly)
  - Quadrilateral  (Catmull-Clark, Doo-Sabin, Mid-edge, Kobbelt)

• **By Limit surface:**
  - Approximating  (Catmull-Clark, Loop, Doo-Sabin, Mid-edge)
  - Interpolating  (Butterfly, Kobbelt)

• **By Refinement rule:**
  - Vertex insertion  (Catmull-Clark, Loop, Butterfly, Kobbelt)
  - Corner cutting  (Doo-Sabin, Mid-edge)
Catmull-Clark Scheme

- **Face point:**
  the average of all the points defining the old face.

- **Edge point:**
  the average of two old vertices and two new face points of the faces adjacent to the edge.

- **Vertex point:**
  \[
  (F + 2E + (n-3)V) / n
  \]
  - \( F \): the average of the new face points of all faces adjacent to the old vertex.
  - \( E \): the average of the midpoints of all adjacent edges.
  - \( V \): the old vertex.
Catmull-Clark Scheme

Initial mesh

Step 1

Step 2

Limit surface
Modified Catmull-Clark

- Extend Cubic B-splines
  - Easier to implement with existing software
- Quadrilaterals are often better at capturing symmetry
  - Like human body parts
- Quads are convenient for cloth dynamics
Catmull-Clark Subdivision
Catmull-Clark Subdivision
Catmull-Clark Subdivision

(1) \[ e_j^{i+1} = \frac{v^i + e_j^i + f_{j-1}^{i+1} + f_j^{i+1}}{4}, \]

(2) \[ v^{j+1} = \frac{n-2}{n} v^i + \frac{1}{n^2} \sum_j e_j^i + \frac{1}{n^2} \sum_j f_j^{i+1} \]
Mid-edge Scheme
Mid-edge Scheme

(a)

(b)

(c)

(d)
Loop Scheme

- **Box splines**
  - A projection of 6D box onto 2D
  - A quartic polynomial basis function
  - Triangular domain

- Works on triangular meshes

- Is an approximating scheme

- Non-tensor-product splines

- Loop scheme results from a generalization of box splines to arbitrary topology

- Guaranteed to be smooth everywhere except at extraordinary vertices
Box Spline Overview

- Based on 2D Box Spline
  - Defined by projection of hypercube (in 6D) into 2D.
  - Satisfies many properties that B-spline has.
    - Recursive definition
    - Partition of unity
    - Truncated power
  - Natural splitting of a cube into sub-cubes provides the subdivision rule.
Basis Functions for Loop’s Scheme

- **Basis Function - Evaluation**

Assign unit weight to center, zero otherwise, over $\mathbb{Z}^2$ lattice

The Limit $\rightarrow N_{2,2,2}$ Basis
Loop’s Scheme Properties

• **Basis Function – Properties**

1. Support $\Rightarrow$ 2 neighbors from the center
2. $C^4$ continuity within the support
3. Piecewise polynomial
4. $N_{2,2,2}(\bullet - j), j \in Z^2$ form a partition of unity
   i.e. $\Sigma N(x - j) = 1$
Loop’s Scheme Rules

• The Rules
Loop Scheme Rules

\[ B = \frac{3}{8}k, \text{ for } n > 3 \]

\[ B = \frac{3}{16}, \text{ for } n = 3 \]
Loop Scheme Example
Butterfly Subdivision
Butterfly Scheme
Modified Butterfly Scheme

Initial mesh  One refinement step  Two refinement steps
Modified Butterfly Example
Modeling Sharp Features

- Corner
- Crease
- Dart
Piecewise Smooth Subdivision

Hoppe et al. Siggraph 94
Piecewise Smooth Surface

- Piecewise $C^1$-continuous extension [Hoppe 94]
  - Extension of the Loop’s scheme.
Non-uniform Subdivision Schemes

- **Piecewise smooth subdivision schemes**
  - *Hoppe et al. Siggraph 94*

- **Hybrid scheme**
  - *et al. Siggraph 98*

- **NURSS scheme**
  - *Sederburg et al. Siggraph 98*

- **Combined scheme**
  - *Levin Siggraph 99*

- **Edge and vertex insertion scheme**
  - *Habib et al. CAGD 99*
Hybrid Subdivision Scheme

(a)  
(b)  
(c)  
(d)  

DeRose et al.  
Siggraph 98
Sharp Edges

1. Tag Edges as “sharp” or “not-sharp”
   - $n = 0$ – “not sharp”
   - $n > 0$ – sharp

During Subdivision,

2. if an edge is “sharp”, use sharp subdivision rules. Newly created edges, are assigned a sharpness of $n-1$.

3. If an edge is “not-sharp”, use normal smooth subdivision rules.

IDEA: Edges with a sharpness of “n” do not get subdivided smoothly for “n” iterations of the algorithm.
Non-Integer Sharpness

- Density of newly generated mesh increases rapidly.
- In practice, 2 or 3 iterations of subdivision is sufficient.
- Need better “control”.

**IDEA:** Interpolate between smooth and sharp rules for non-integer sharpness values of n.
Hierarchical Editing

Zorin et al. Siggraph 97
Surface Reconstruction

Hoppe et al. Siggraph 94
Local Subdivision Schemes

- Complex data structures required to perform subdivision.
  - Every polygon (triangle, quad,..) must know its neighbors
  - Every vertex must know its neighbors
- Can we do something simpler?
  - Use vertex normal information to help “guess” about neighboring polygons.
  - Subdivide based on the normals.
Local Subdivision (PN Triangles)

- Defined from “triangular bezier” patches.

\[ u,v,w \] are barycentric coordinates
\[ w = 1 - u - v, \quad u,v,w \geq 1 \]

\[
b(u, v) = \sum_{i+j+k=3} b_{ijk} \frac{3!}{i! j! k!} u^i v^j w^k
\]

Bezier basis function

Curved PN Triangles

Alex Vlachos  Joerg Peters  Chas Boyd  Jason Mitchell
Computing the Control Mesh

\[ b_{300} = P_1 \]
\[ b_{030} = P_2 \]
\[ b_{003} = P_3 \]
\[ w_{ij} = (P_j - P_i) \cdot N_i \]
\[ b_{210} = \left(2P_1 + P_2 - w_{12}N_1 \right) / 3 \]
\[ b_{120} = \left(2P_2 + P_1 - w_{21}N_2 \right) / 3 \]
\[ b_{021} = \left(2P_2 + P_3 - w_{23}N_2 \right) / 3 \]
\[ b_{012} = \left(2P_3 + P_2 - w_{32}N_3 \right) / 3 \]
\[ b_{102} = \left(2P_3 + P_1 - w_{31}N_3 \right) / 3 \]
\[ b_{201} = \left(2P_1 + P_3 - w_{13}N_1 \right) / 3 \]
\[ E = \left(b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201}\right) / 6 \]
\[ V = \left(P_1 + P_2 + P_3 \right) / 3 \]
\[ b_{111} = E + \left(E - V \right) / 2 \]
PN Triangles

- Interpolating Scheme.
- Example..
Local Subdivision

• **Advantages**
  – Easy to implement
    • No complex data structures
  – Easy to integrate into existing graphics applications
  – Hardware amenable
  – Looks good

• **Disadvantages**
  – No guarantees on higher level continuity.
  – Is limited in the amount of curvature it can provide.
  – In some sense it is a hack and not as “correct”.

Subdivision as Matrices

- Subdivision can be expressed as a matrix $S_{mask}$ of weights $w$.
  - $S_{mask}$ is very sparse
  - *Never Implement this way!*
  - Allows for analysis
    - Curvature
    - Limit Surface

$$S_{mask}P = \hat{P}$$

$S_{mask}$ Weights  Old Control Points  New Points
What about Continuity

- Subdivision mask weights $w$ are derived from splines, such as B-Splines.
  - Subdivision surfaces converge to spline surfaces with $C^2$ continuity everywhere.**
  - Too lengthy to cover here, but there is lots of literature.

**Math works out except at “Extraordinary Vertices”.

Most Subdivision Schemes have and “ideal” valence for which it can be shown that the limit surface will converge to a spline surface.
Comparison

- Catmull-Clark yields the nicest surface.
- Loop is more asymmetric.
- Mod. Butterfly is the worst.
Comparison

- Extreme shrink for Loop and Catmull-Clark.
Comparison

Midedge

Doo-Sabin

• The increasing shrinkage with increasing smoothness.

Biquadric
• Similar results
• Interpolating schemes are sensitive to the presence of sharp features, and may produce low quality surfaces unless the initial mesh is smooth enough.
Comparison

Initial mesh | Loop | Catmull-Clark | Catmull-Clark, after triangulation
Comparison

– Loop and Catmull-Clark appear to be the best choices for most applications.
  • Loop seems to be more reliable.

– Quadrilateral scheme
  • Natural texture mapping for quads.
  • Natural number of symmetries?

– Curvature continuity
  • No $C^1$ with small support.
Subdivision

– Pro
  • No Trimming
  • Connectivity and Smoothness Guaranteed

– Con
  • Not much studies like NURBS

Subdivision Surface = 

<table>
<thead>
<tr>
<th></th>
<th>polygons</th>
<th>B-splines</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ flexible</td>
<td>-faceted</td>
<td>-restrictive</td>
</tr>
<tr>
<td>-faceted</td>
<td>+smooth</td>
<td></td>
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</tbody>
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“Geri’s Game”

Subdivision Surfaces in the Making of Geri’s Game

DeRose et al. Siggraph 98
Subdivision in Production Environment

- Traditionally spline patches (NURBS) have been used in production for character animation.
- Difficult to control spline patch density in character modeling.

Subdivision in Character Animation
Tony Derose, Michael Kass, Tien Troung
(SIGGRAPH ’98)

(Geri’s Game, Pixar 1998)
Gery’s Head
Catmull-Clark Surface Modeling

- Subdivision produces smooth continuous surfaces.
- How can “sharpness” and creases be controlled in a modeling environment?

**ANSWER:** Define new subdivision rules for “creased” edges and vertices.

1. Tag Edges sharp edges.
2. If an edge is sharp, apply new sharp subdivision rules.
3. Otherwise subdivide with normal rules.
Modeling Fillets and Blends

- **Infinitely sharp creases**

  Without Crease

  ![Image without crease](image1.png)

  With Crease

  ![Image with crease](image2.png)
Semi-sharp Creases

- Modify averaging rules
- Hybrid subdivision
Integer Sharpness $s$

- Subdivide $s$ times using sharp rules
- Use smooth rules to the limit surface

Example $s = 2$
Variable Sharpness

Model courtesy of Jason Bickerstaff
Texturing

Scalar Fields provide texture coordinates.

\[ S(s,t) = (x(s,t), y(s,t), z(s,t)) \]
Texturing

- Specify parameters independent of subdivision level
- Assign parameters at control vertices.
- Subdivide using same rules.
- Interpolating using Laplacian smoothing or Painting an intensity map
Implementation Issues

– Subdivision surfaces now implemented in RenderMan.

– Regular mesh regions -> B-splines.

– Using B-splines allows
  • Efficiency in memory usage
  • Reduce the total amount of splitting
  • Forward algorithms are available to dice B-spline patches

– An advantage of semi-sharp creases
  • Never tear
• Pixar Developments make subdivision surfaces very practical and useful

• Subdivision > NURBS
  – More control, accuracy
  – Time saved, To be refined locally
  – Remove two obstacles by developing semi-sharp creases and scalar fields
  – An efficient data structure and cloth energy function well suited to physical and cloth simulation

• Now part of Renderman
Subdivision Splines

- We treat subdivision as a novel method to produce spline-like models in the limit.
- Key components for spline models:
  - Control points, basis functions over their parametric domain, parameterization, piecewise decomposition
- Parameterization is done naturally via subdivision.
- The initial control mesh serves as the parametric domain.
- Basis functions are available for regular settings as well as irregular settings.
- Control points for one patch are in the vicinity of its parametric domain from its initial control vertices.
- Subdivision-based spline formulation is fundamental for physics-based geometric modeling and design, finite element analysis, simulation, and the entire CAD/CAM processes.
Chaikin Curve Example
Interpolation Curve Example
Parameterization
Butterfly Surface Example
Control Vertices for Butterfly Surface
Control Vertices for Surface Patches
Butterfly Patches
Butterfly Basis Function
Catmull-Clark Surface Example
Catmull-Clark Patches
Catmull-Clark Basis Function
Simple Sculpting Examples

original object | deformation | cutting

extrusion | fixed regions

extrusion | fixed regions

CSE528 Lectures
Chair Example --- Finite Element Simulation

Initial control lattice

Finite element structure after a few subdivisions

Deformed object

Photo-realistic rendering
Sculpting Tools

- carving
- extrusion
- detail editing
- joining
- sharp features
- deformation
Sculpting Tools

- **inflation**
  - Material mapping

- **deflation**
  - Material probing

- **curve-based design**
  - Physical window

**CSE528 Lectures**
Sculpting Tools

- Pushing
- Sweeping
- Curve-based Join
- Curve-based Cutting
- Feature Deformation
- Multi-face Extrusion
Interactive Sculpting
More Examples
Volume Editing and Visualization

original lattice

deformed lattice

original volume

deformed volume
Sculpted CAD Models
Subdivision Solids

Fig. 22. Jet engine model comprised of two disconnected parts.

(a) (b) (c)

Fig. 23. Material properties can be interpolated smoothly throughout the entire volumetric domain. (a) Control mesh with color. (b-c) Model after three levels of subdivision.
Scenes and Sculptures
Other Applications
Rendering – Adaptive Tessellation
Meshless Geometric Subdivision
Point-based Graphics

- Core: unstructured point cloud
- Points with attributes:
  - color, normal, etc.
- Advantages:
  - acquisition
  - multiresolution
  - storage
- Drawback: meshing + visualization
Modeling + Visualization enabled by Subdivision Surface Fitting