Triangular Bezier Patch

- Triangular Bezier Surface

$$\mathrm{s}(u,v) = \sum_{i,j,k>=0} \mathbf{p}_{i,j,k} B^n_{i,j,k}(r,s,t),$$

where $i+j+k=n$, $r+s+t=1$, and they are local barycentric coordinates

- Basis functions are Bernstein polynomials of degree n

$$B^n_{i,j,k}(r,s,t) = \frac{n!}{i!j!k!} r^i s^j t^k$$

- How many basis functions: $\frac{1}{2}(n+1)(n+2)$

- How many control points: $\frac{1}{2}(n+1)(n+2)$

- r,s,t: local barycentric coordinates in the uv-plane

- Partition of unity

$$\sum_{i,j,k>=0} B^n_{i,j,k}(r,s,t) = 1$$

- **Positivity**

$$B^n_{i,j,k}(r,s,t) >= 0, r, s, t \in [0,1]$$

- **Recursion**

$$B^n_{i,j,k}(r,s,t) = rB^{n-1}_{i-1,j,k} + sB^{n-1}_{i,j-1,k} + tB^{n-1}_{i,j,k-1}$$

- **Recursive evaluation**

$$\mathbf{p}^0_{i,j,k} = \mathbf{p}_{i,j,k}$$

$$\mathbf{p}^l_{i,j,k} = r\mathbf{p}^{l-1}_{i+1,j,k} + s\mathbf{p}^{l-1}_{i,j+1,k} + t\mathbf{p}^{l-1}_{i,j,k+1}$$

where $i+j+k=n-l$**, and** $i,j,k >= 0$

$$\mathbf{s}(u,v) = \mathbf{p}^n_{0,0,0}$$

(refer to the diagram)

- **Efficient algorithms**

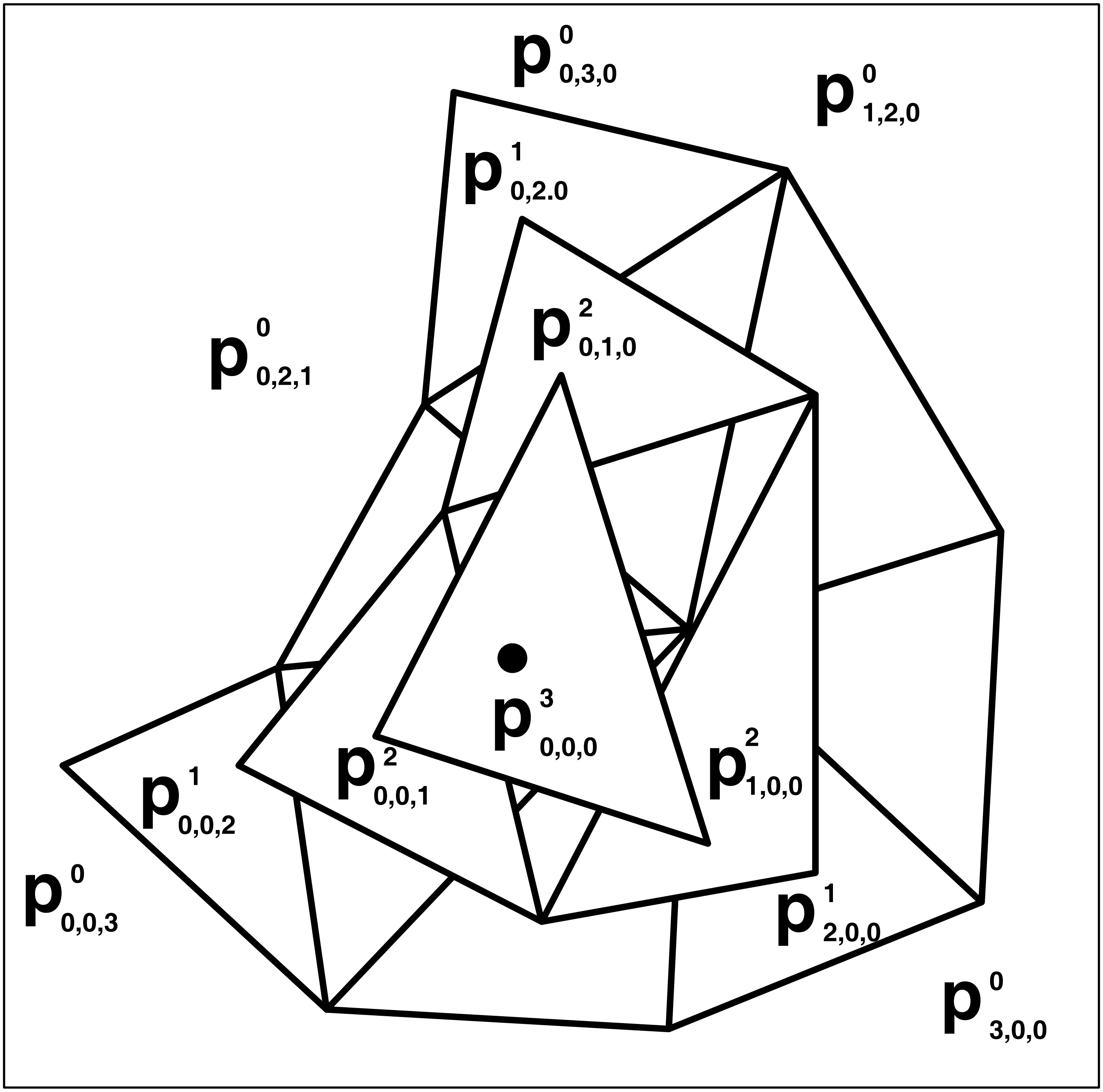
- **(Directional) derivatives**

- **Degree elevation**

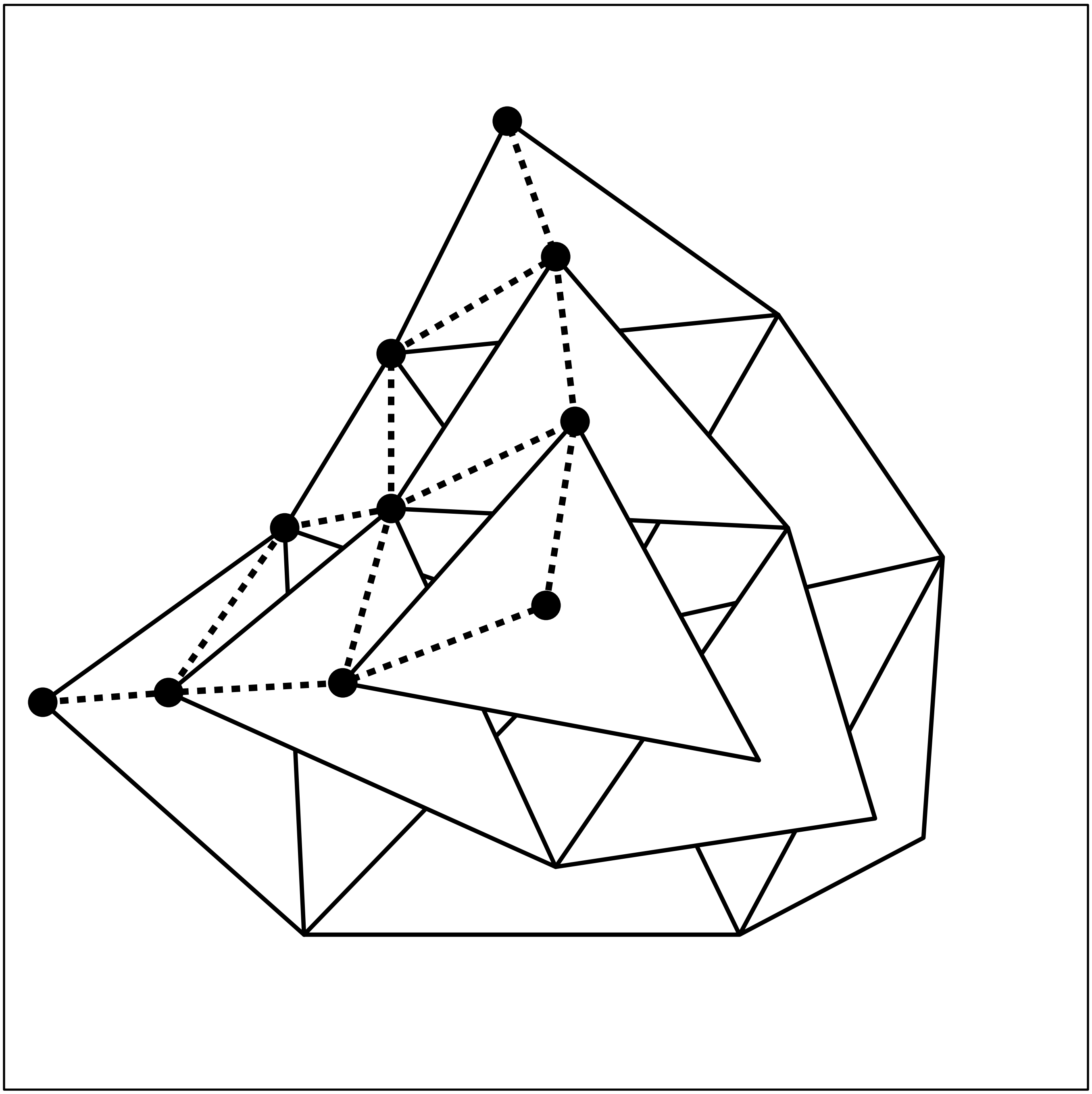
- **Subdivision**

- Composite surfaces
- Continuity across adjacent patches
- Integral computation
- Triangular splines over regular triangulation
- Transform triangular splines to a set of piecewise triangular Bezier patches
- Interpolation/approximation using triangular splines

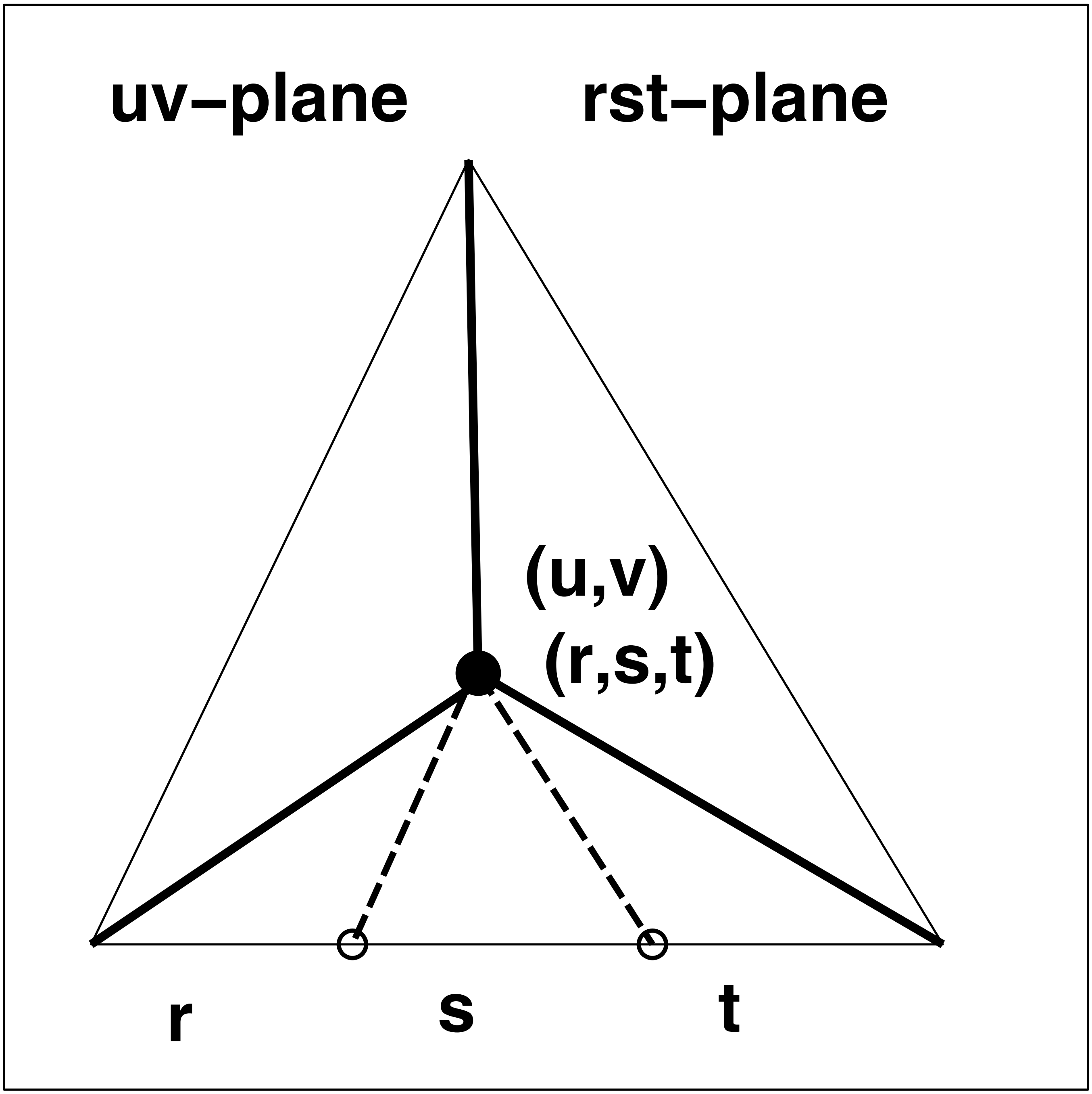
Recursive Evaluation



Triangular Subdivision



Triangular Domain



Basis functions (Cubic)