Subdivision: Its Past, Present, and Future

Hong Qin
Department of Computer Science
State University of New York at Stony Brook
Tel: (631) 632-8450; Fax: (631) 632-8334
qin@cs.sunysb.edu
http://www.cs.sunysb.edu/~qin

Course Overview

- Fundamental idea and simple examples
- Motivation for subdivision
- Historic review on existing techniques
  - Curves, surfaces
  - Comparison
- Subdivision splines
- Current work on subdivision solids
- On-going research topics and possible future research directions (if time permits)

Course Overview

- Fundamental idea and simple examples
- Motivation for subdivision
- Historic review on existing techniques
  - Curves, surfaces
  - Comparison
- Subdivision splines
- Current work on subdivision solids
- On-going research topics and possible future research directions (if time permits)

Chaikin's Algorithm

- A set of control points to define a polygon

$$\mathbf{p}_0^0, \mathbf{p}_1^0, \mathbf{p}_2^0, \ldots, \mathbf{p}_n^0$$

- Subdivision process (more control vertices)
- Rules (corner chopping)

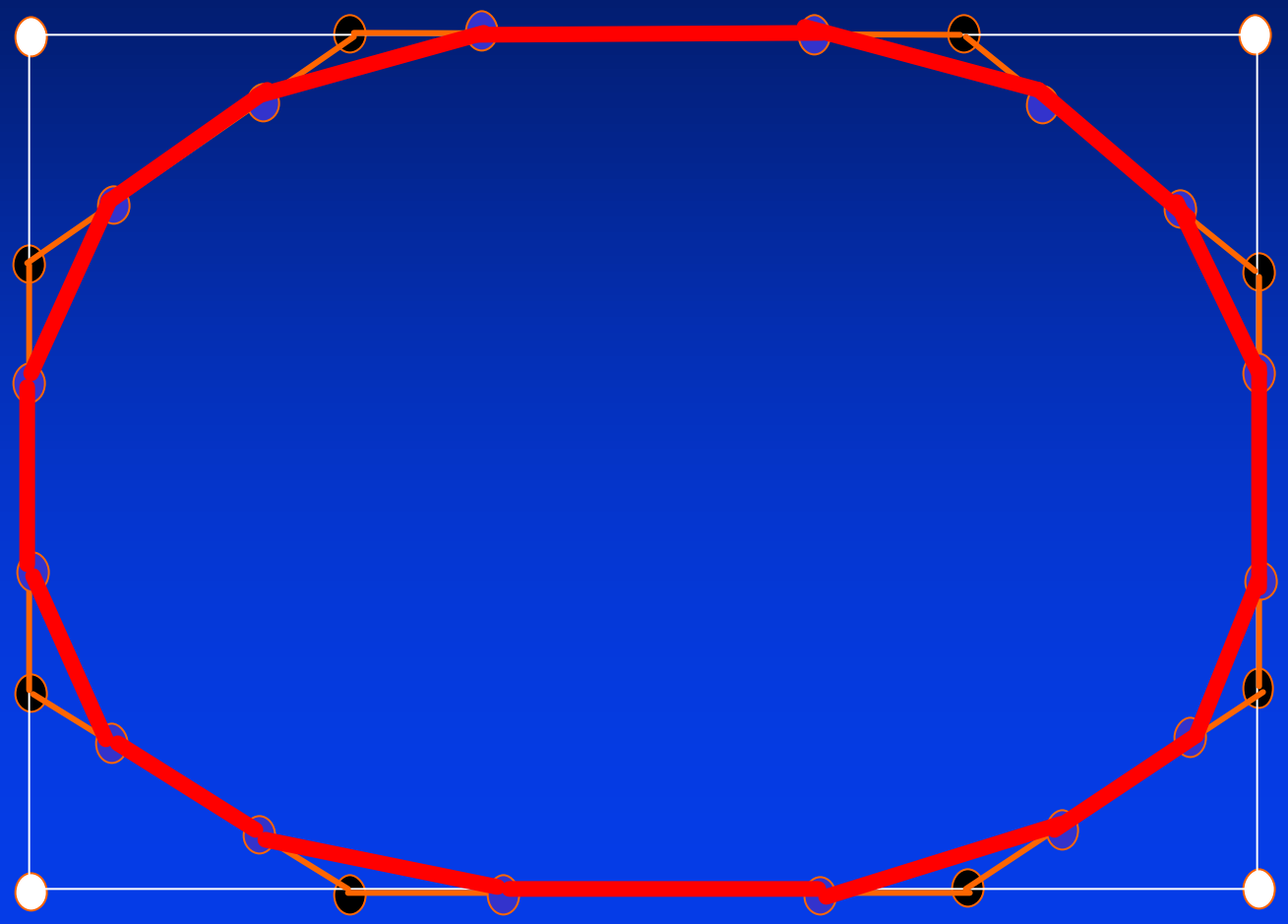
$$\mathbf{p}_{2i}^{k+1} = \frac{3}{4}\mathbf{p}_i^k + \frac{1}{4}\mathbf{p}_{i+1}^k$$

$$\mathbf{p}_{2i+1}^{k+1} = \frac{1}{4}\mathbf{p}_i^k + \frac{3}{4}\mathbf{p}_{i+1}^k$$

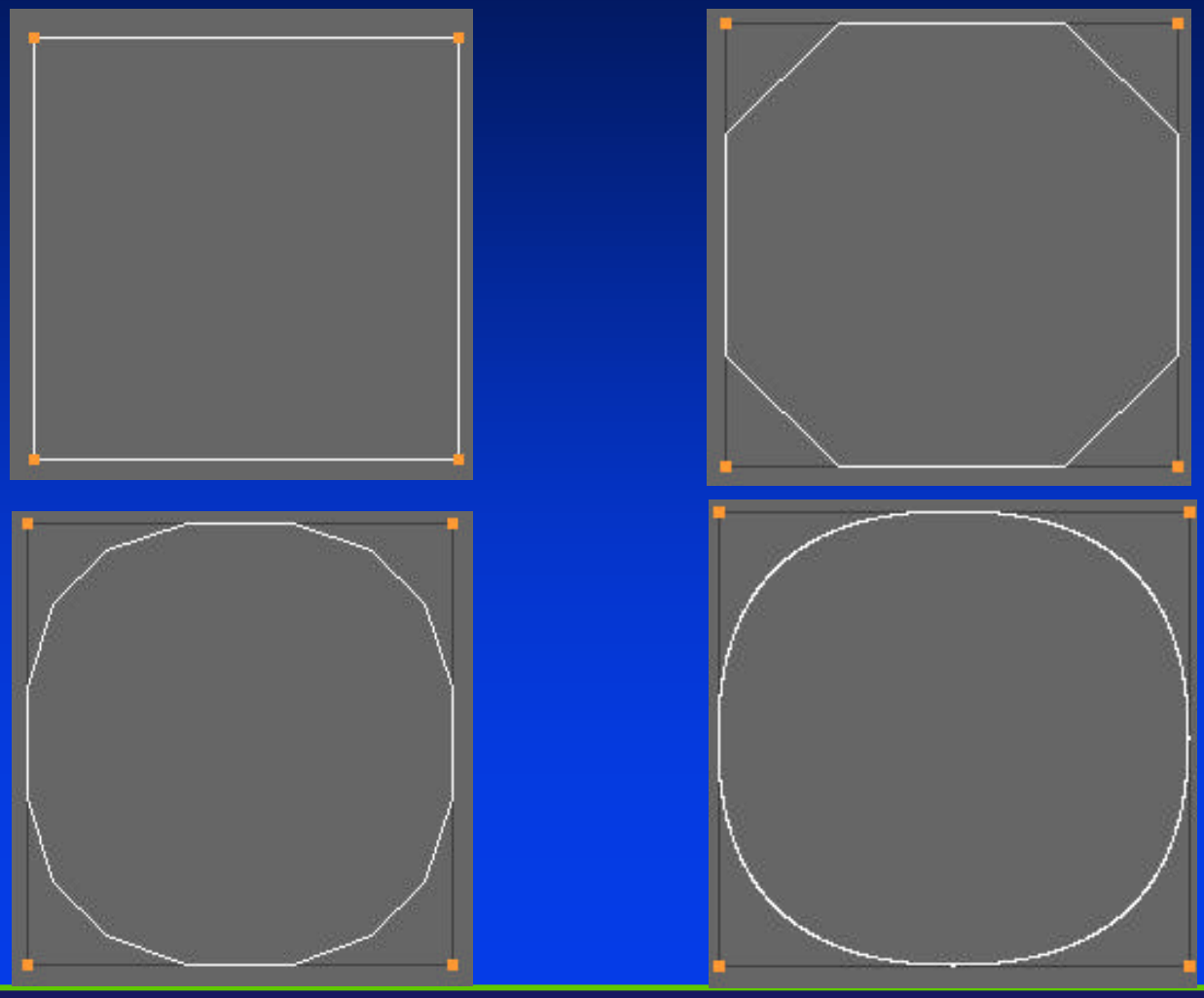
$$\mathbf{p}_0^k, \mathbf{p}_1^k, \mathbf{p}_2^k, \ldots \mathbf{p}_{2^k n}^k$$

- Properties:
  - quadratic B-spline curve, C1 continuous, tangent to each edge at its mid-point

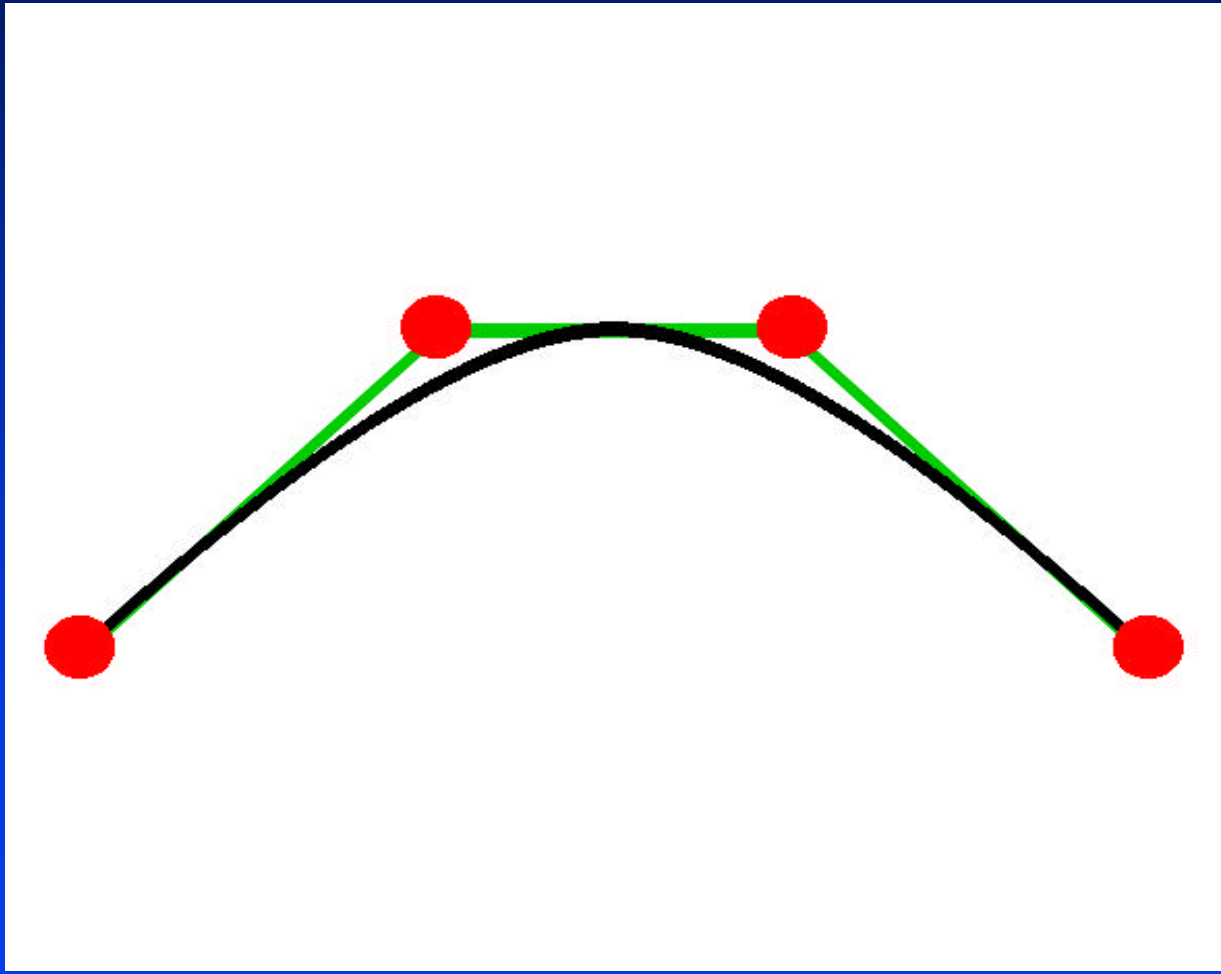
Chaikin's Algorithm



B-spline via Subdivision



Quadratic Spline



Cubic Spline

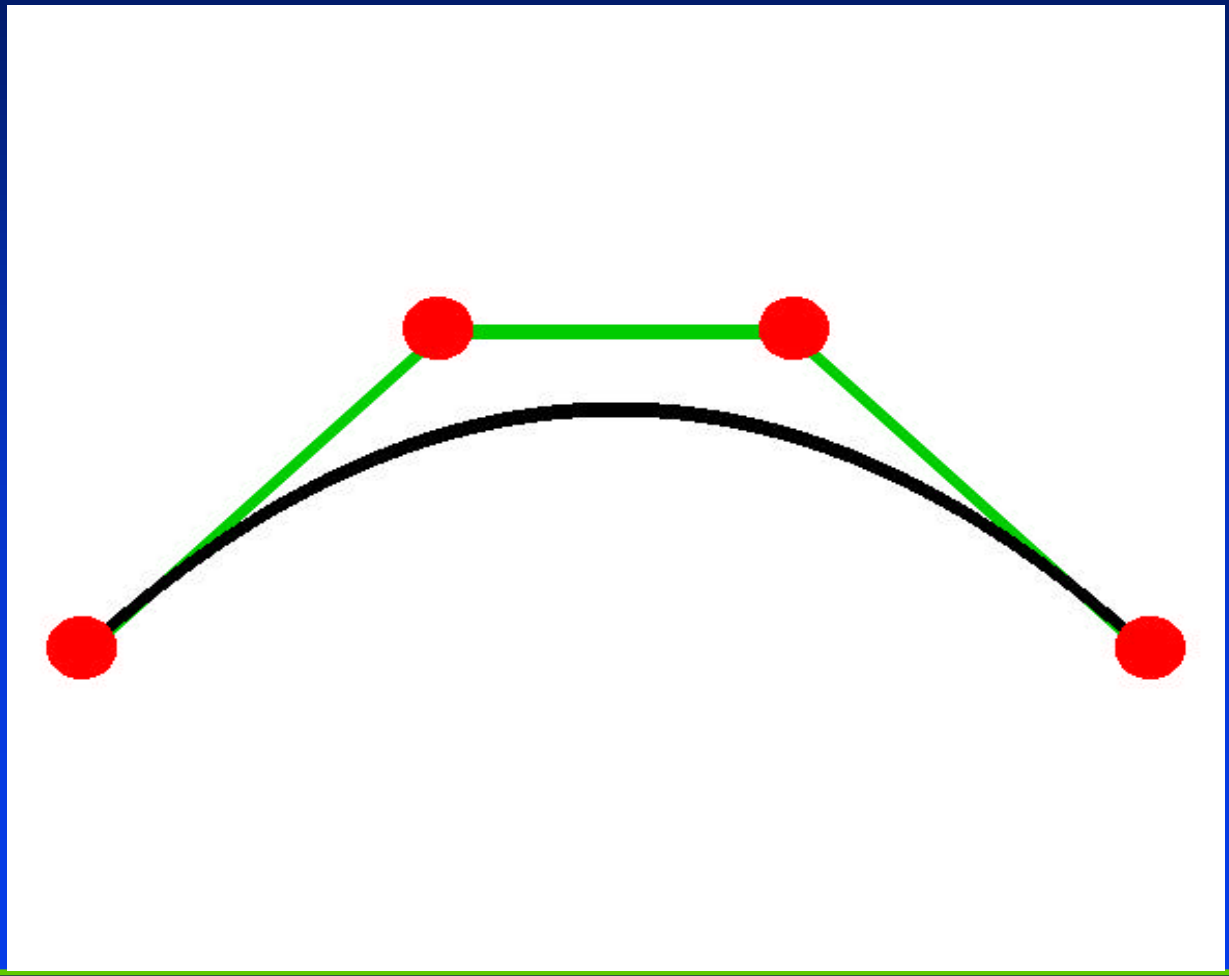
- Subdivision rules

$$\mathbf{p}_{2i}^{k+1} = \frac{1}{2}\mathbf{p}_i^k + \frac{1}{2}\mathbf{p}_{i+1}^k$$

$$\mathbf{p}_{2i+1}^{k+1} = \frac{1}{4}\left(\frac{1}{2}\mathbf{p}_i^k + \frac{1}{2}\mathbf{p}_{i+2}^k\right) + \frac{3}{4}\mathbf{p}_{i+1}^k$$

- C2 cubic B-spline curve
- Corner-chopping
- No interpolation

Cubic Spline



Curve Interpolation

- Control points $\mathbf{p}_{-2}^{0}, \mathbf{p}_{-1}^{0}, \mathbf{p}_{0}^{0}, \ldots \mathbf{p}_{n+2}^{0}$
- Rules:

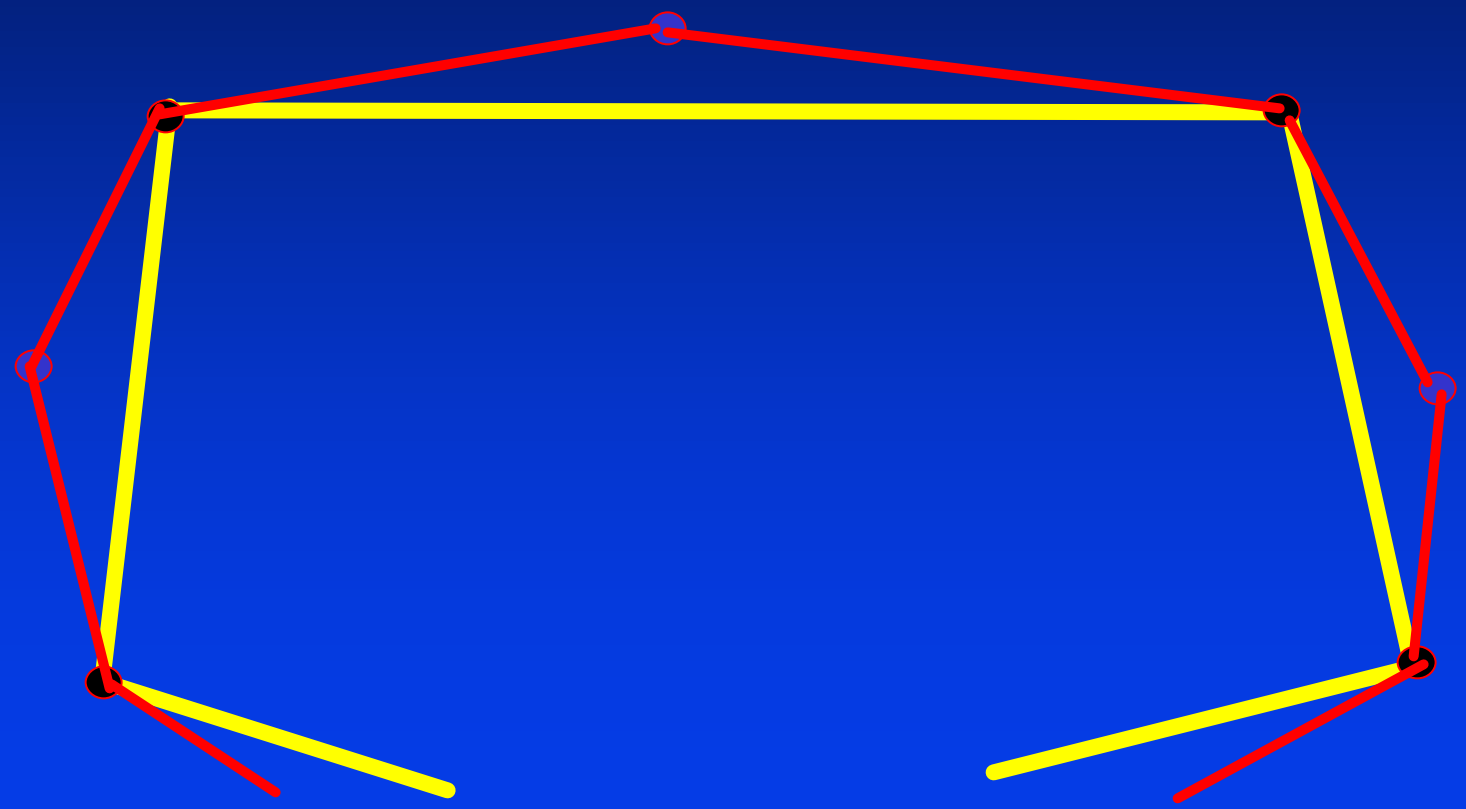
$$\mathbf{p}_{2i}^{k+1} = \mathbf{p}_{i}^{k}, -1 \le i \le 2^{k} n + 1$$

$$\mathbf{p}_{2i+1}^{k+1} = \left(\frac{1}{2} + w\right)\left(\mathbf{p}_{i}^{k} + \mathbf{p}_{i+1}^{k}\right) - w\left(\mathbf{p}_{i-1}^{k} + \mathbf{p}_{i+2}^{k}\right),$$

$$-1 \le i \le 2^{k} n$$

- At each stage, we keep all the OLD points and insert NEW points "in between" the OLD ones
- Interpolation!
- The behaviors and properties of the limit curve depend on the parameter w
- Generalize to SIX-point interpolatory scheme!

Curve Interpolation



Basic Idea of Subdivision

- Start from an initial "control polygon" or "control polyhedron" and subdivide
- Apply procedural rules (take weighted averages) to geometrically round off objects (recursive refinement)
- A smooth curve (surface) is generated in the limit of this recursive subdivision process
- Limit object may approximate (lie near) or interpolate (pass through) the control structure
- Can represent shapes of arbitrary topology

Subdivision Curve

- Chaikin's algorithm is "corner cutting"
- Subdivision started as a rendering technique, NOT as a powerful modeling tool as we are perceiving at present!
- From polygon to smooth curve
- Simple rules --- affine combination
- Generalization
  - Other rules for curve generation
  - From polyhedron to smooth surface

Subdivision Surface

- From polyhedron to smooth surface

  (1) a polyhedral network of vertices, edges, and faces

  (2) apply a set of refinement rules on the current control mesh

  (3) a new polyhedron with more and smaller faces/edges and more vertices than the original one

  (4) recursive procedure for (2)-(3)

  (5) smooth surface in the limit

Catmull-Clark Subdivision

- NEW face points --- the average of all OLD points defining the face
- New edge points --- the average of the following four points: (i) two OLD vertices defining the edge + (ii) two NEW face points of the faces adjacent to the edge
- New Vertex points --- from the average:

$$\frac{F}{n} + \frac{2E}{n} + \frac{(n-3)V}{n}$$

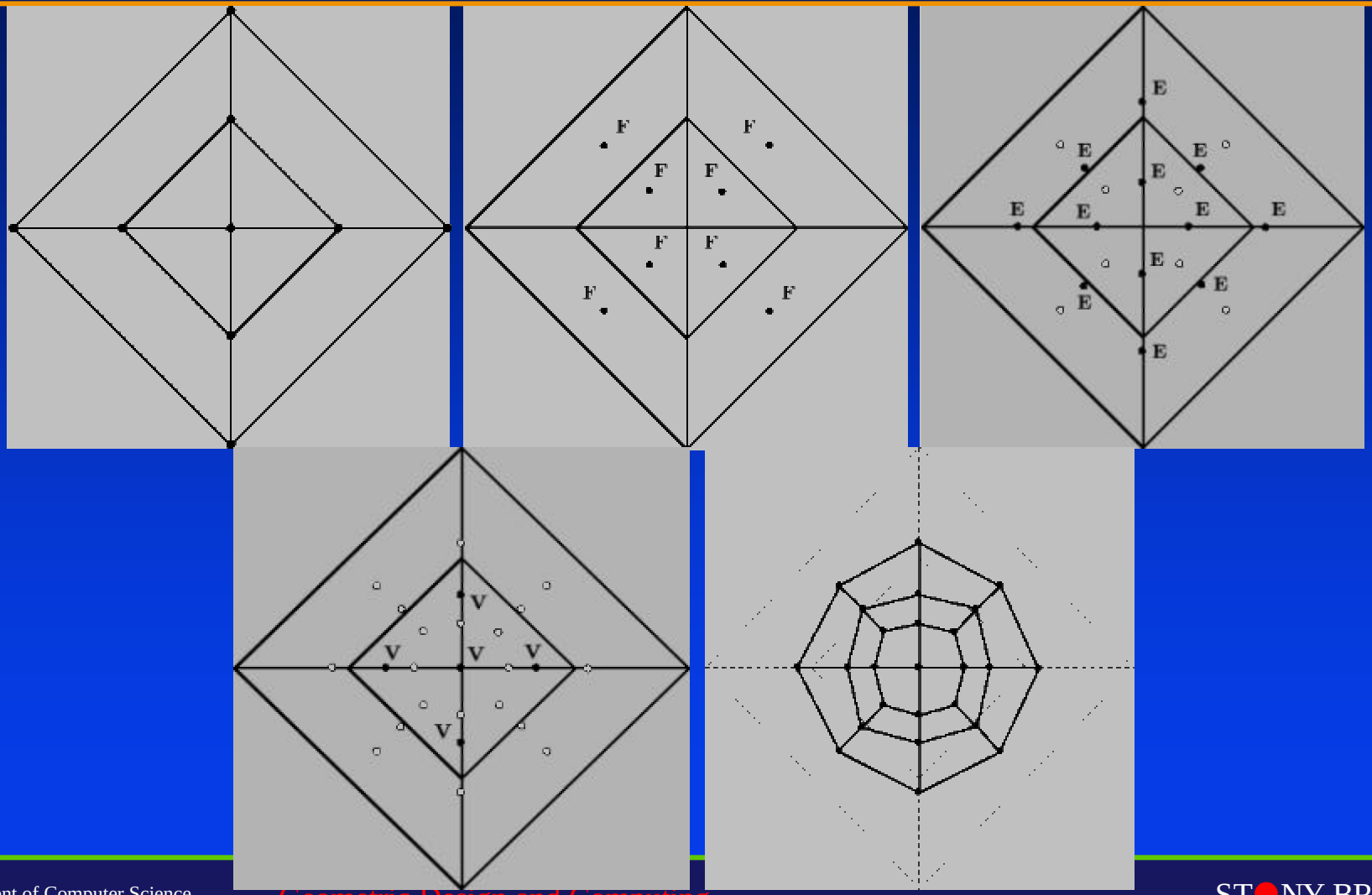
F: the average of the NEW face points of all faces adjacent to the OLD vertex point

E: the average of the midpoints of all edges incident on the OLD vertex

V: OLD vertex point

n: the number of the edges incident on the vertex

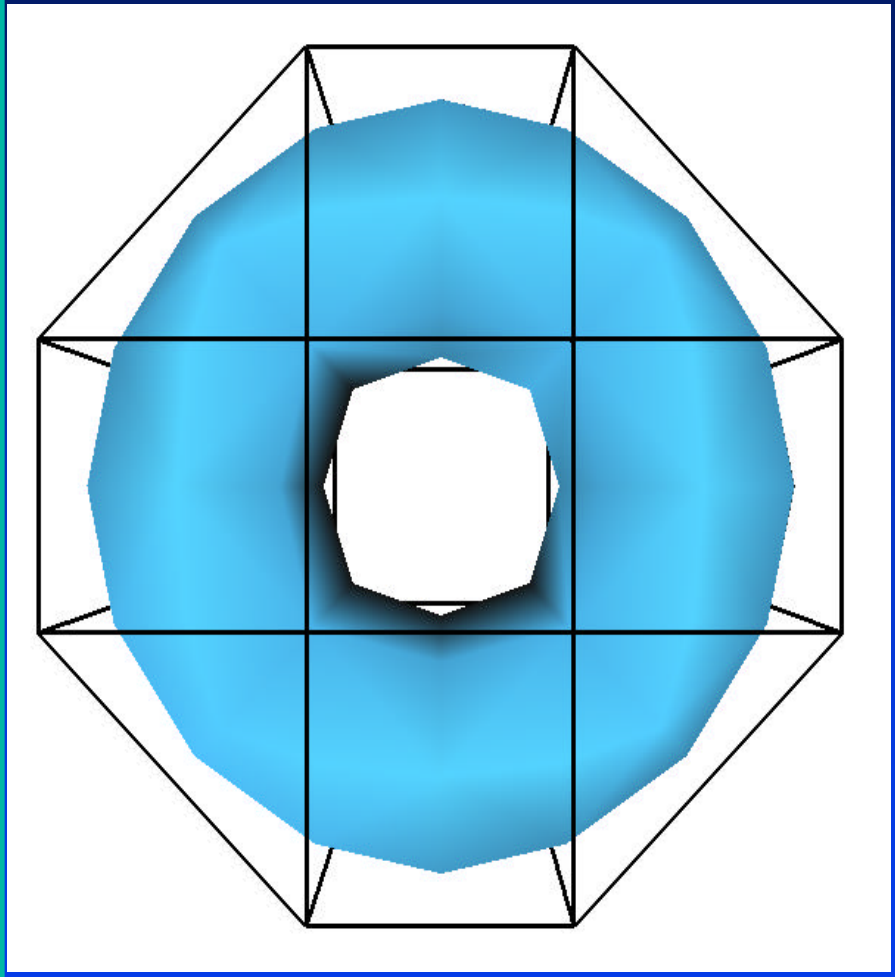
Catmull-Clark Surface

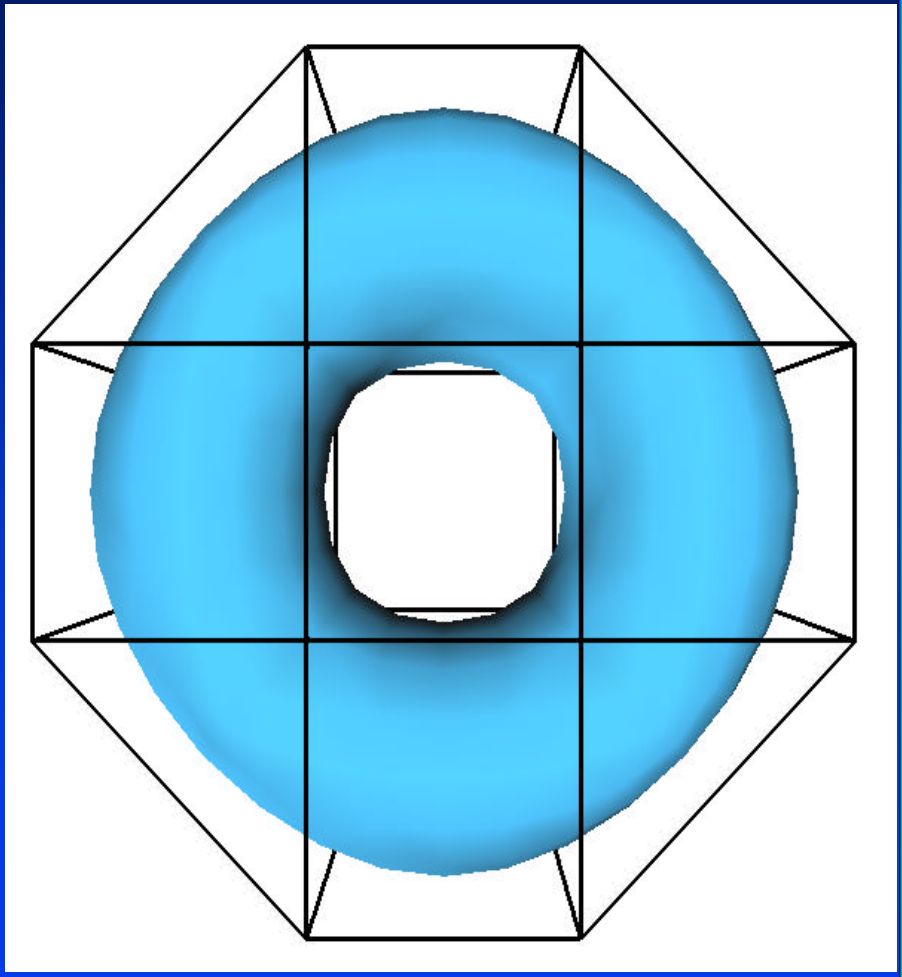


Catmull-Clark Rules

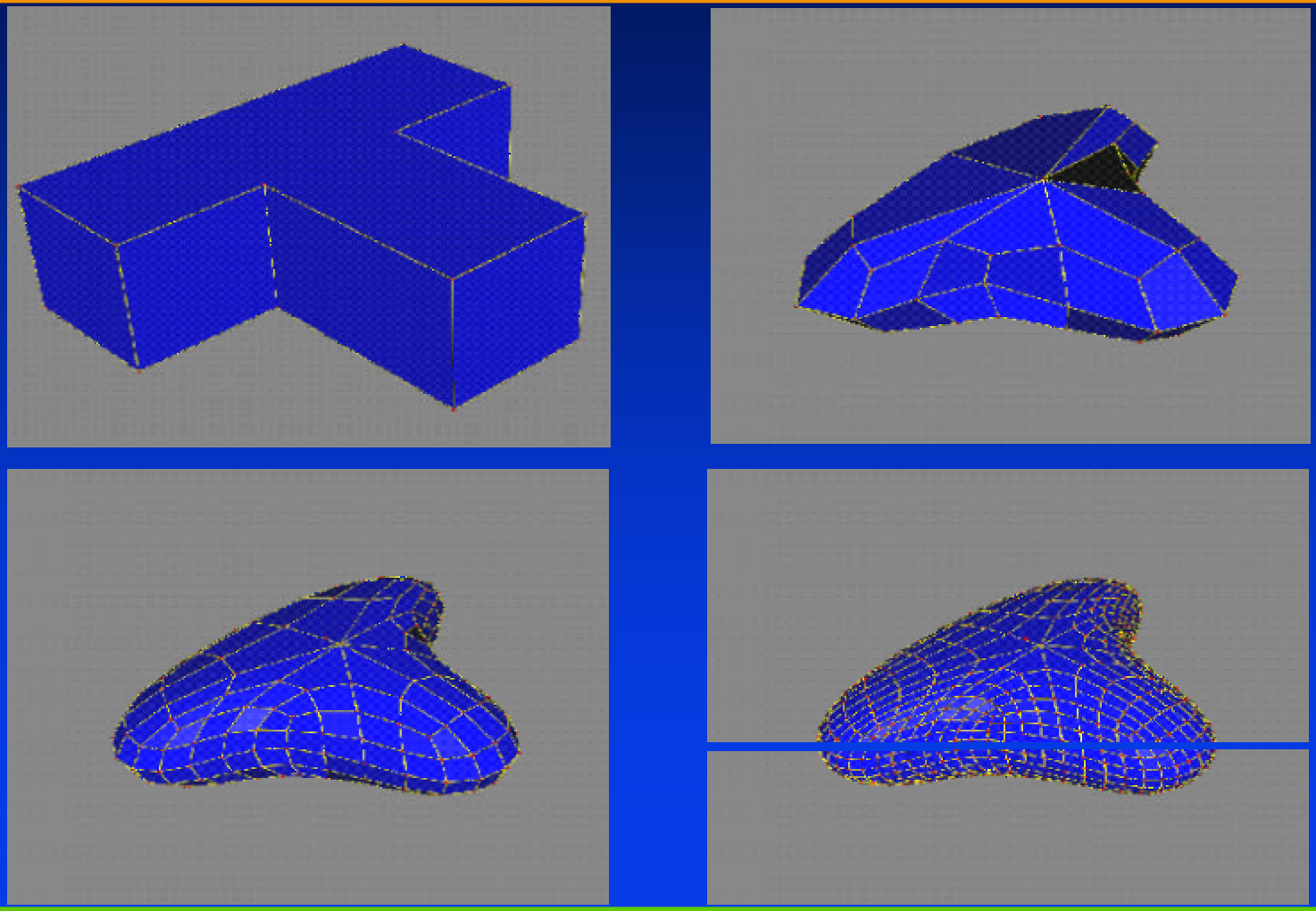
- NEW edge
  - (NEW face point, NEW edge point)
  - (NEW vertex point, NEW edge point)
- NEW face
  - Enclosed by NEW edges

Catmull-Clark Subdivision





Catmull-Clark Subdivision



Catmull-Clark Properties

- Rectangular meshes lead to B-spline surface
- Non-rectangular meshes lead to (1) a set of B-spline patches, (2) extraordinary points
- A finite number of extraordinary points
- Curvature-continuous except at extraordinary points
- Tangent-plane-continuity at extraordinary points
- No global parametric domains!
- Geometric objects of arbitrary topology

Course Overview

- Fundamental idea and simple examples
- Motivation for subdivision
- Historic review on existing techniques
  - Curves, surfaces
  - Comparison
- Subdivision splines
- Current work on subdivision solids
- On-going research topics and possible future research directions (if time permits)

Parametric Splines

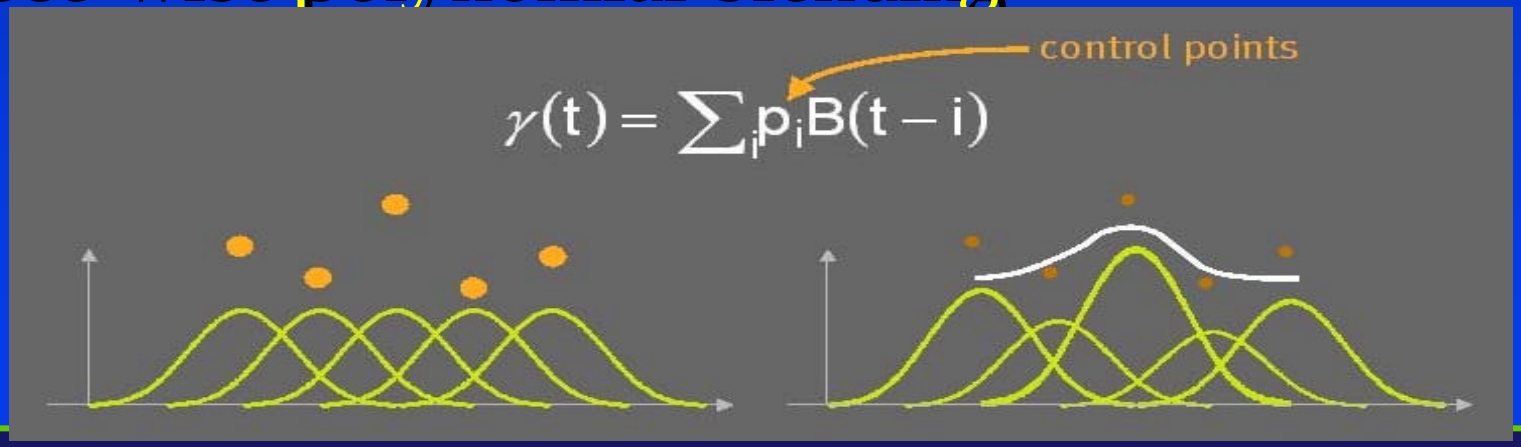
- Parametric curve functions

$$x = x(u), y = y(u), z = z(u)$$

- Parametric surface functions

$$x = x(u,v), y = y(u,v), z = z(u,v)$$

- Piece-wise polynomial blending



Relevant Primitives

- Splines
  - High-level control
  - Compact analytical representations
  - Difficult to maintain and manage inter-patch smoothness constraints
  - Expensive trimming needed to model features
  - Slow rendering for large models
- Polygonal meshes
  - Very general
  - Can describe very fine detail accurately
  - Direct hardware implementation
  - Heavy, massive representation
  - A simplification algorithm is always needed

Concise Summary

- Subdivision advantages:
  - Arbitrary topology
  - Hierarchical structure, level of detail control, scalability
  - Unified representation for splines and meshes, uniformity
  - Numerical quality
  - Code simplicity
- Disadvantages:
  - Difficult for analysis of properties like smoothness and continuity

Course Overview

- Fundamental idea and simple examples
- Motivation for subdivision
- Historic review on existing techniques
  - Curves, surfaces
  - Comparison
- Subdivision splines
- Current work on subdivision solids
- On-going research topics and possible future research directions (if time permits)

Background

- How NEW vertices are constructed DIFFERS tremendously!
- A large variety of subdivision rules
  - Sabin & Doo approach
  - Butterfly scheme
  - Loop scheme
- Classification of subdivision schemes
  - Stationary schemes
  - Uniform/semi-uniform, non-uniform schemes
- Analysis of stationary schemes
  - Subdivision matrix, formula for limit position and normal

Prior Work on Subdivision

- Curves
  Chaikin ’74; Dyn et al. ’86, ’87, ’88
- Surfaces
  Catmull and Clark ’78; Doo and Sabin ’78;
  Loop ’87; Dyn ’90; Kobbelt ’96; Lounsbery ’94;
  Welch and Witkin ’92; Zorin ’96; DeRose ’98;
  Sederberg et al ‘98; Stam ’98; Levin ’99
- Solids
  MacCracken and Joy ’96 (*but*, for free-form deformation!)

and many more!

Historical Perspectives

Phase I: fundamental ideas, create smooth curves from arbitrary mesh

- de Rham, 1947
- Chaikin, 1974

Phase II: generalize spline surfaces to arbitrary topology

- Catmull and Clark, 1978
- Doo and Sabin, 1978

Phase III: extensive applications in animation and entertainment industry

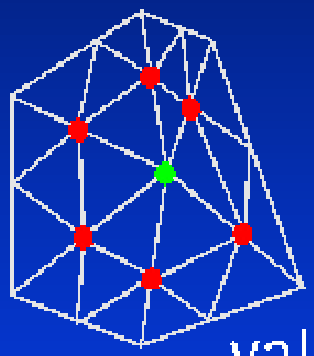
- Pixar Studio, "Geri's Game",1998

Phase IV: serious scientific tools in CAGD, CAD, and CIM

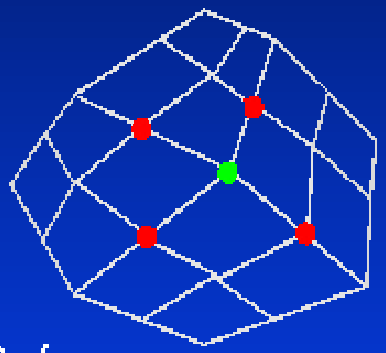
Extraordinary Vertices

Triangular meshes

Quad meshes

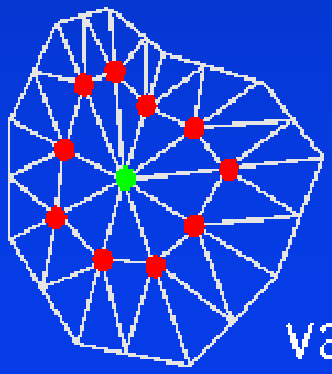


regular

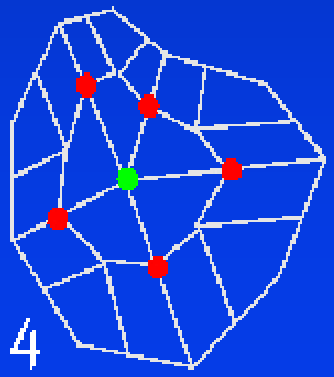


valence 6

valence 4



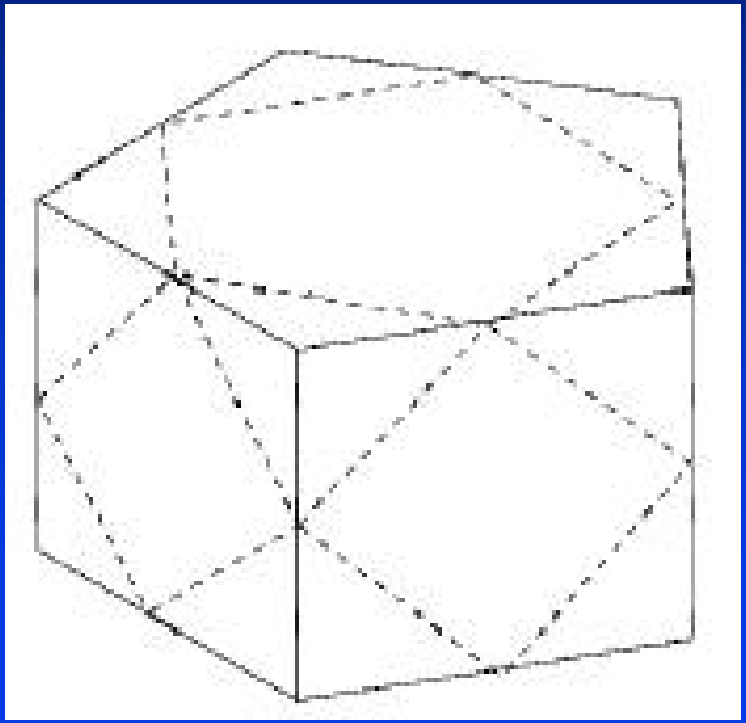
extraordinary



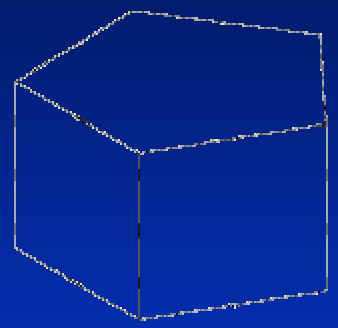
valence $\neq 6$

valence $\neq 4$

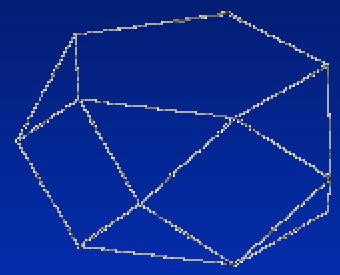
Mid-edge Scheme



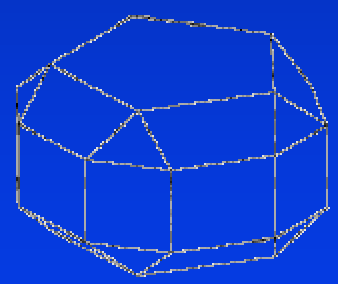
Mid-edge Scheme



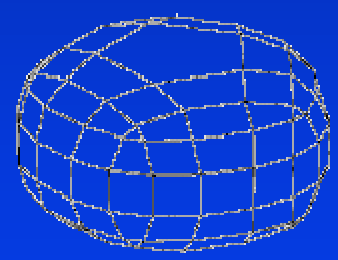
(a)



(b)

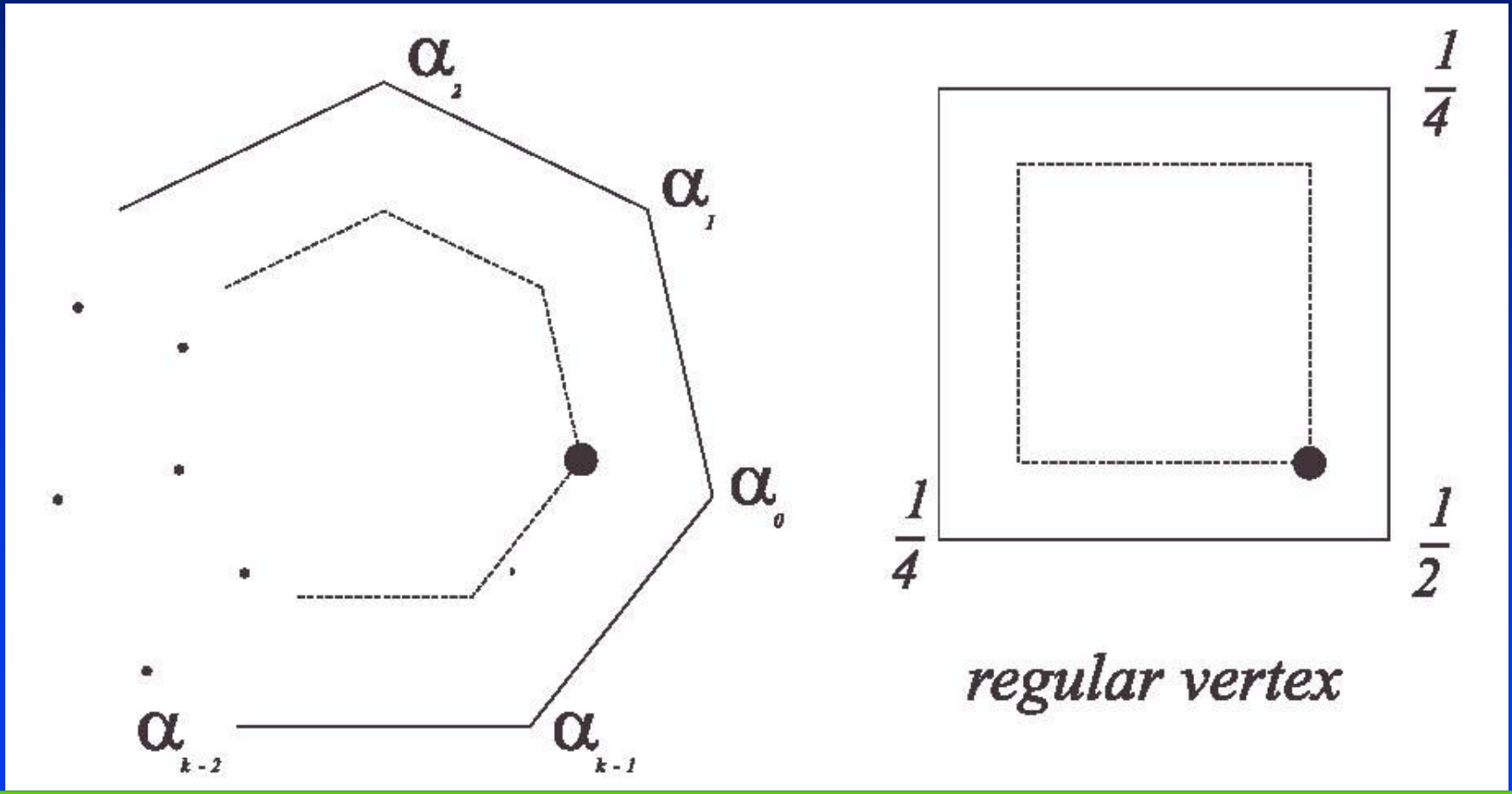


(c)

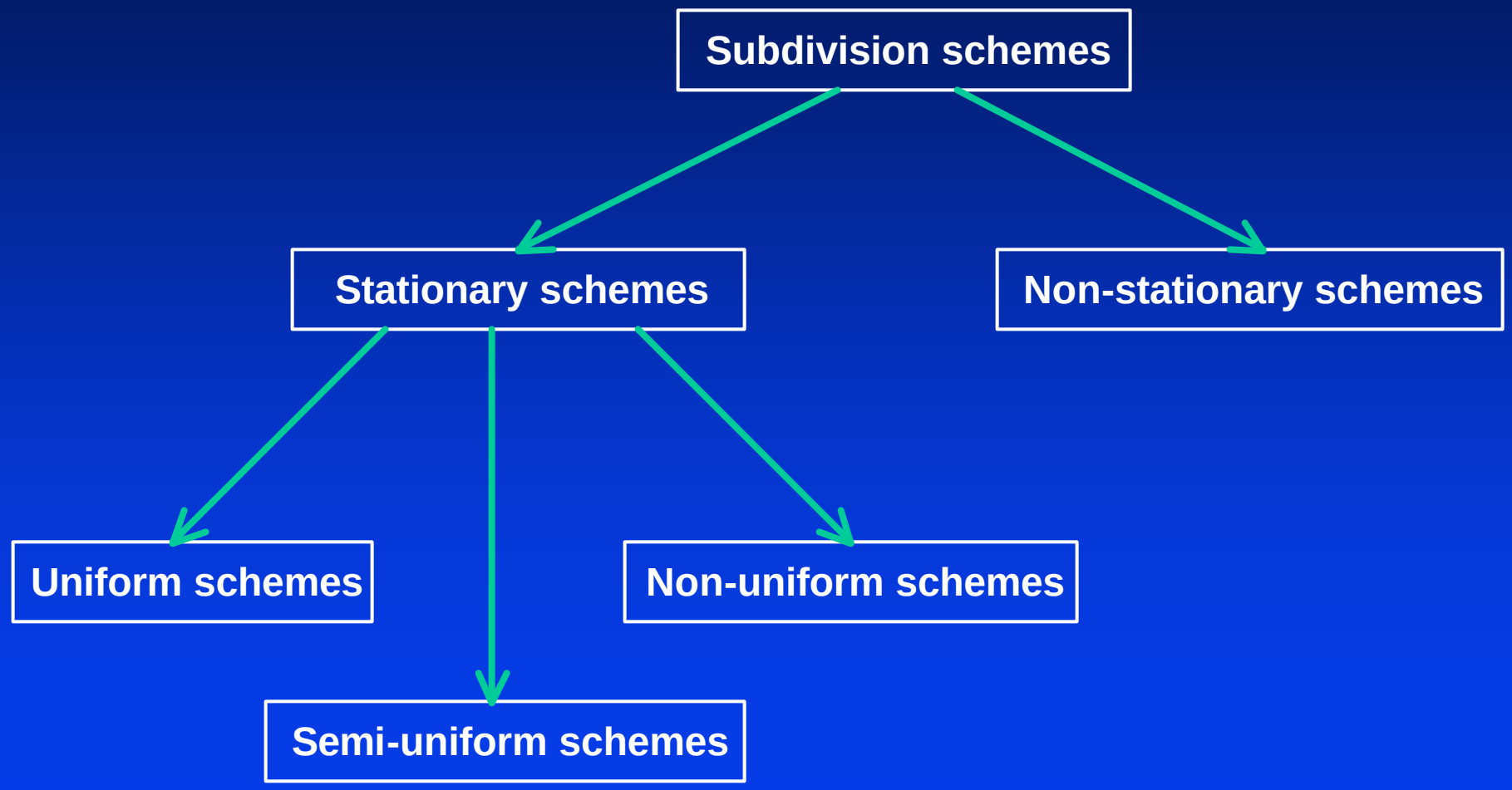


(d)

Mid-edge Scheme



Classification of Subdivision Schemes



Classification

- Stationary schemes
  - Same rules at all levels
- Non-stationary schemes
  - Different rules at different levels

Stationary Subdivision

- Uniform schemes --- fixed rules everywhere on the mesh
- Semi-uniform schemes --- rules depend on the local connectivity
- Non-uniform schemes --- rules depend on both the connectivity and the geometry

Sabin & Doo Algorithm

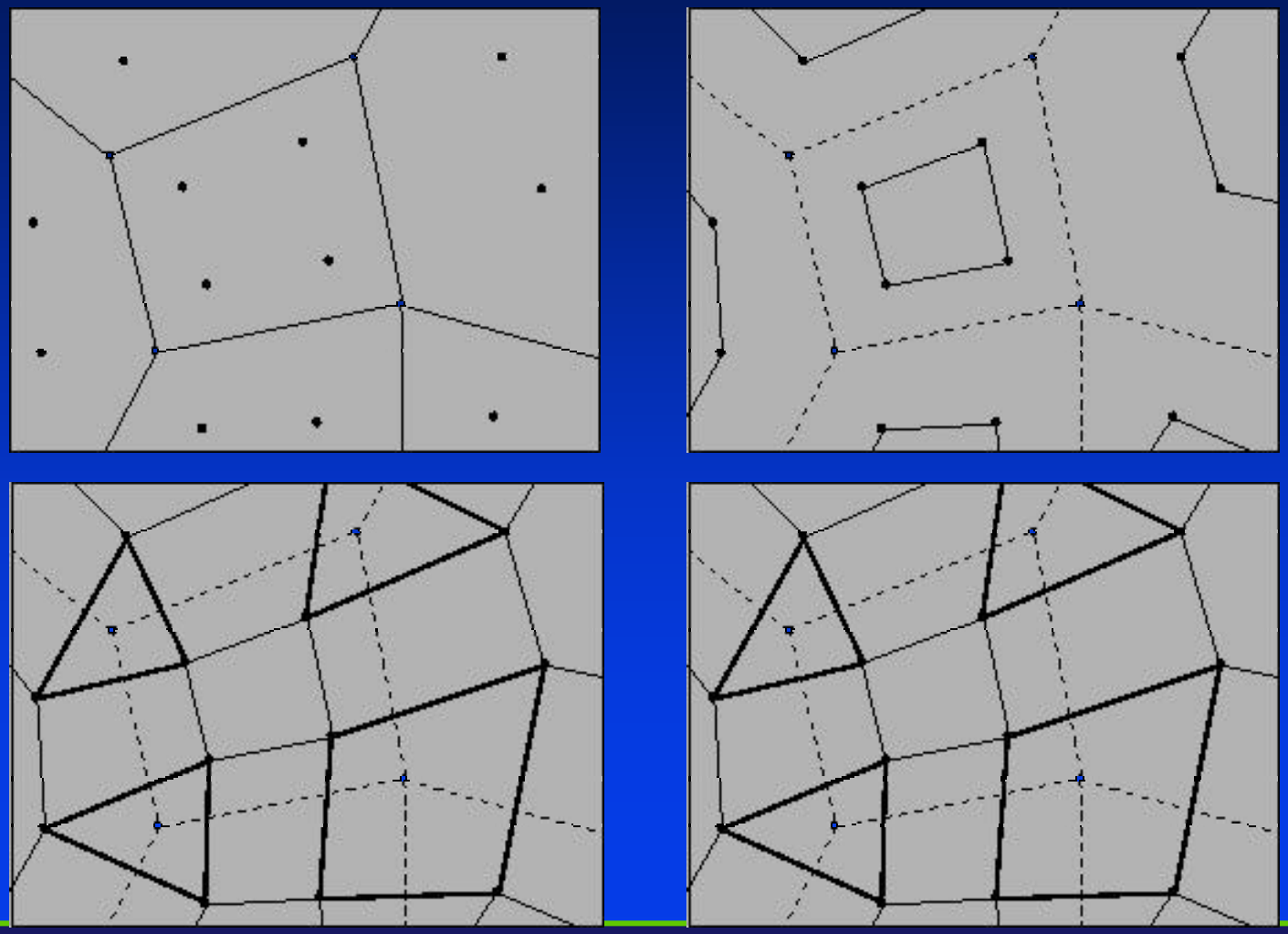
- Subdivision rules:

$$\mathbf{v}_i = \sum_{j=1}^{n} a_{ij} \mathbf{w}_j$$

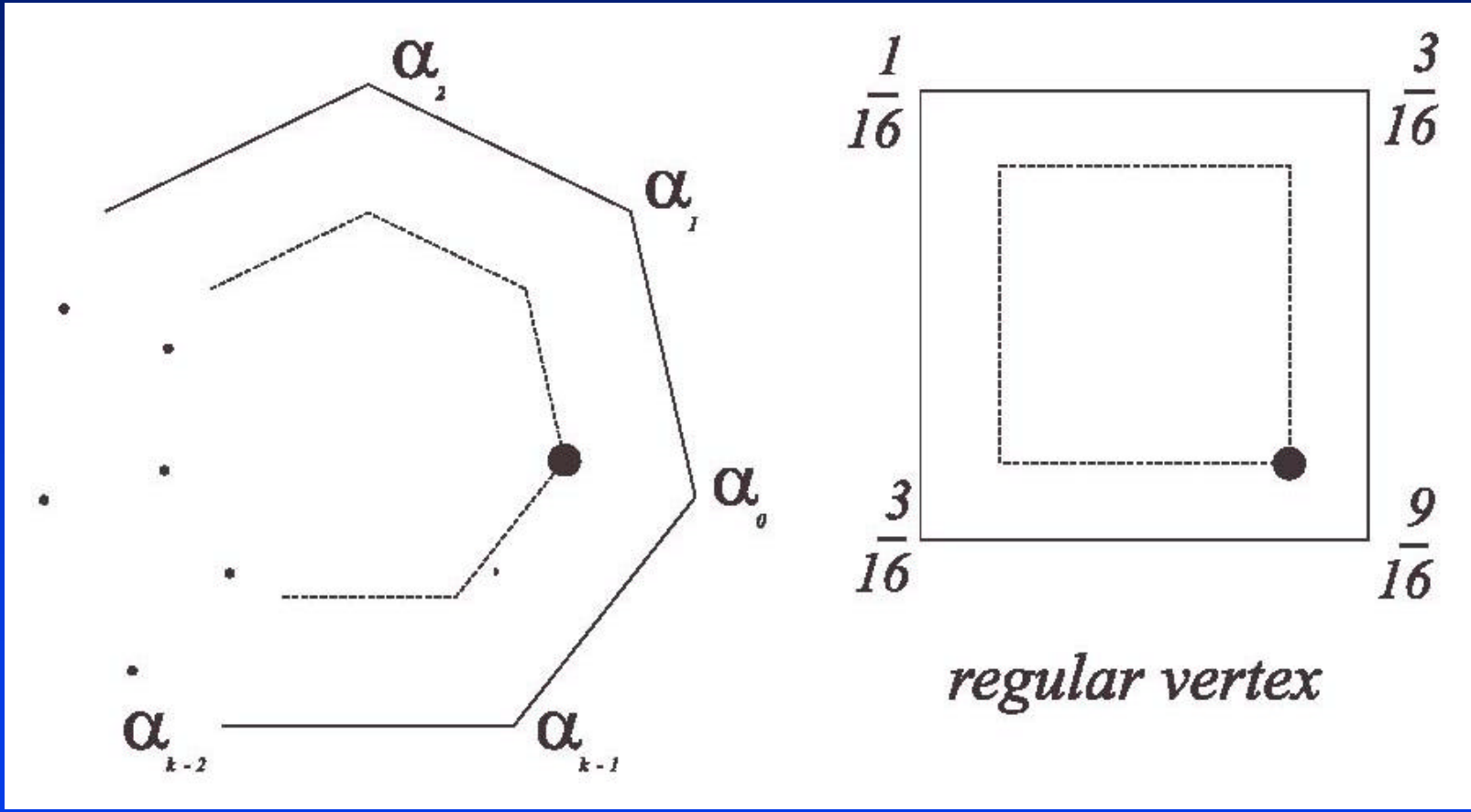
$$a_{ij} = \begin{cases} \dfrac{n+5}{4n} & i = j \\ \dfrac{3 + 2\cos\left(\dfrac{2(i-j)p}{n}\right)}{4n} & otherwise \end{cases}$$

- New faces after subdivision
  - V-face, E-face, F-face
- Properties
  - The limit surface is C1
  - Interpolate the centroids of all faces at every subdivision
  - Generalize biquadratic B-splines

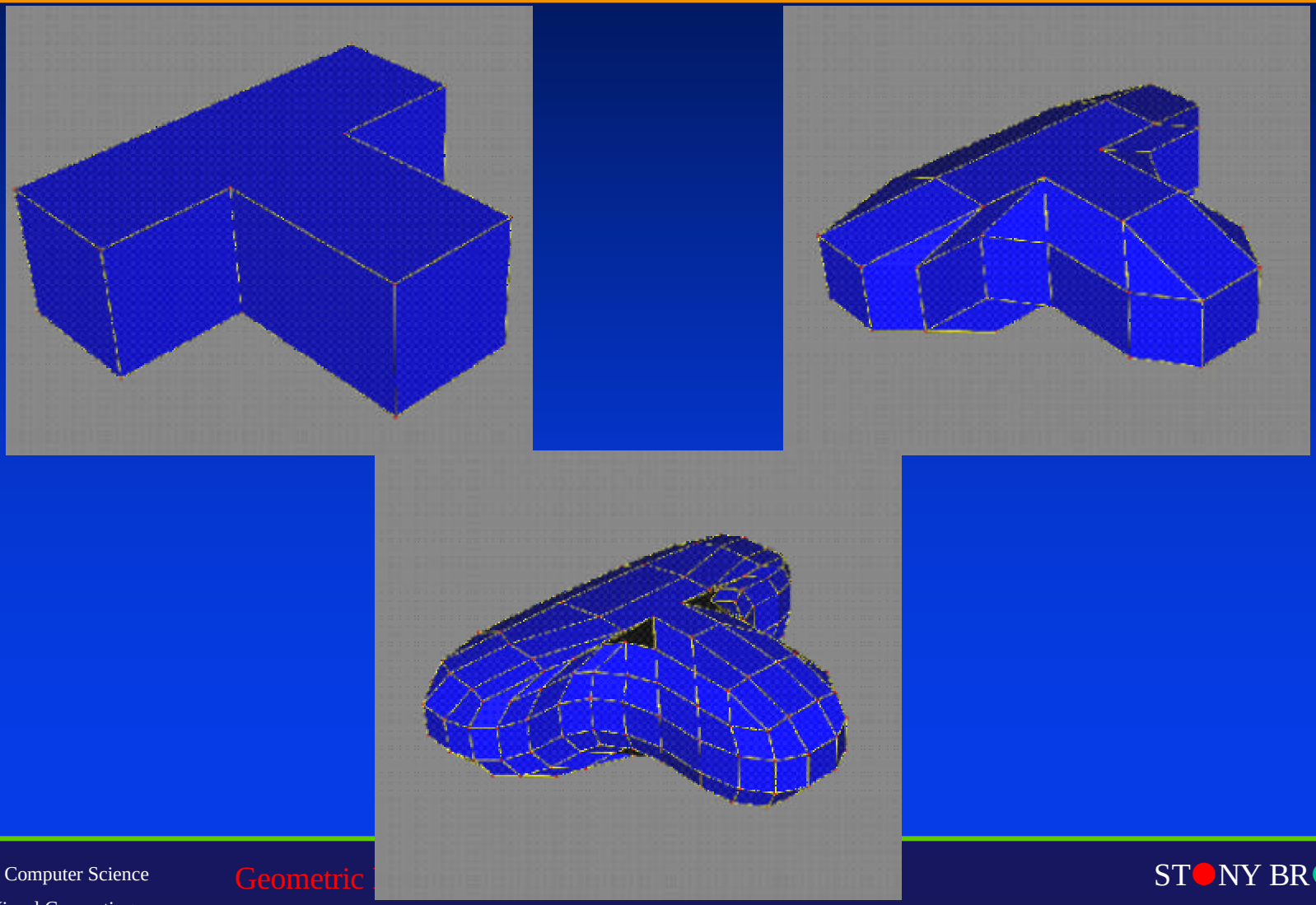
Sabin & Doo Scheme



Sabin & Doo Scheme



Sabin & Doo Scheme

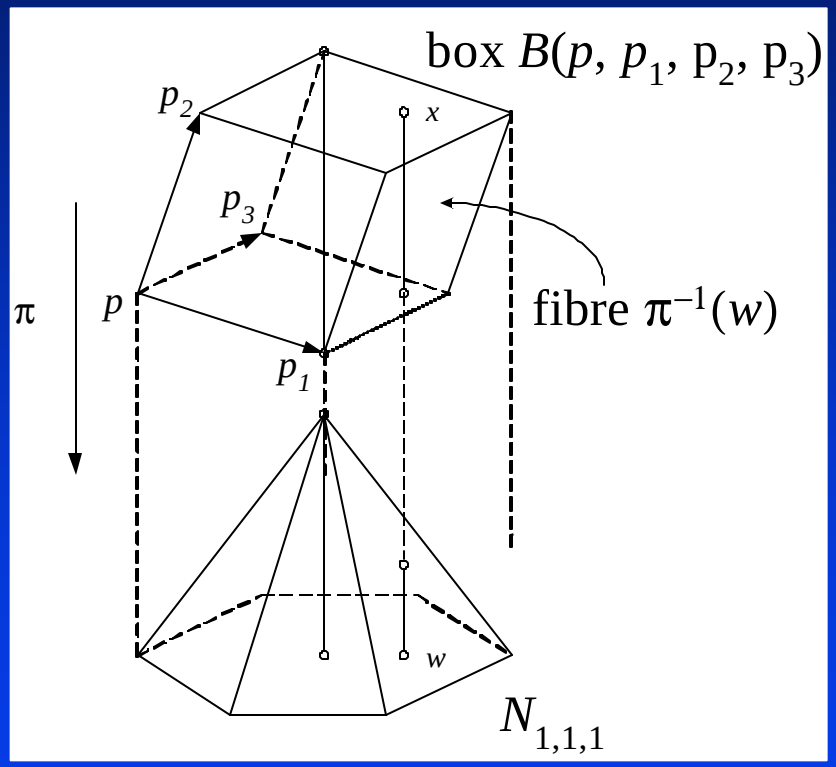


Loop Scheme

- Box splines
  - A projection of 6D box onto 2D
  - A quartic polynomial basis function
  - Triangular domain
- Non-tensor-product splines
- Loop scheme results from a generalization of box splines to arbitrary topology

Box Spline Overview

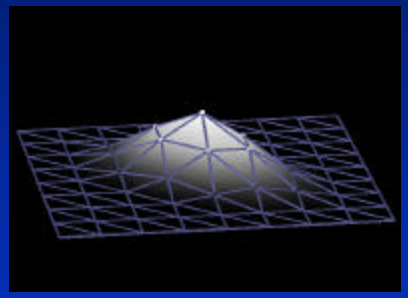
- Based on 2D Box Spline
  - Defined by projection of hypercube (in 6D) into 2D.
  - Satisfies many properties that B-spline has.
    - Recursive definition
    - Partition of unity
    - Truncated power
  - Natural splitting of a cube into sub-cubes provides the subdivision rule.

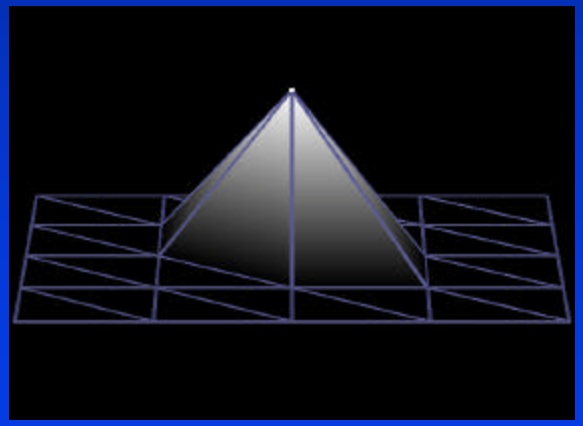


Basis Functions for Loop's Scheme

- Basis Function - Evaluation

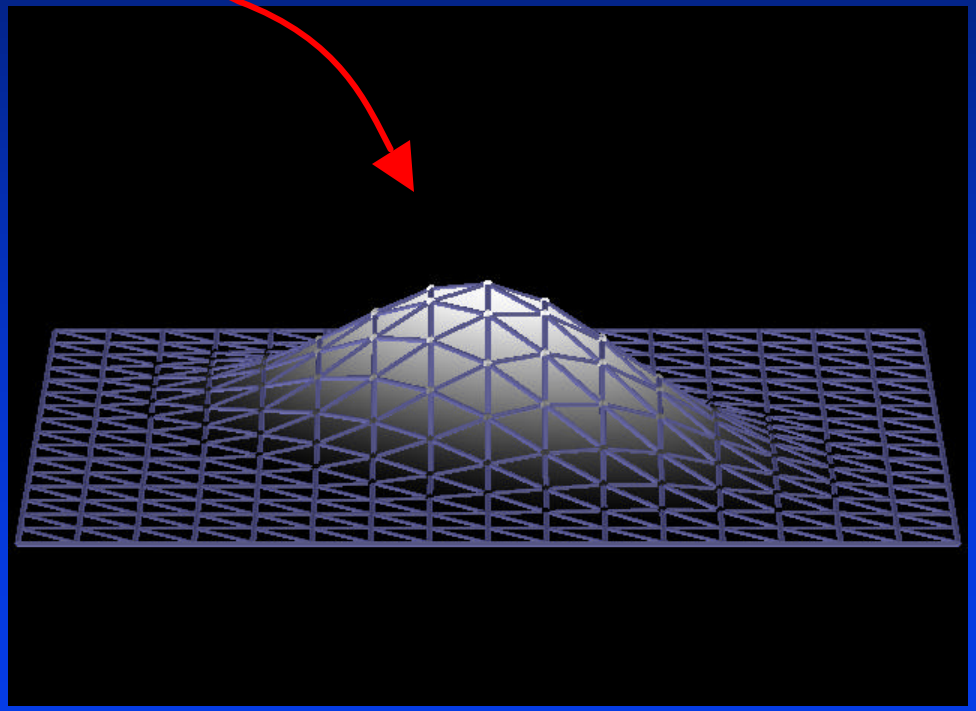
Successive Subdivision





Assign unit weight to center, zero otherwise, over Z^2 lattice

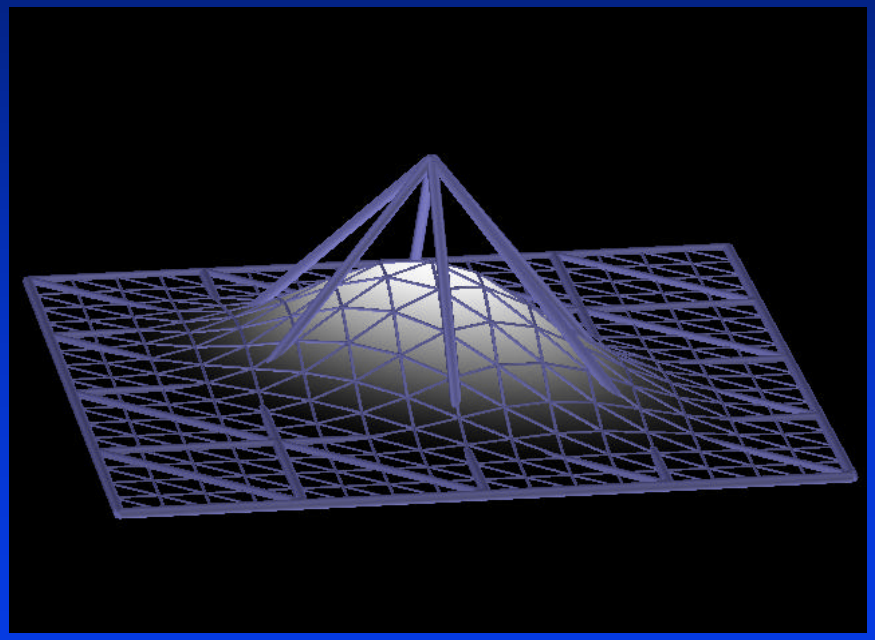
The Limit → $N_{2,2,2}$ Basis



Loop's Scheme Properties

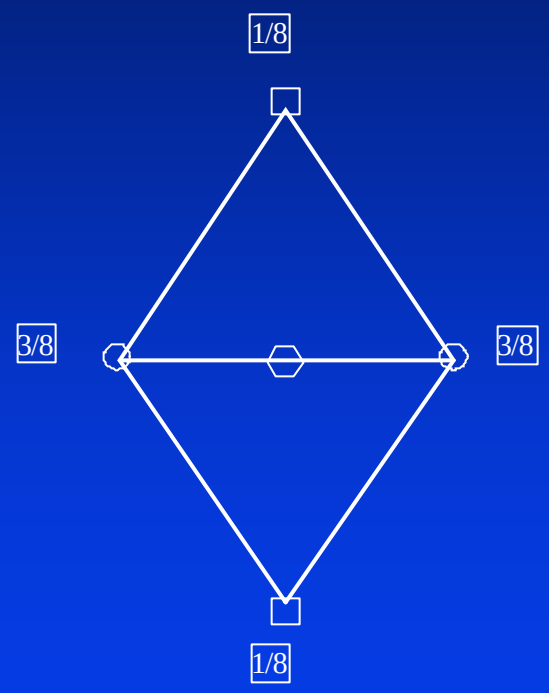
- Basis Function – Properties

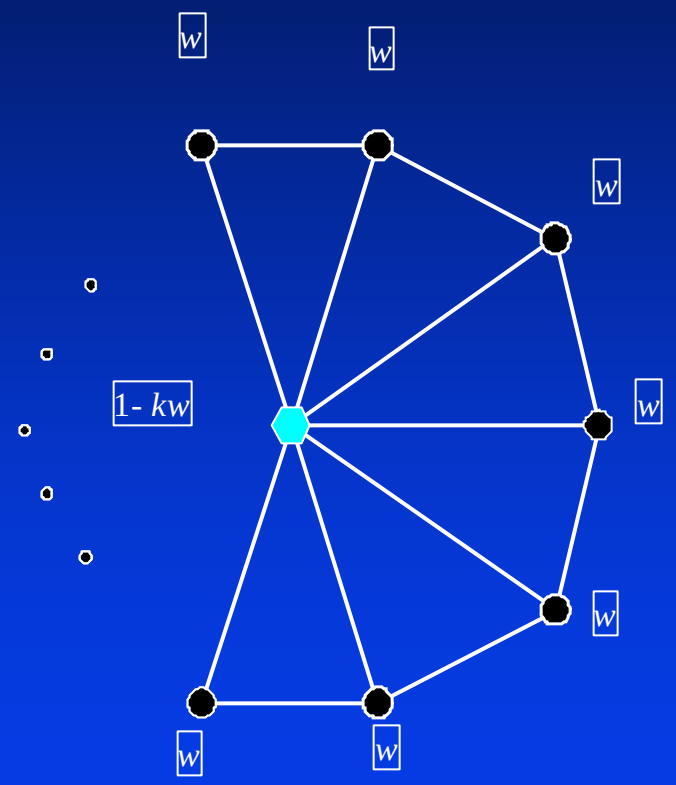
1. Support → 2 neighbors from the center
2. C^4 continuity within the support
3. Piecewise polynomial
4. $N_{2,2,2}(\bullet - j)$, $j \in Z^2$ form a partition of unity i.e. $S\ N(x - j) = 1$



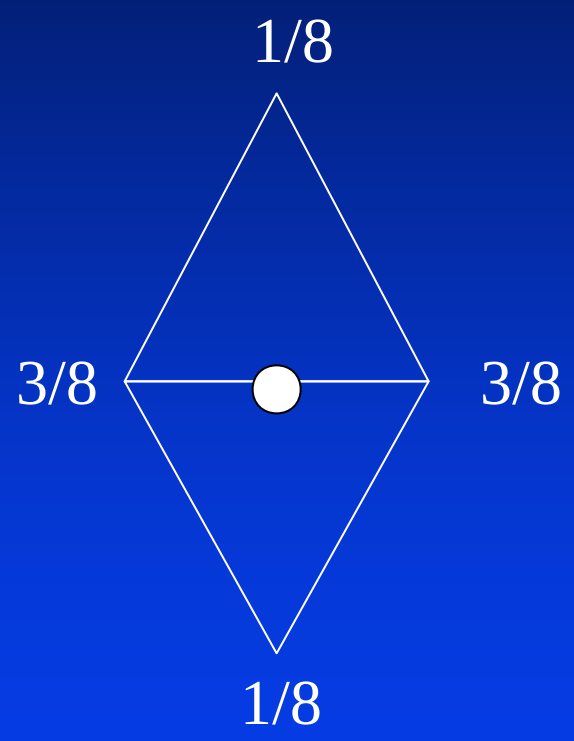
Loop's Scheme Rules

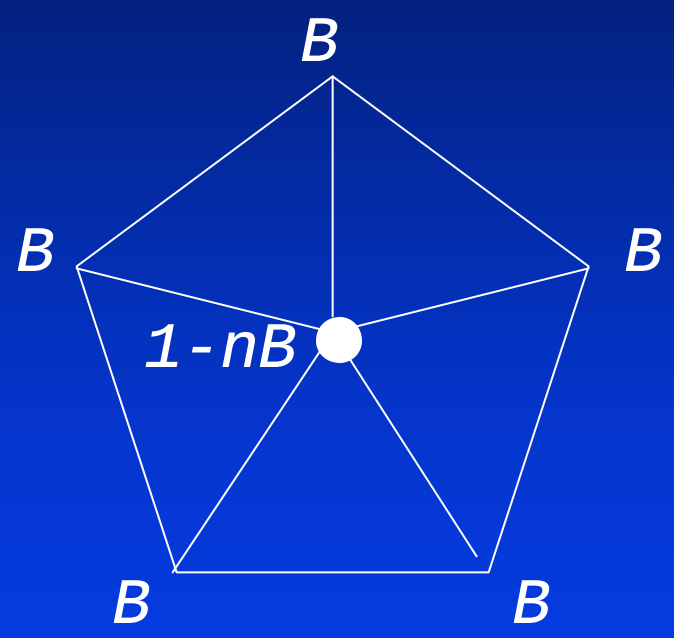
- The Rules





Loop Scheme Rules

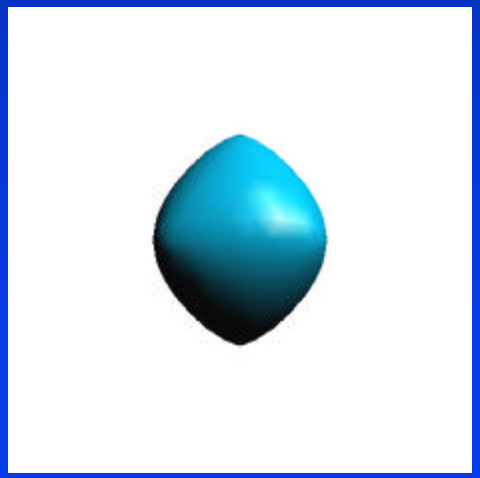




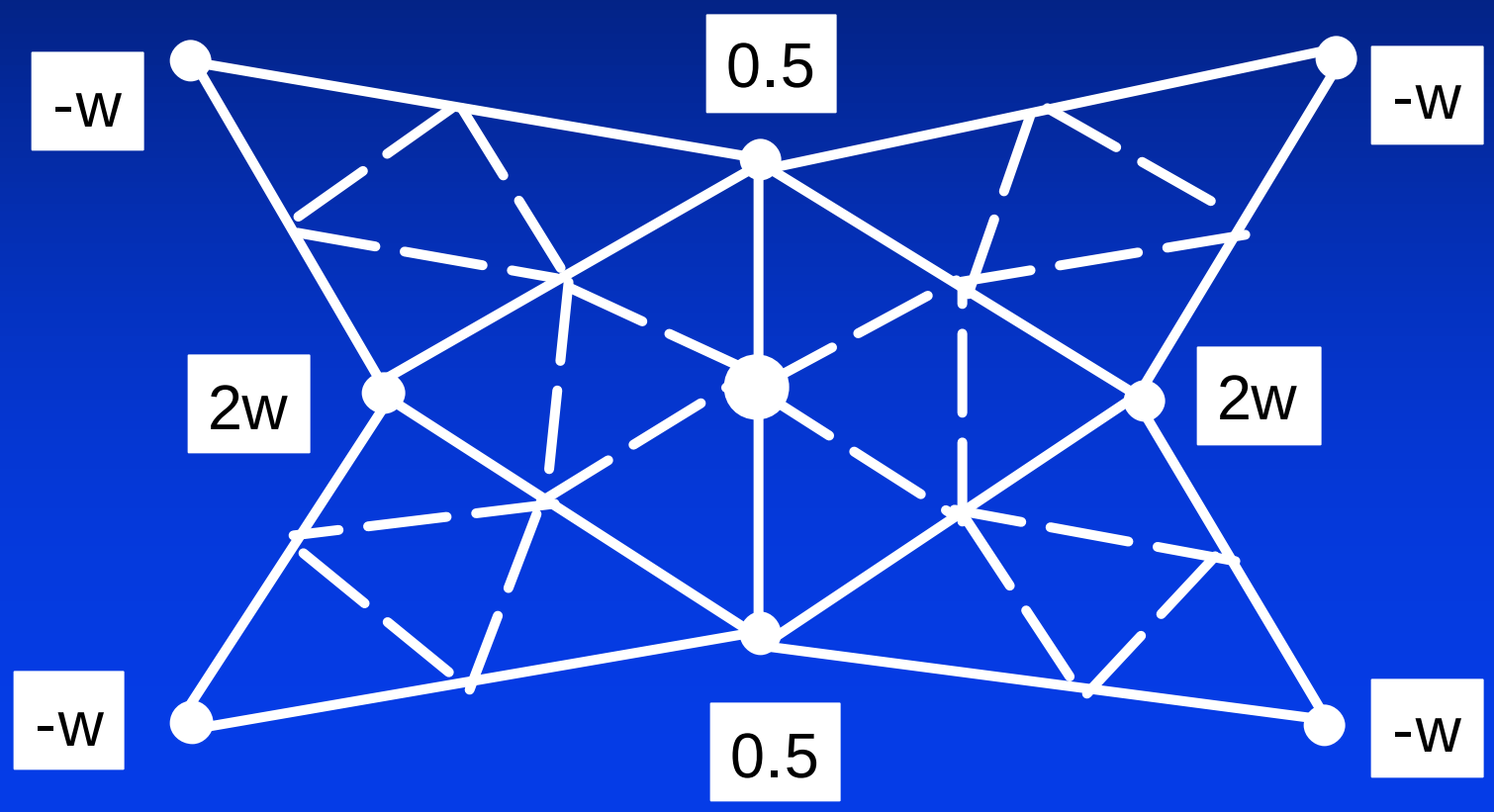
$B = 3/8k$, for $n>3$

$B = 3/16$, for $n>3$

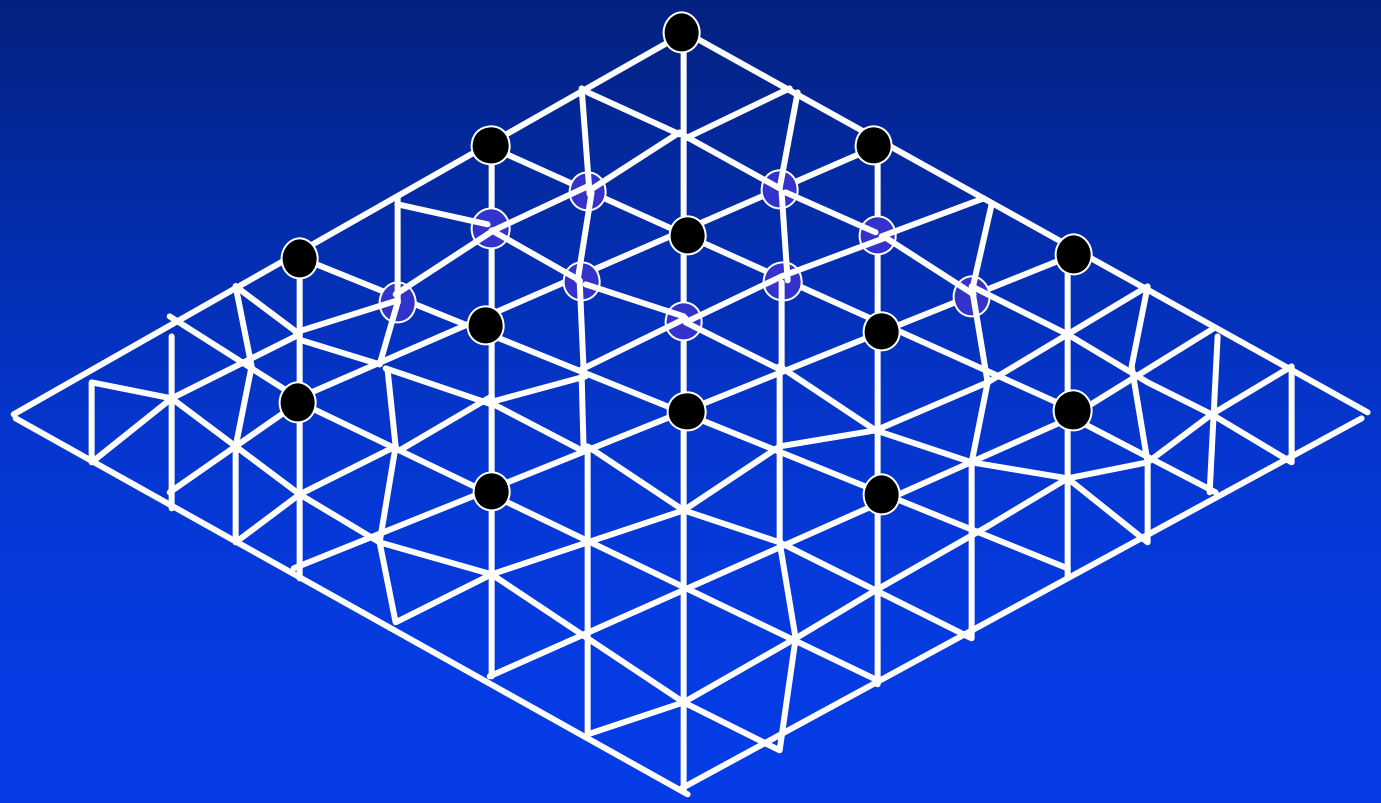
Loop Scheme Example



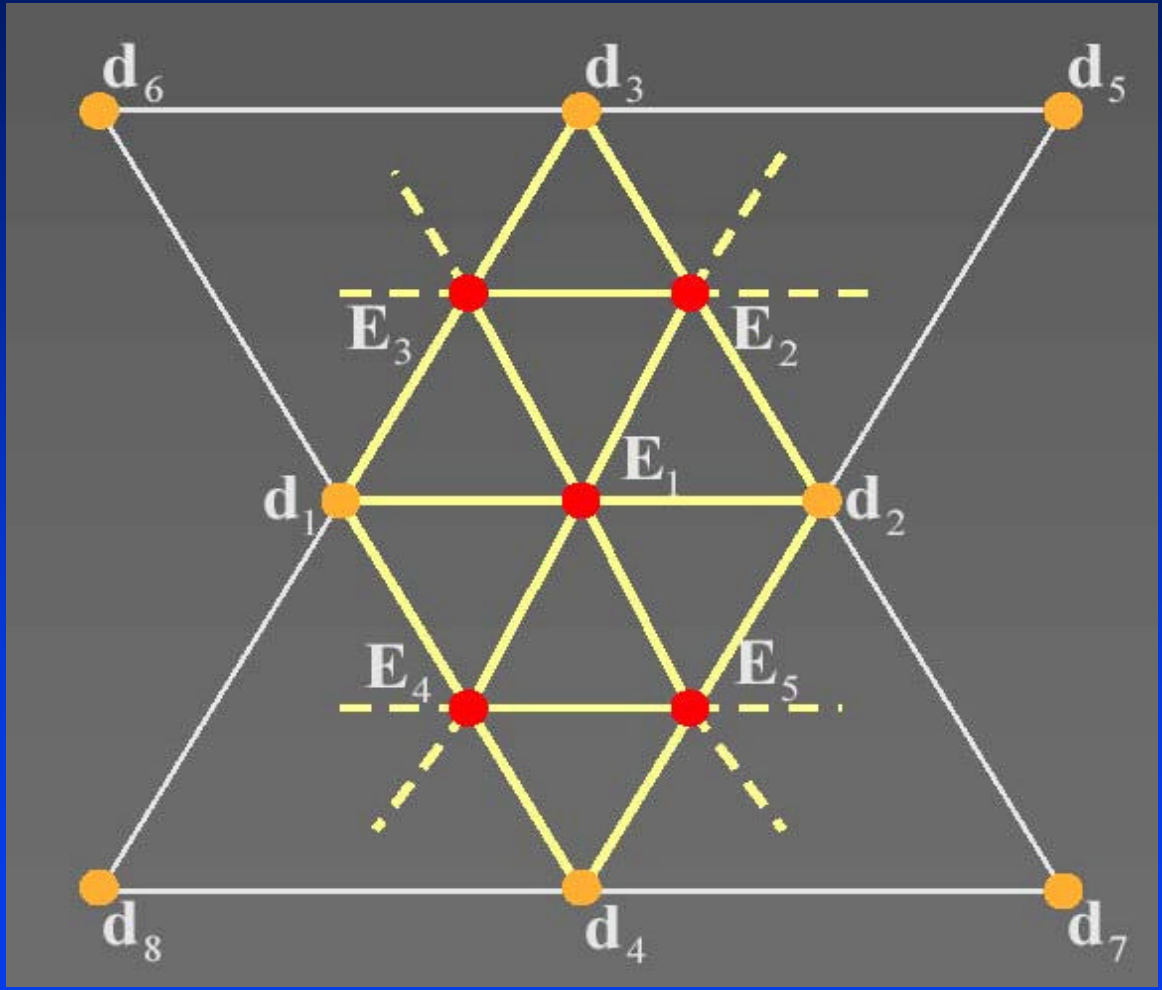
Butterfly Subdivision



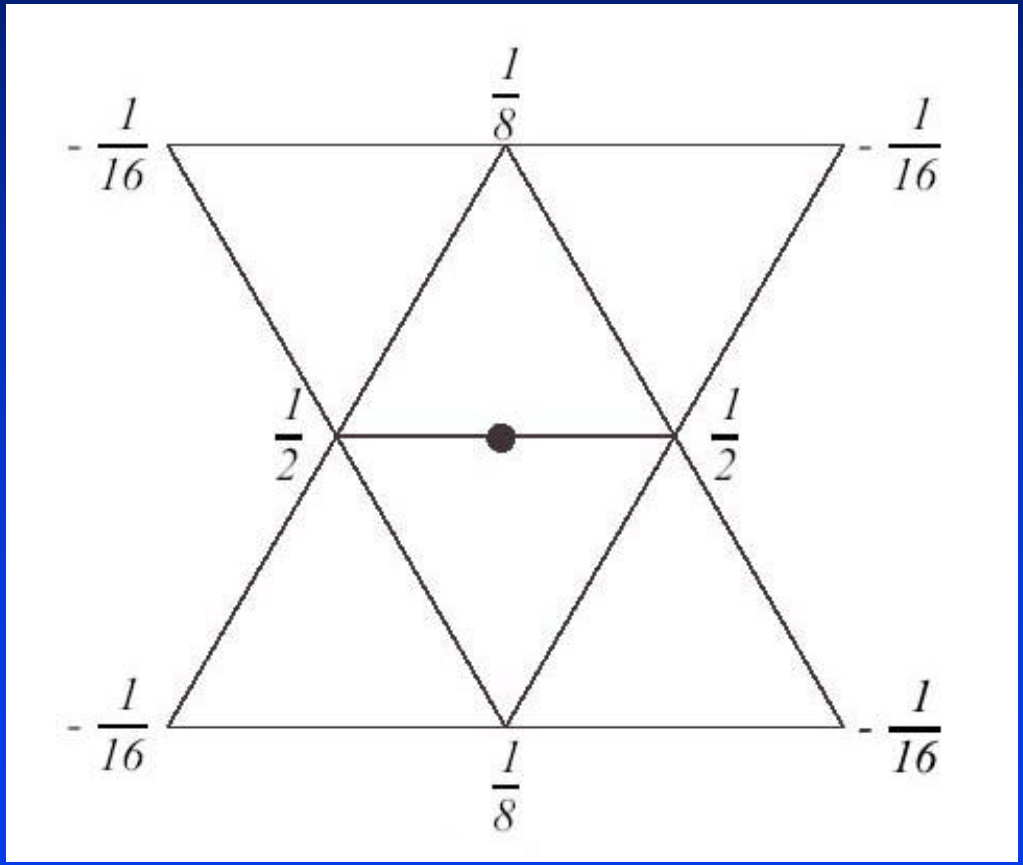
Butterfly Scheme



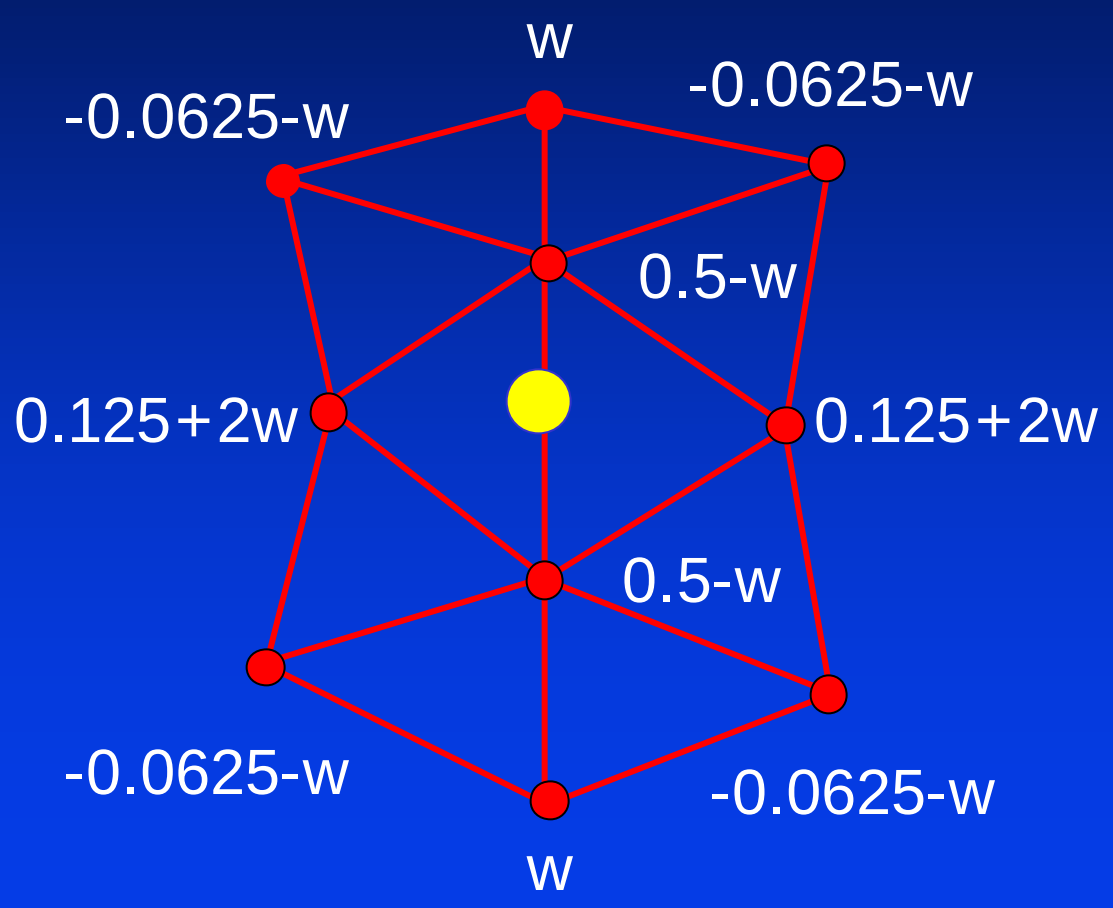
Modified Butterfly Scheme

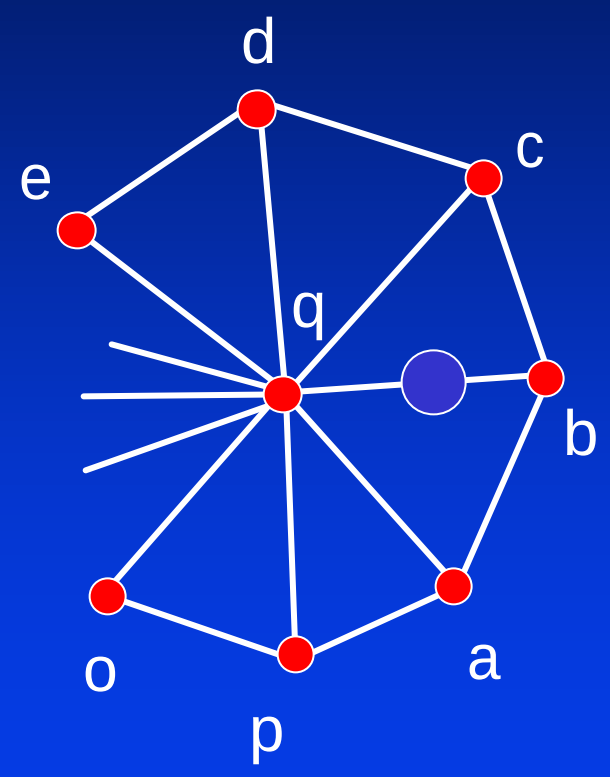


Modified Butterfly Scheme

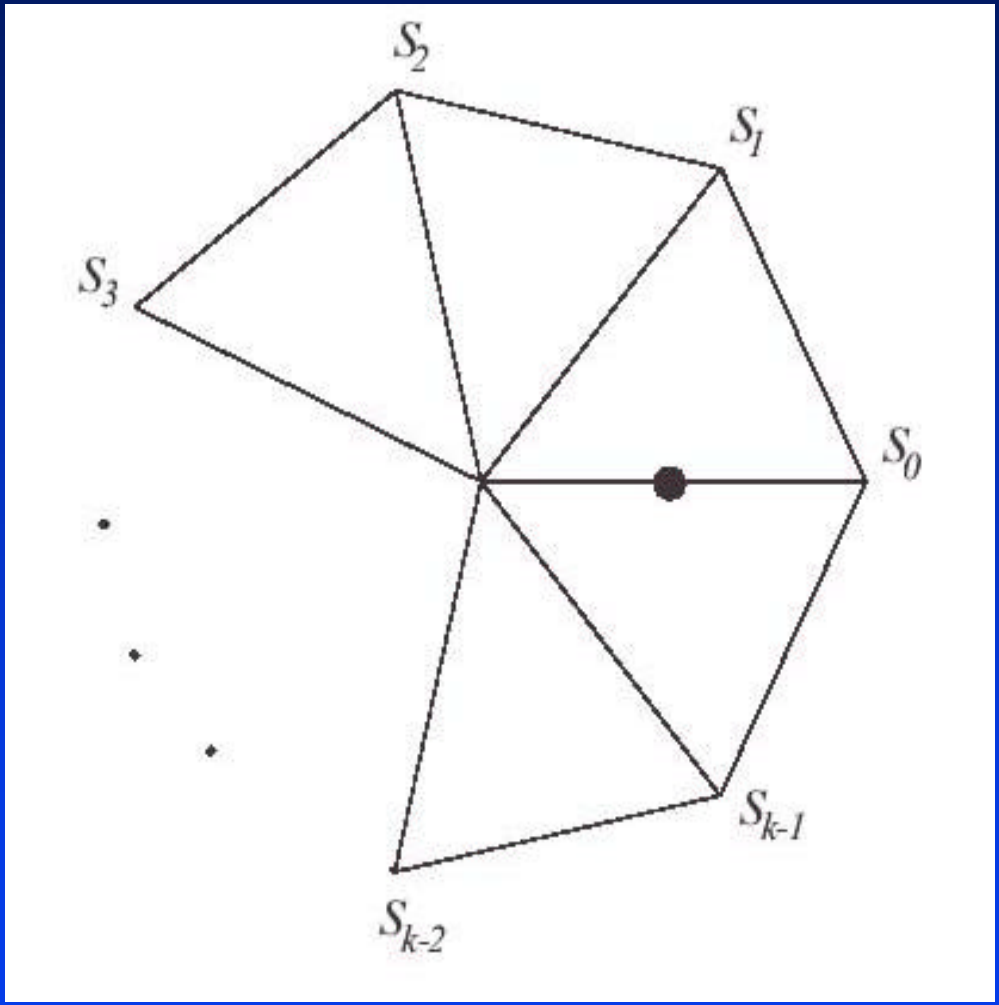


Modified Butterfly Scheme

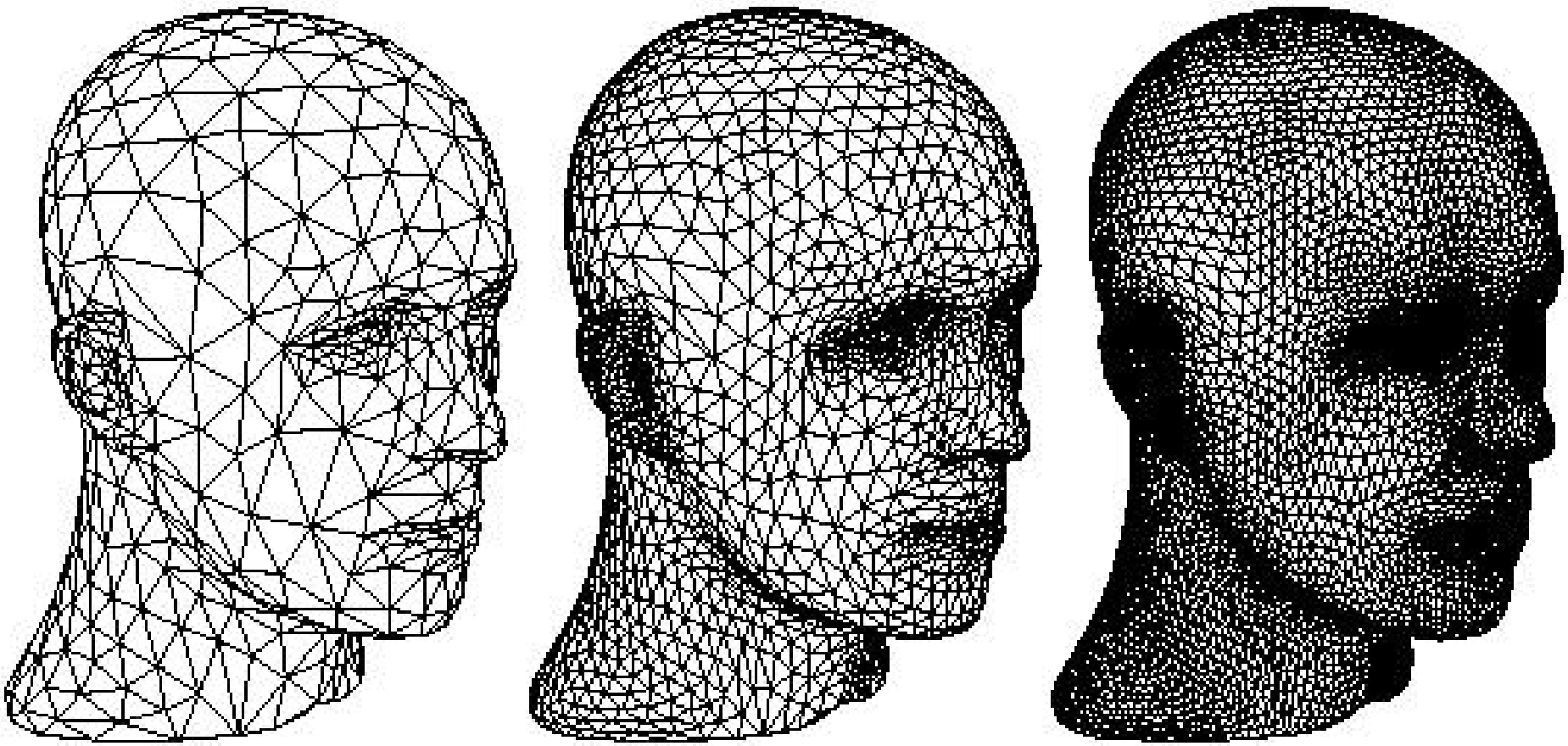




Modified Butterfly Scheme

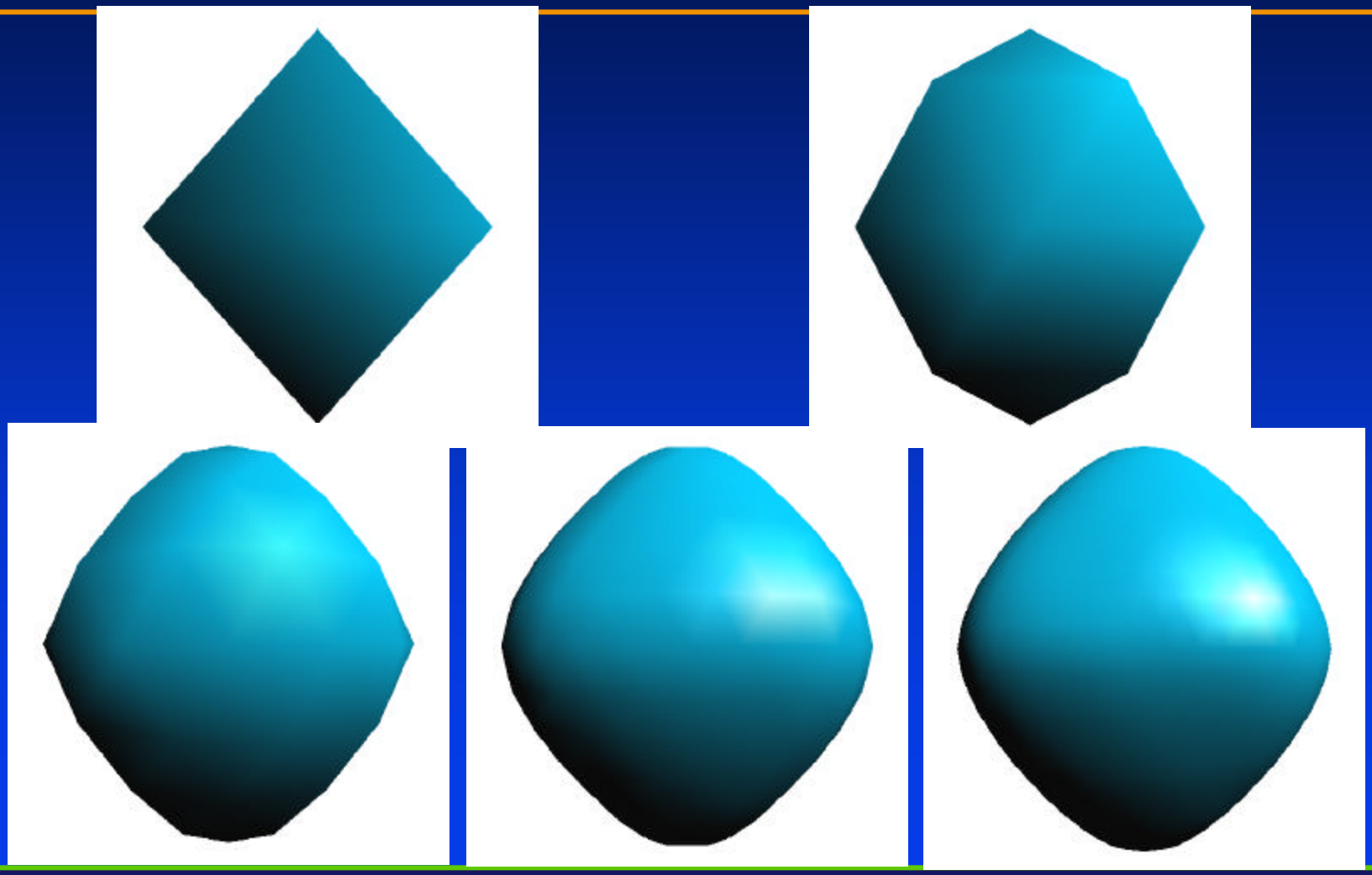


Modified Butterfly Scheme



Initial mesh One refinement step Two refinement steps

Modified Butterfly Example



Uniform/Semi-uniform Schemes

- Catmull-Clark scheme
  - *Catmull and Clark, CAD 1978*
- Doo-Sabin scheme
  - *Doo and Sabin, CAD 1978*
- Loop scheme
  - *Loop, Master's Thesis, 1987*
- Butterfly scheme
  - *Dyn, Gregory and Levin, ACM TOG 1990.*
- Mid-edge scheme
  - *Habib and Warren, SIAM on Geometrrc Design 1995*
- Kobbelt scheme
  - *Kobbelt, Eurographics 1996*

Classification

- By Mesh type:
  - Triangular (Loop, Butterfly)
  - Quadrilateral (Catmull-Clark, Doo-Sabin, Mid-edge, Kobbelt)
- By Limit surface:
  - Approximating (Catmull-Clark, Loop, Doo-Sabin, Mid-edge)
  - Interpolating (Butterfly, Kobbelt)
- By Refinement rule:
  - Vertex insertion (Catmull-Clark, Loop, Butterfly, Kobbelt)
  - Corner cutting (Doo-Sabin, Mid-edge)

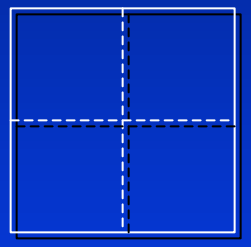
Summary

Vertex insertion

Corner cutting

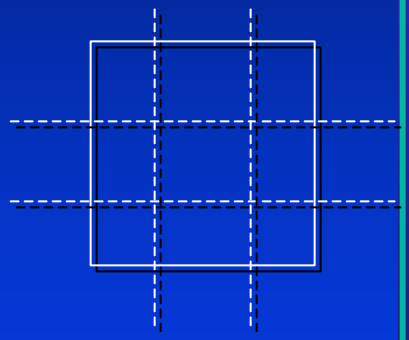
Approximating **Interpolating**

Catmull-Clark **Kobbelt**

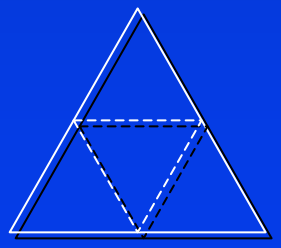


Doo-Sabin

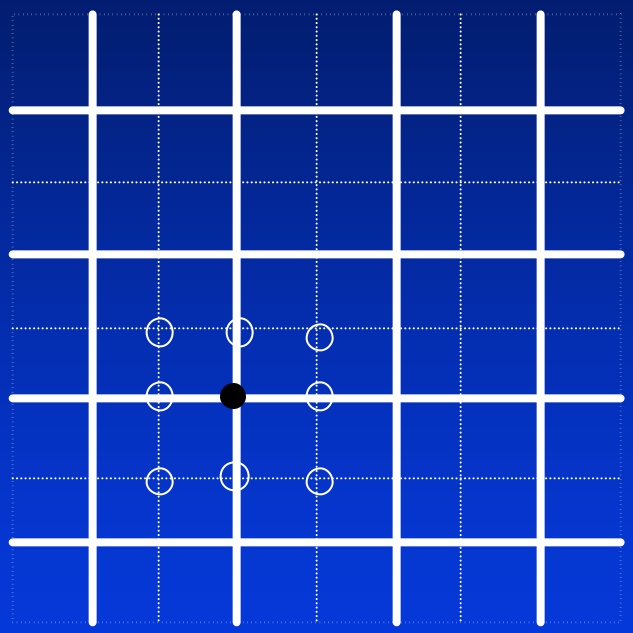
Mid-edge



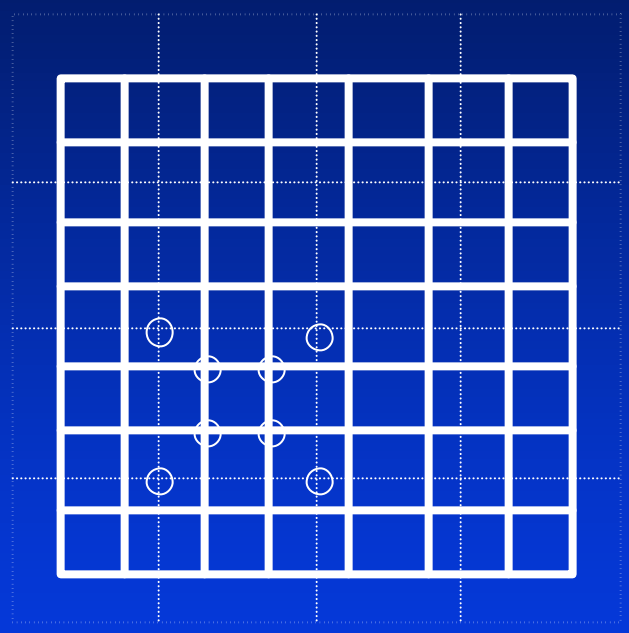
Loop **Butterfly**



Mesh Refinement Type

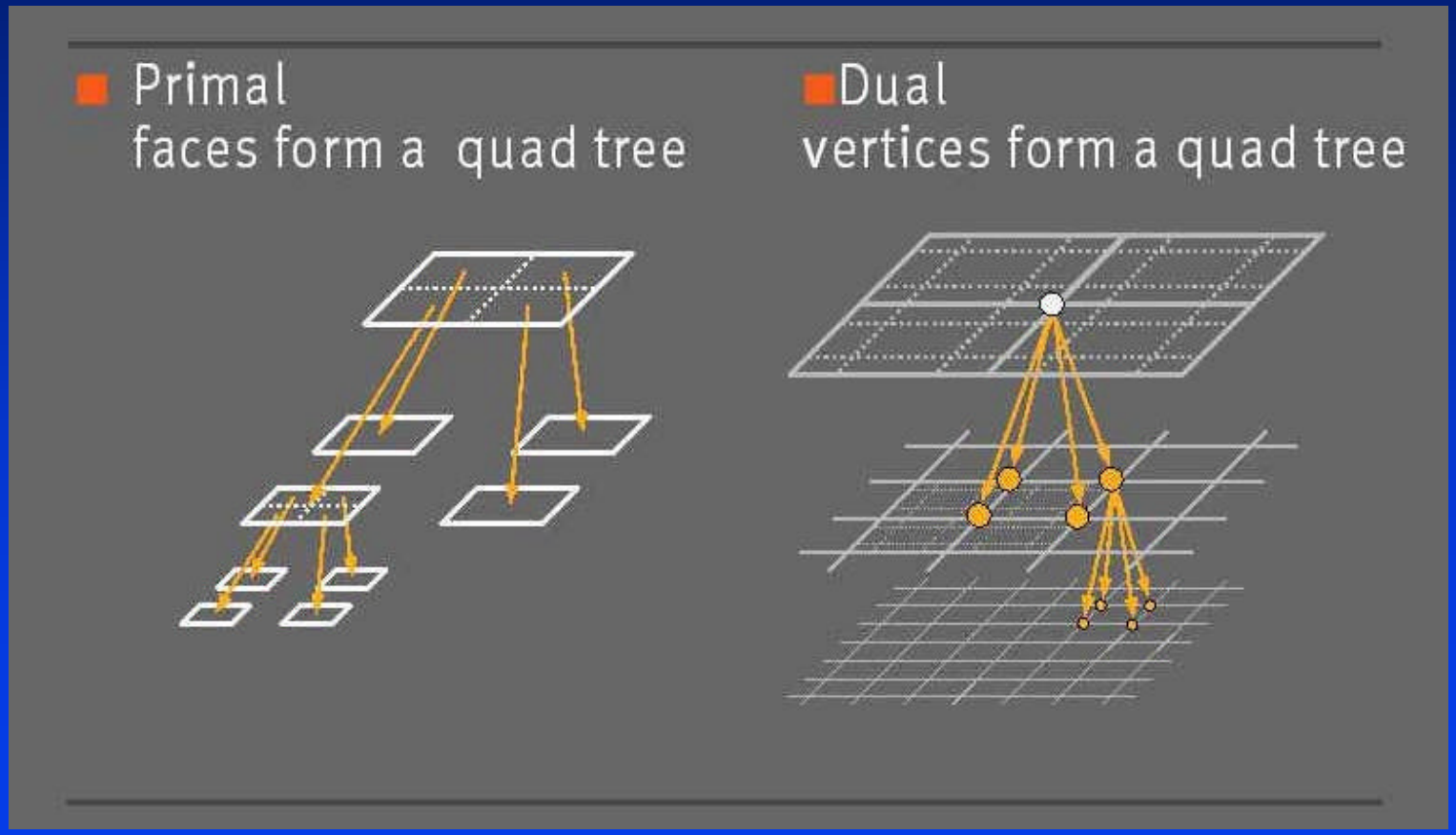


Vertex insertion



Corner cutting

Face vs. Vertex Refinement



Continuity

Schemes	Continuity at Regular Vertex	Continuity at Extraordinary Point
Catmull-Clark	C2	C1
Doo-Sabin	C1	C1
Loop	C2	C1
Butterfly	C1	C1 except k=3 or k>7
Modified Butterfly	C1	C1
Mid-edge	C1	C1

Summary

Schemes	Mesh Type	Refinement Rule	Approximation /Interpolation
Catmull-Clark	Quadrilateral	Vertex Insertion	Approximation
Doo-Sabin	Quadrilateral	Corner Cutting	Approximation
Loop	Triangular	Vertex Insertion	Approximation
Butterfly	Triangular	Vertex Insertion	Interpolation
Mid-edge	Quadrilateral	Corner Cutting	Approximation
Kobbelt	Quadrilateral	Vertex Insertion	Interpolation

Analysis Tool

- Analysis of stationary schemes
- Subdivision matrix
- Analysis of properties (convergence, continuity, smoothness)

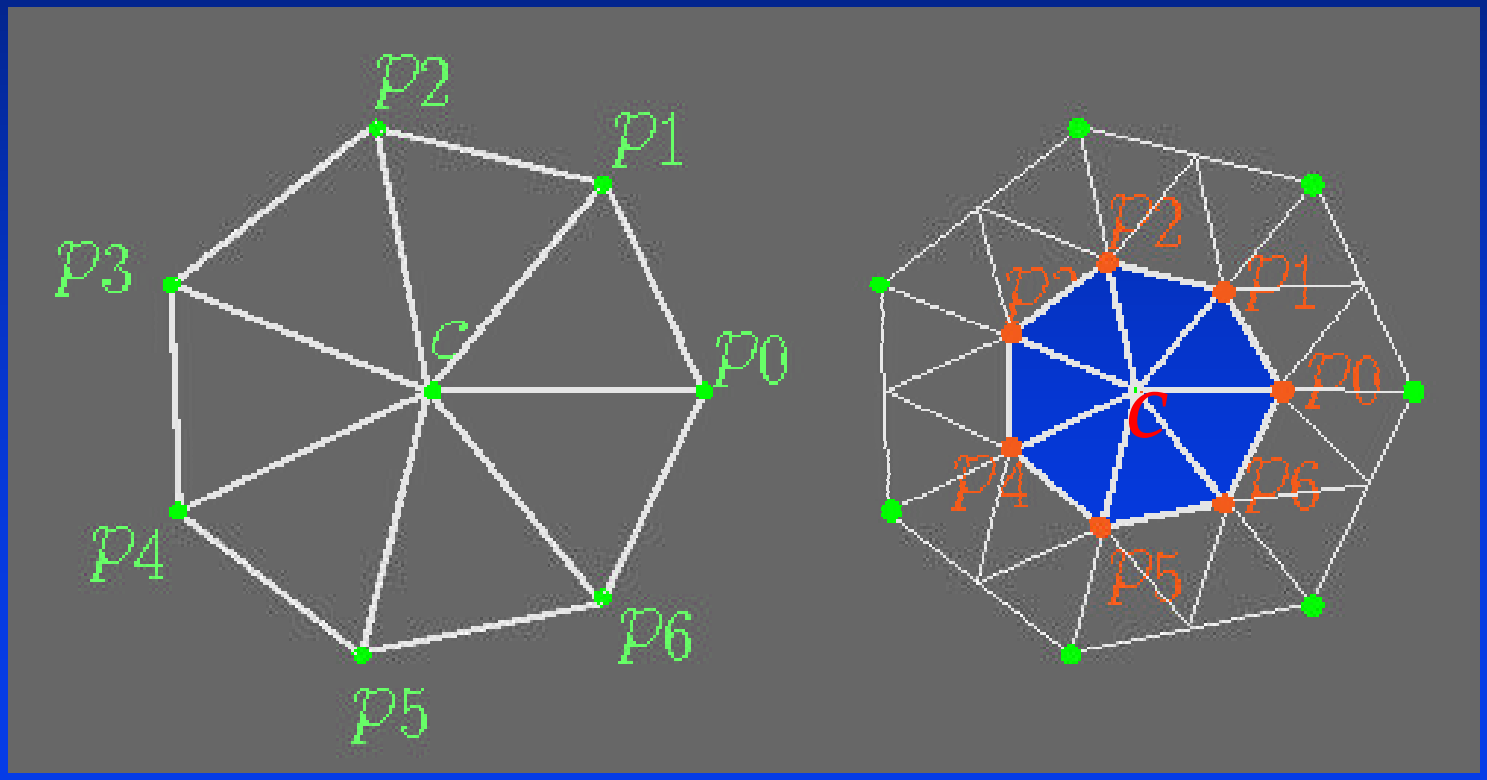
  Doo and Sabin 78, Ball and Storry 86, Reif 95, Zorin 97
- Exact formula for the limit position and normal

  Halstead et al. Siggraph 93
- Explicit evaluation of surfaces at arbitrary points of the domain

  Jos Stam Siggraph 98, Qin et al. TVCG 98

Subdivision Matrix

$$p^{j+1} = S \cdot p^{j}$$



Limit Position

$$p^{j+1} = S \cdot p^{j}$$

S is the subdivision matrix

$$p^{0} = \sum_{i} a_i x_i$$

x_i are the eigenvectors of S

$$p^{j} = (S)^{j} p^{0}$$

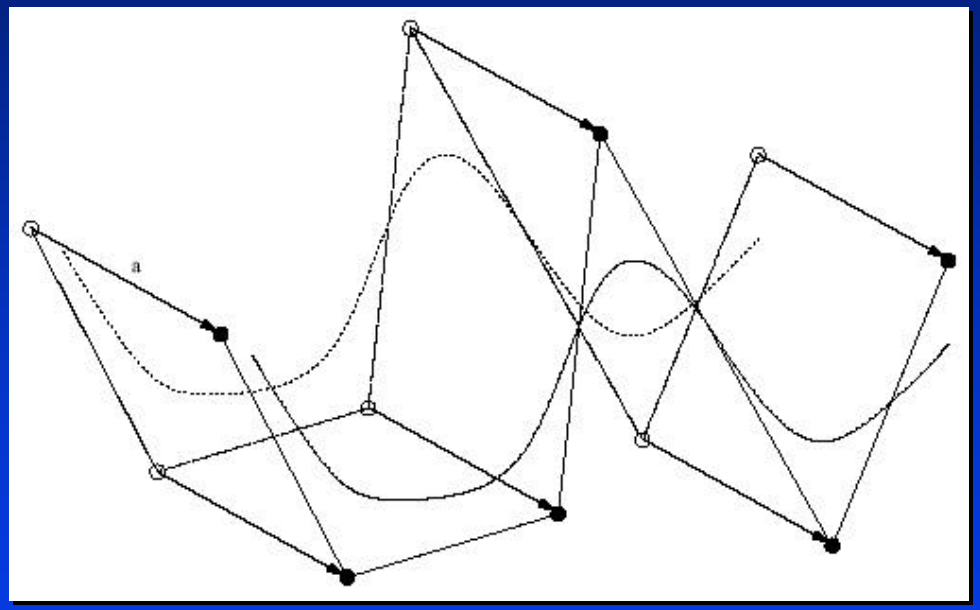
$$= \sum_{i} (\lambda_i)^{j} a_i x_i$$

λ_i are the eigenvalues

We can prove: $\lambda_0 = 1$ and $?_i < 1,\ i = 1, 2, \ldots$

Then, $$p^{j} = a_0 x_0 + \sum_{i=1} (\lambda_i)^{j} a_i x_i$$

Eigenvalue Analysis



By linearity of S,

$$S(p^j + 1\cdot a) = Sp^j + S(1\cdot a)$$
$$= p^{j+1} + S(1\cdot a)$$

By translation invariance,

$$S(p^j + 1\cdot a) = Sp^j + 1\cdot a$$
$$= p^{j+1} + 1\cdot a$$

So, $S(1\cdot a) = 1\cdot a$

$$l_0 = 1!$$

Normal

$$p^j = \sum_i (l_i)^j a_i x_i$$

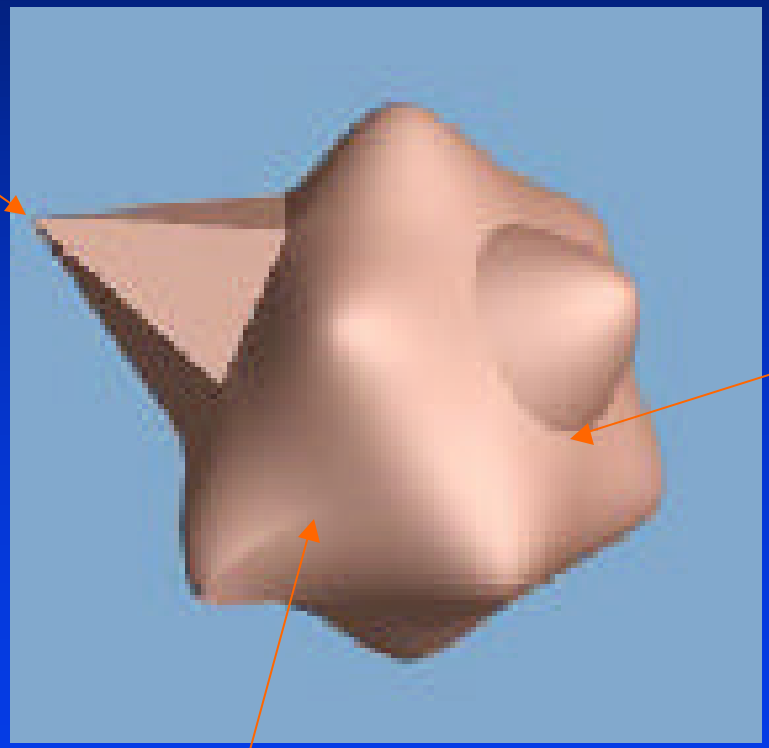
Let $a_0 = 0$ and assume $l = l_1 = l_2 > l_3$

Then, $$\frac{p^j}{(l)^j} = a_1 x_1 + a_2 x_2 + \frac{(l_3)^j}{l} x_3 \cdots$$

a_1 and a_2 span the tangent plane

The normal $n = a_1 \times a_2$

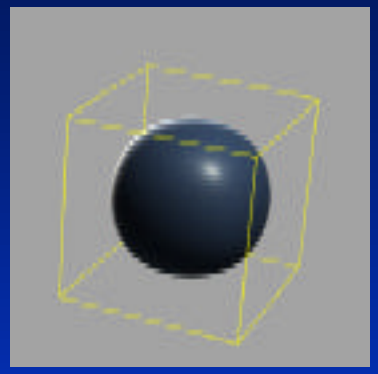
Modeling Sharp Features



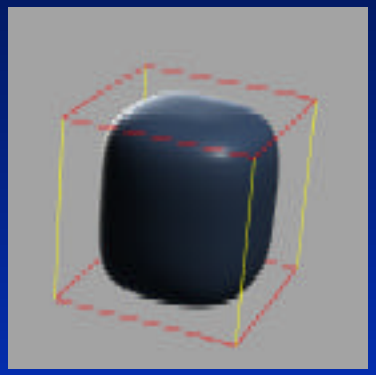
Non-uniform Subdivision Schemes

- Piecewise smooth subdivision schemes
  - *Hoppe et al. Siggraph 94*
- Hybrid scheme
  - *et al. Siggraph 98*
- NURSS scheme
  - *Sederburg et al. Siggraph 98*
- Combined scheme
  - *Levin Siggraph 99*
- Edge and vertex insertion scheme
  - *Habib et al. CAGD 99*

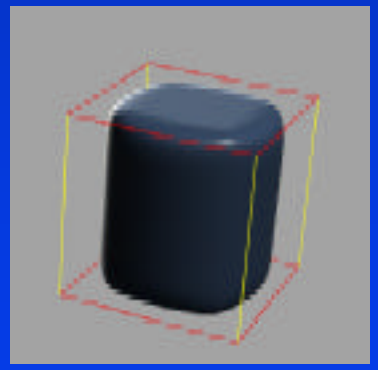
Hybrid Subdivision Scheme



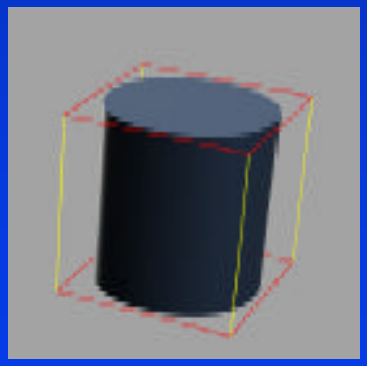
(a)



(b)



(c)



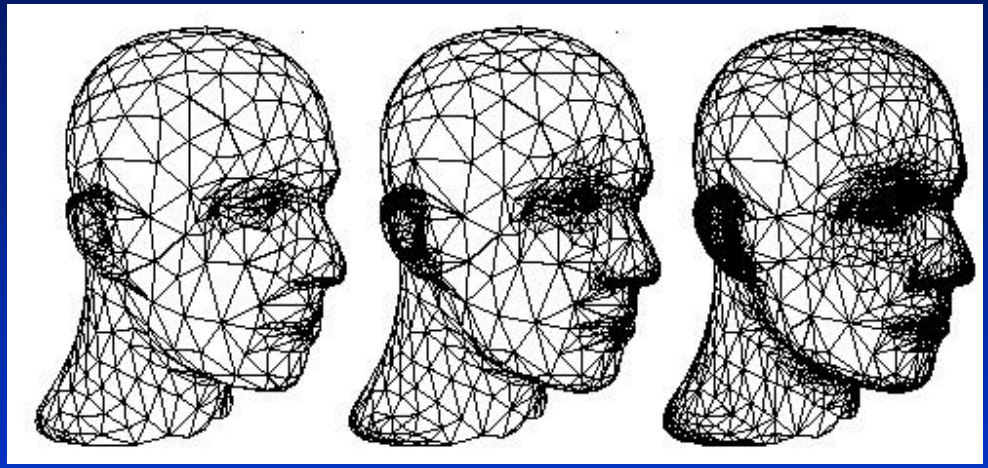
(d)

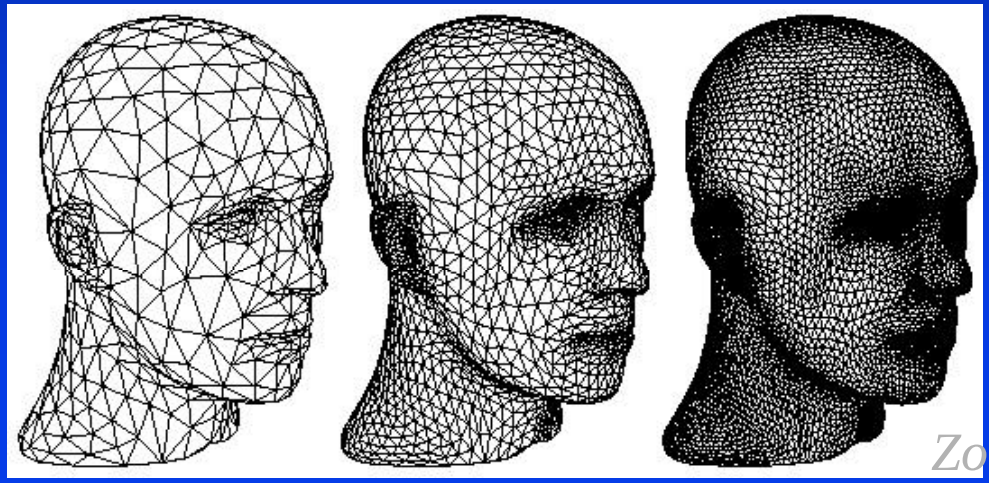
DeRose et al.
Siggraph 98

Prior Algorithms & Techniques

- Hierarchical editing
  - Forsey 88, Lounsbery 94, Zorin 97, Pulli 97
- Boundary interpolation and manipulation
  - Nasri 87, Halstead 93, Levin 99
- Fair surface design
  - Halstead 93, Taubin 95, Koebblt 96, Warren 97
- Surface reconstruction
  - Hoppe et al. 94
- Physics-based modeling approach
  - Qin and Mandal 98

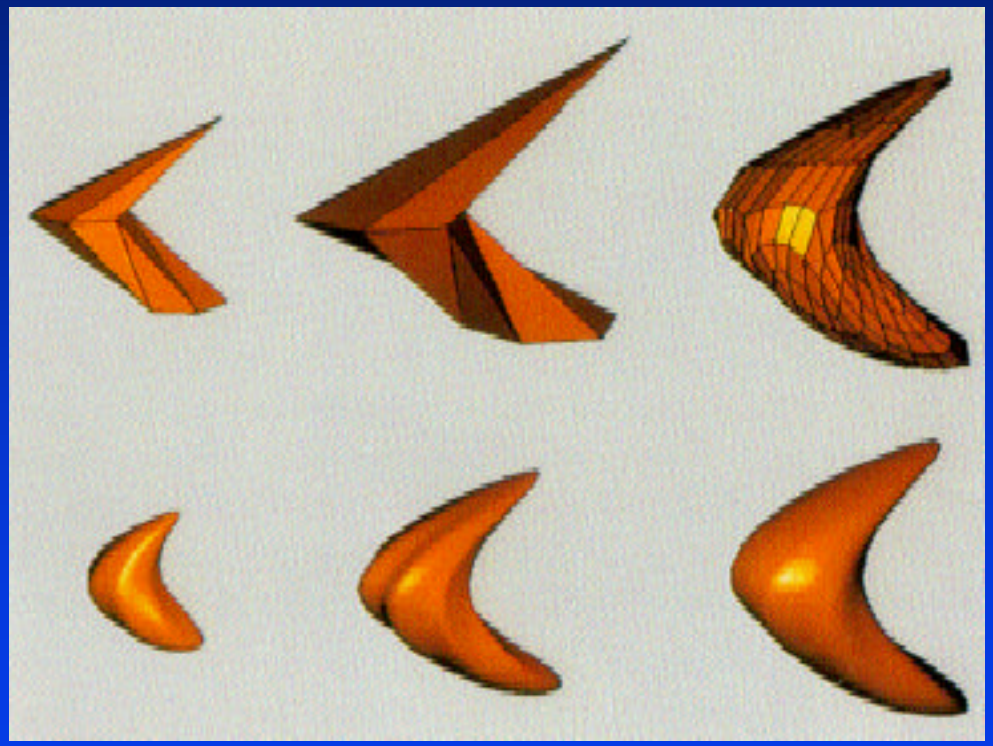
Hierarchical Editing





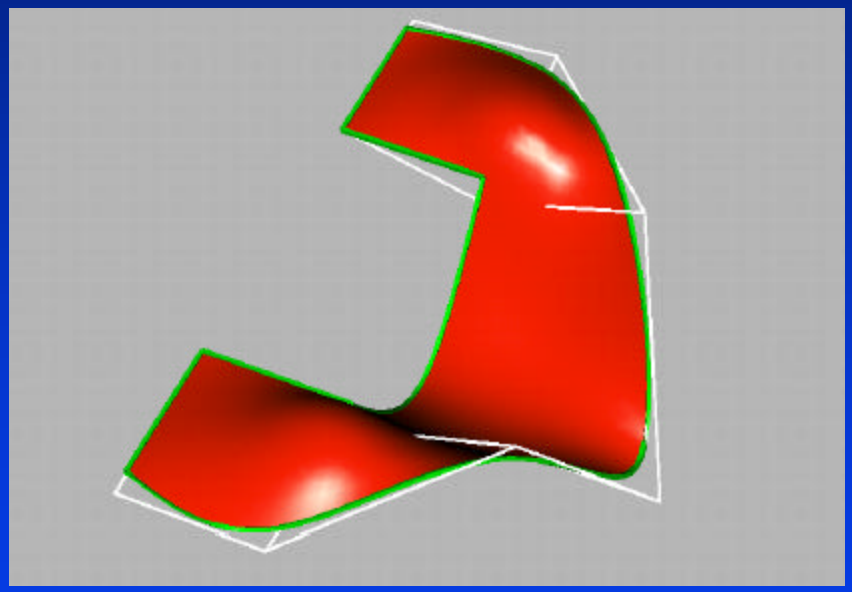
Zorin et al. Siggraph 97

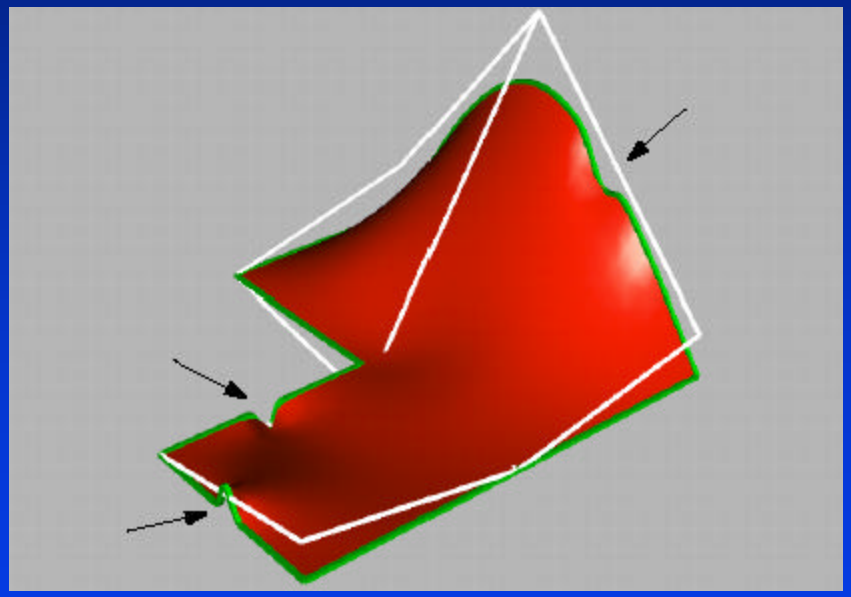
Fair Surface Design



Halstead et al. Siggraph 93

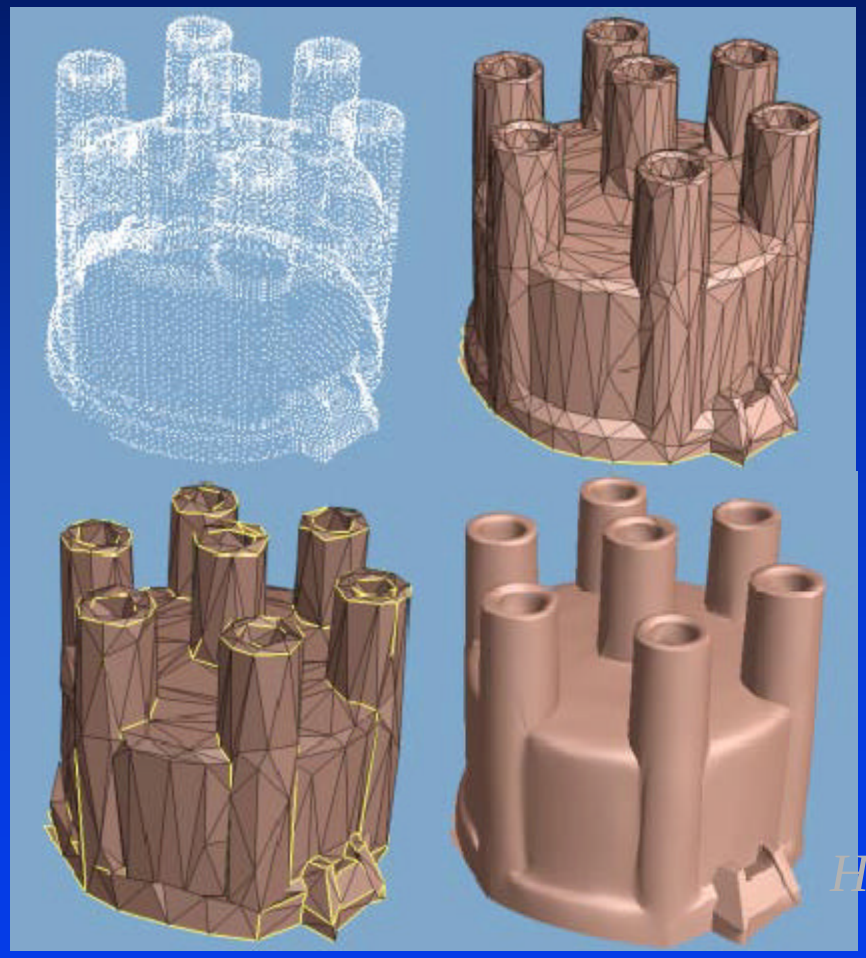
Boundary Control





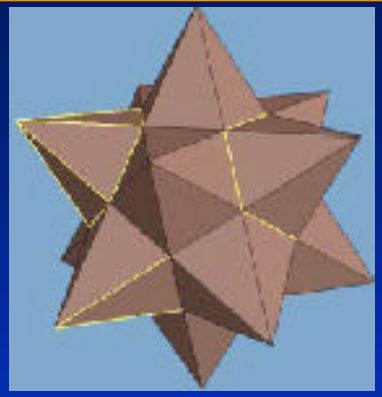
Levin, Siggraph 99

Surface Reconstruction

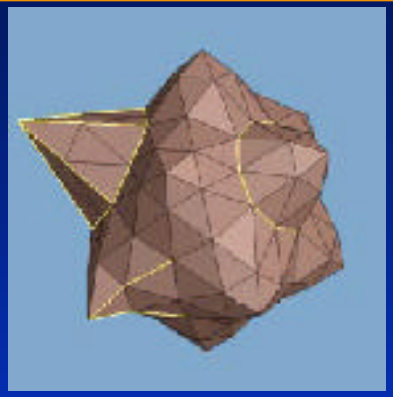


Hoppe et al. Siggraph 94

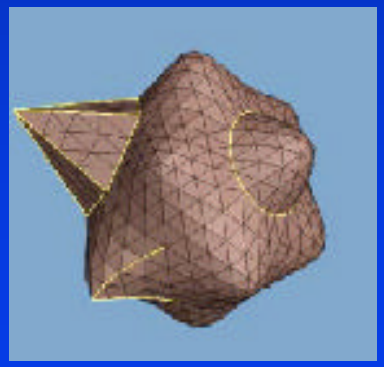
Piecewise Smooth Subdivision



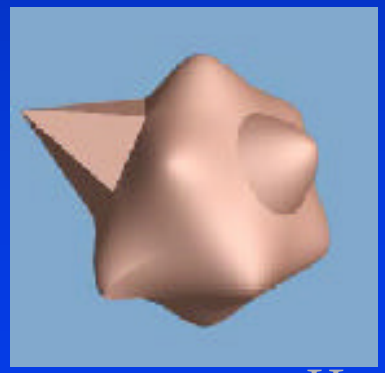
(a)



(b)



(c)



(d)

Hoppe et al. Siggraph 94

“Geri’s Game”

DeRose et al. Siggraph 98

Our Objectives

- One key goal is to promote subdivision representation and techniques to become the next-generation industry standard!
- I will re-visit this aspect later!

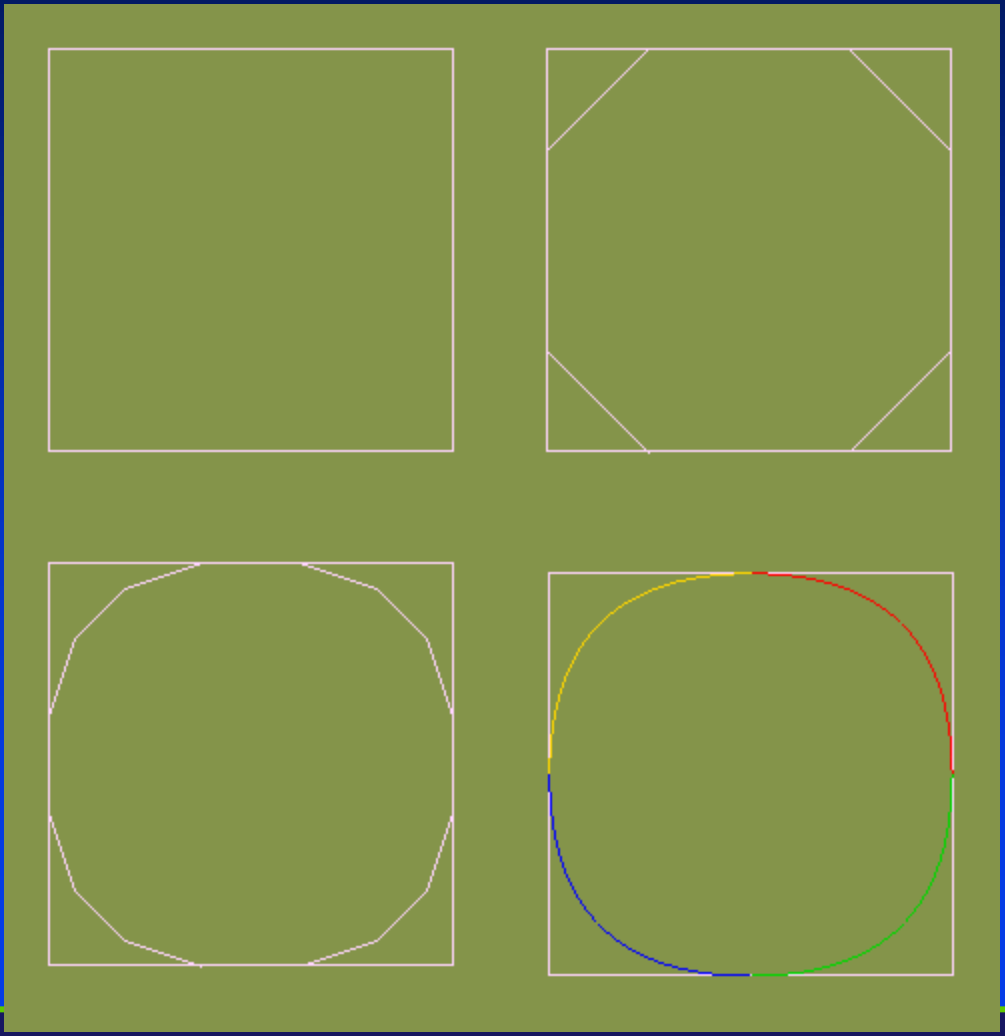
Course Overview

- Fundamental idea and simple examples
- Motivation
- Historic review on existing techniques
  - Curves, surfaces
  - Comparison
- Subdivision splines
- Current work on subdivision solids
- On-going research topics and possible future research directions (if time permits)

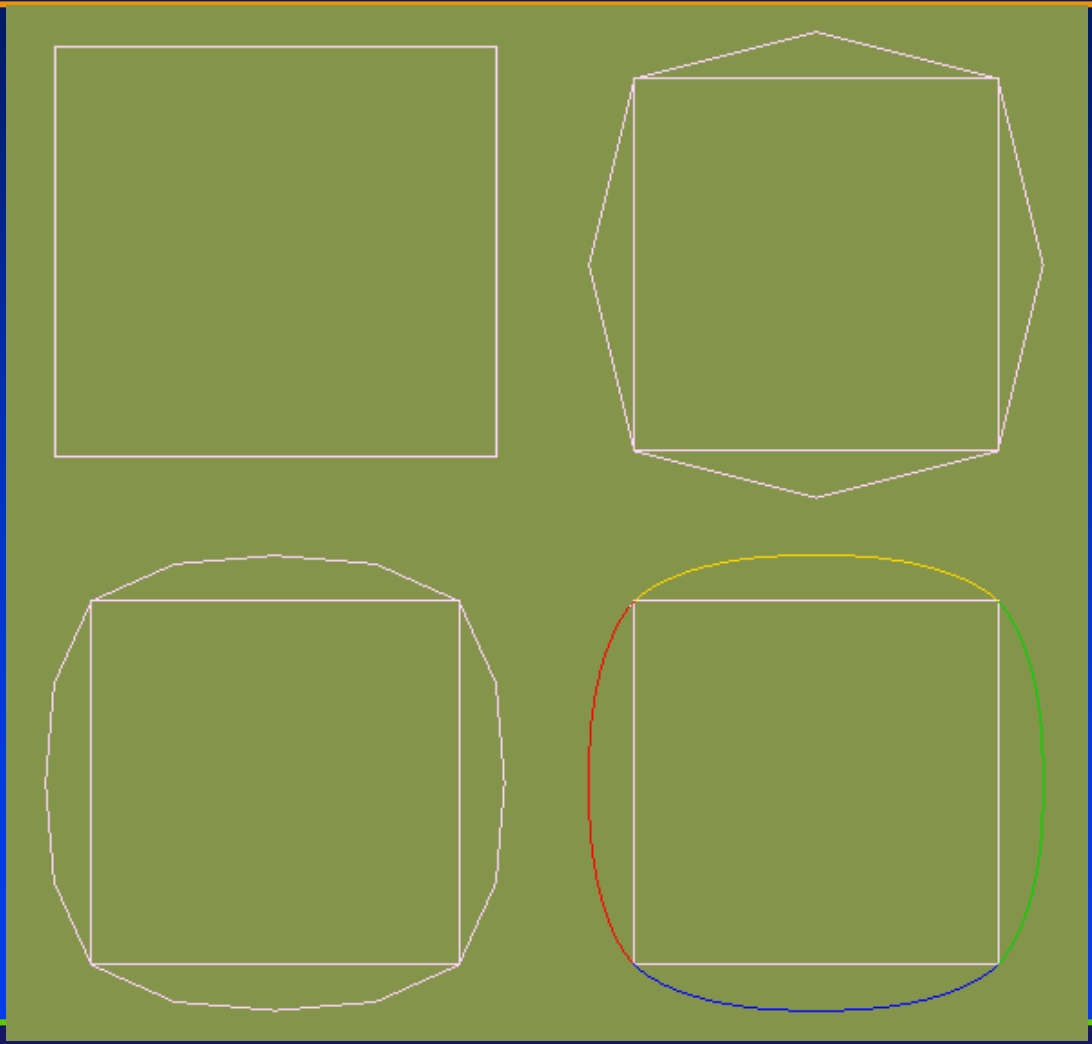
Subdivision Splines

- We treat subdivision as a novel method to produce spline-like models in the limit
- Key components for spline models
  - Control points, basis functions over their parametric domain, parameterization, piecewise decomposition
- Parameterization is done naturally via subdivision
- The initial control mesh serves as the parametric domain
- Basis functions are available for regular settings as well as irregular settings
- Control points for one patch are in the vicinity of its parametric domain from its initial control vertices
- Subdivision-based spline formulation is fundamental for physics-based geometric modeling and design, finite element analysis, simulation, and the entire CAD/CAM processes

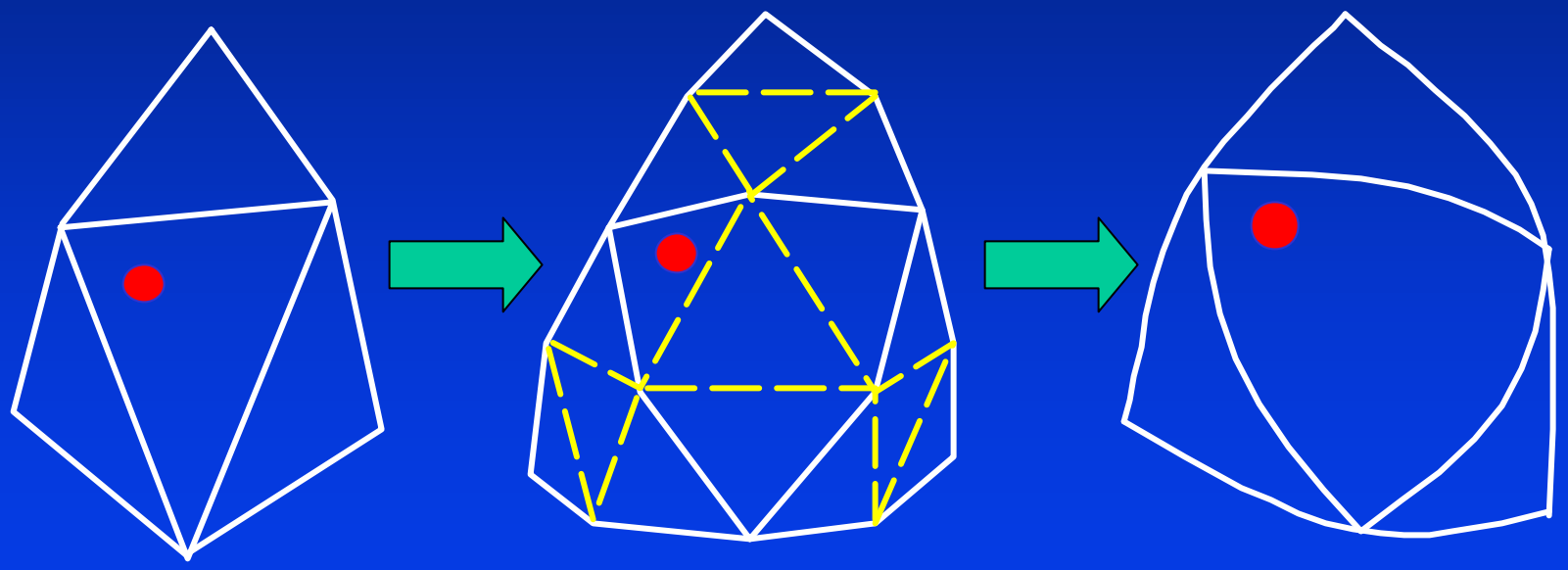
Chaikin Curve Example



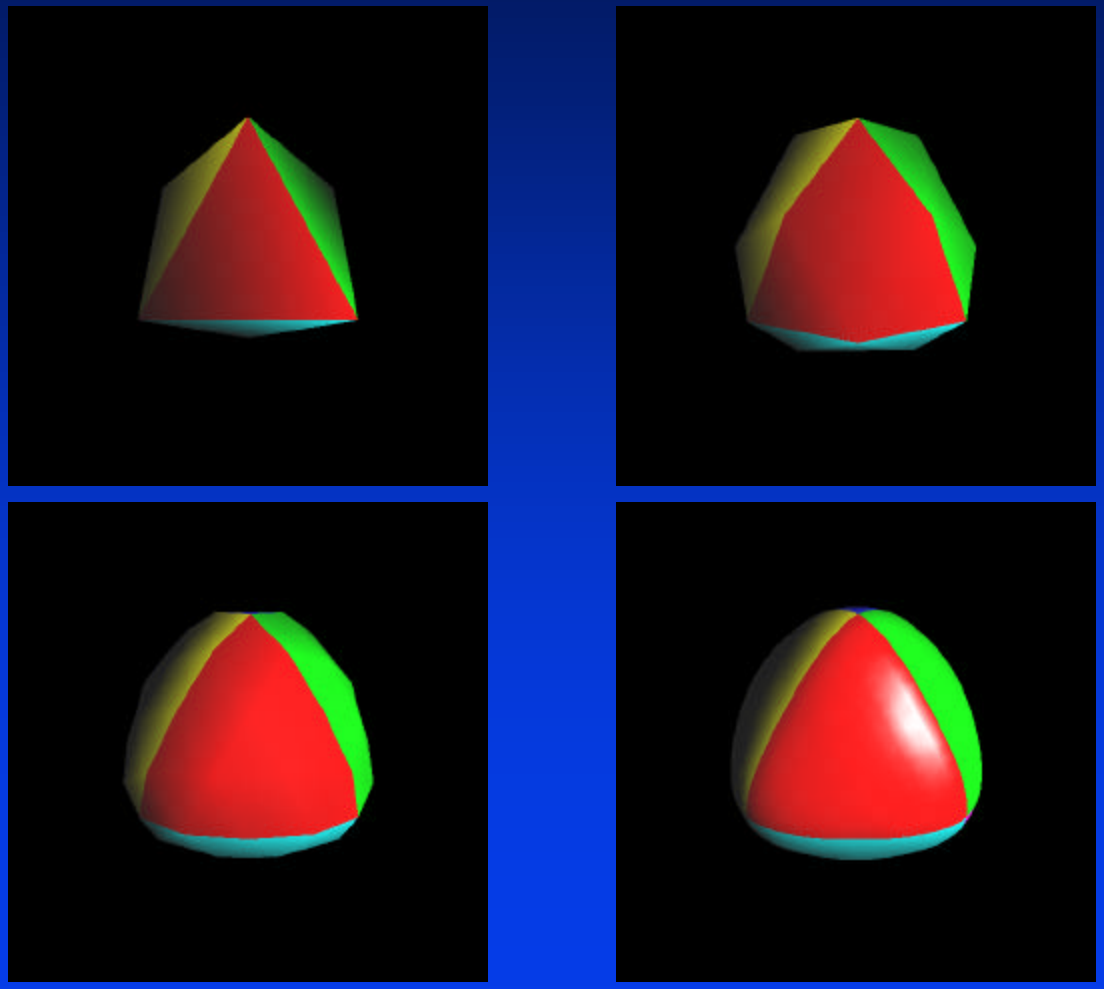
Interpolation Curve Example



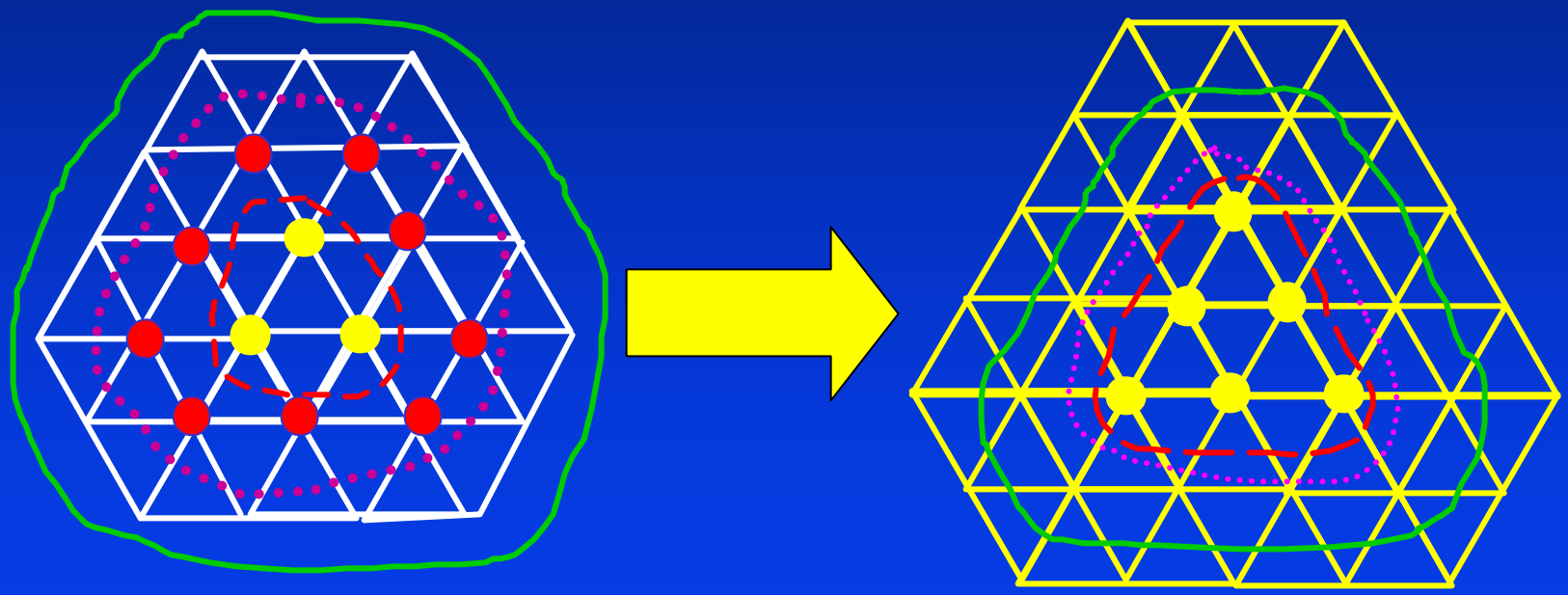
Parameterization



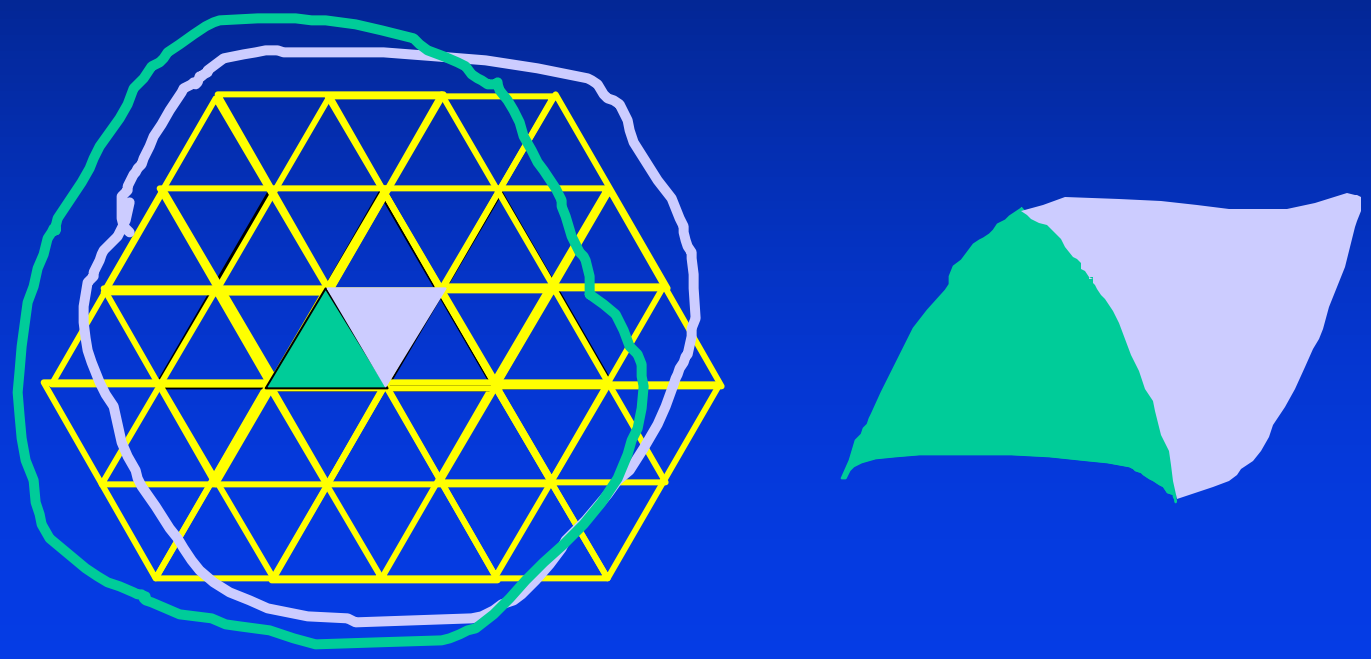
Butterfly Surface Example



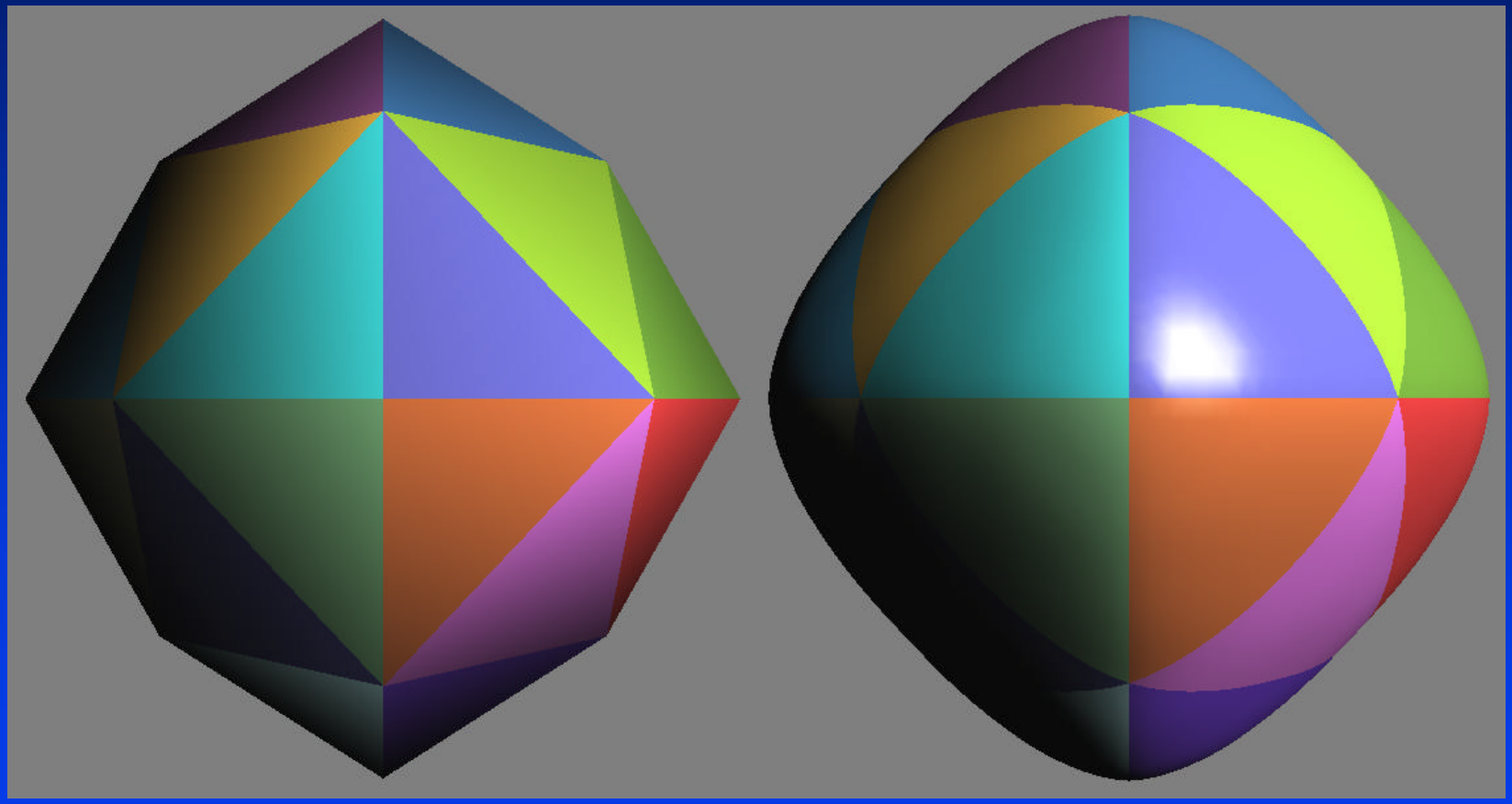
Control Vertices for Butterfly Surface



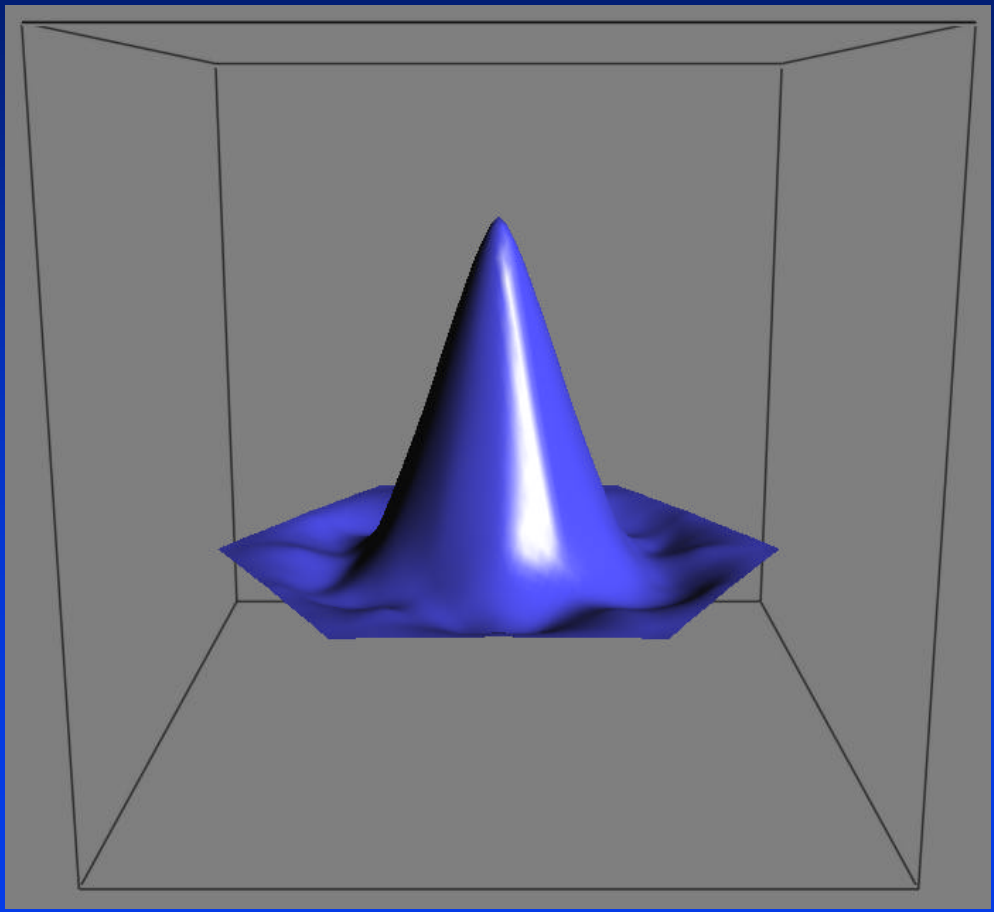
Control Vertices for Surface Patches



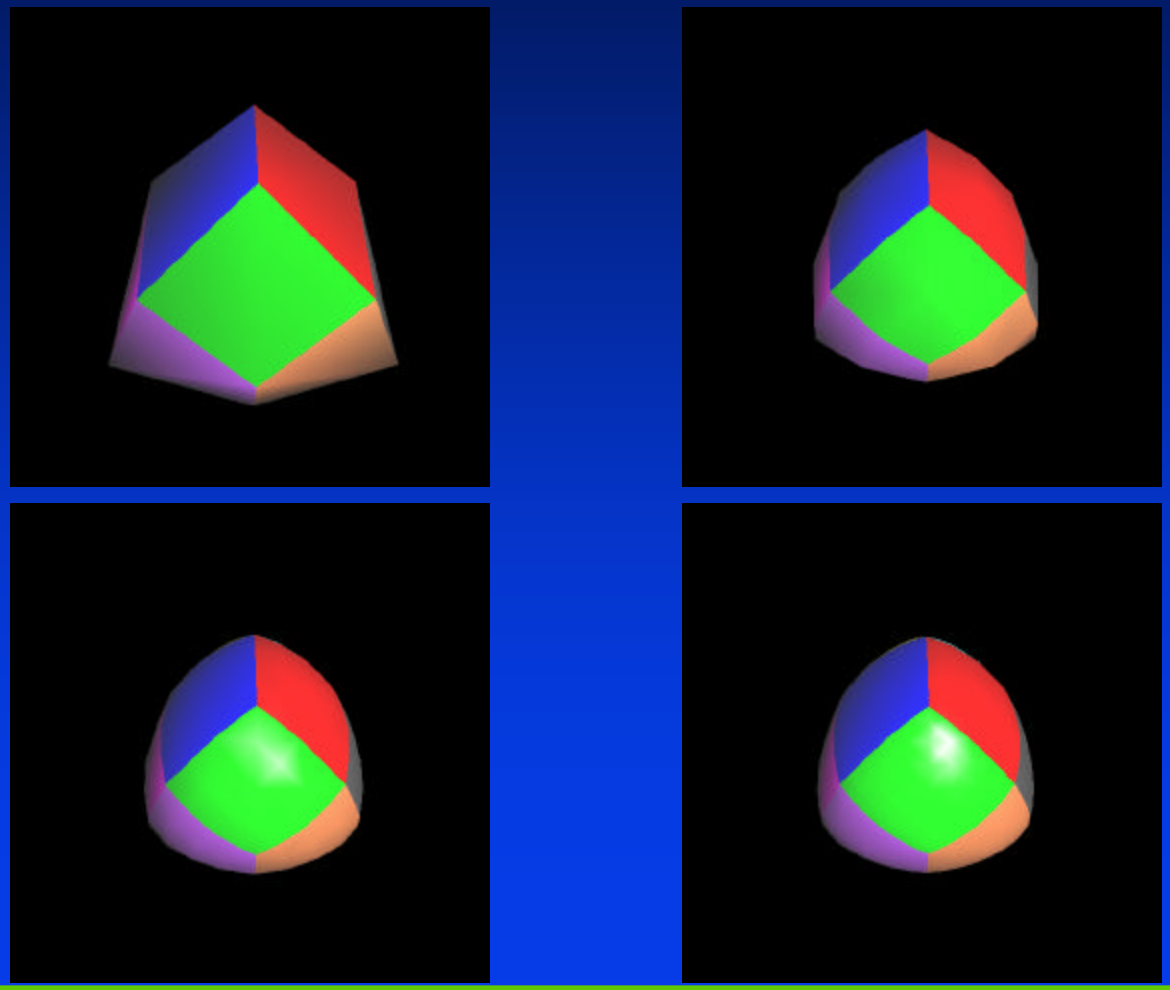
Butterfly Patches



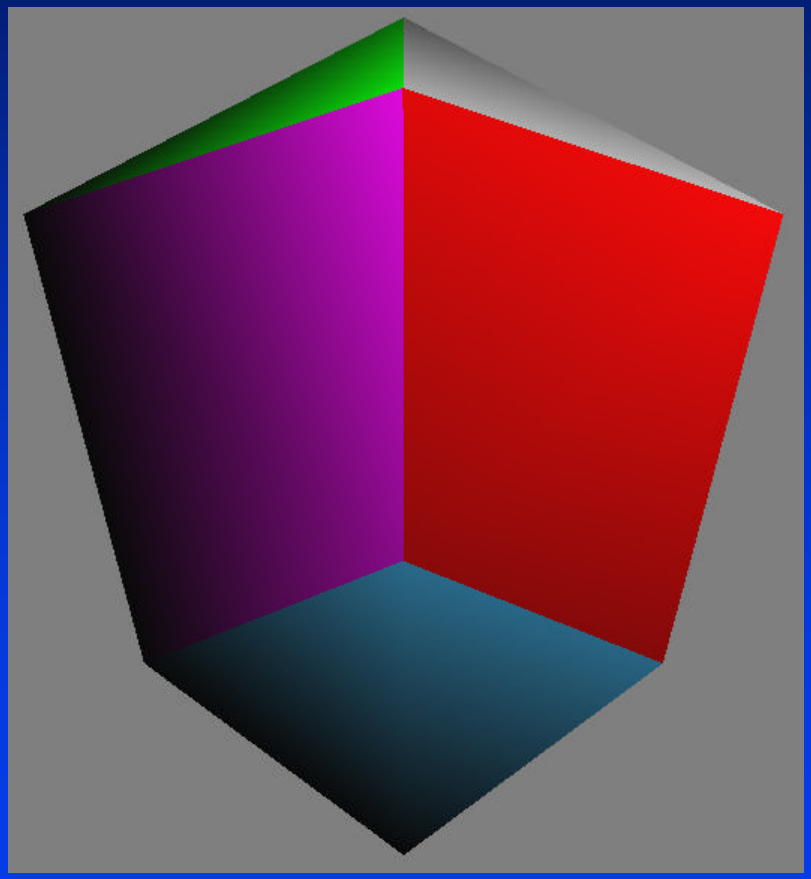
Butterfly Basis Function

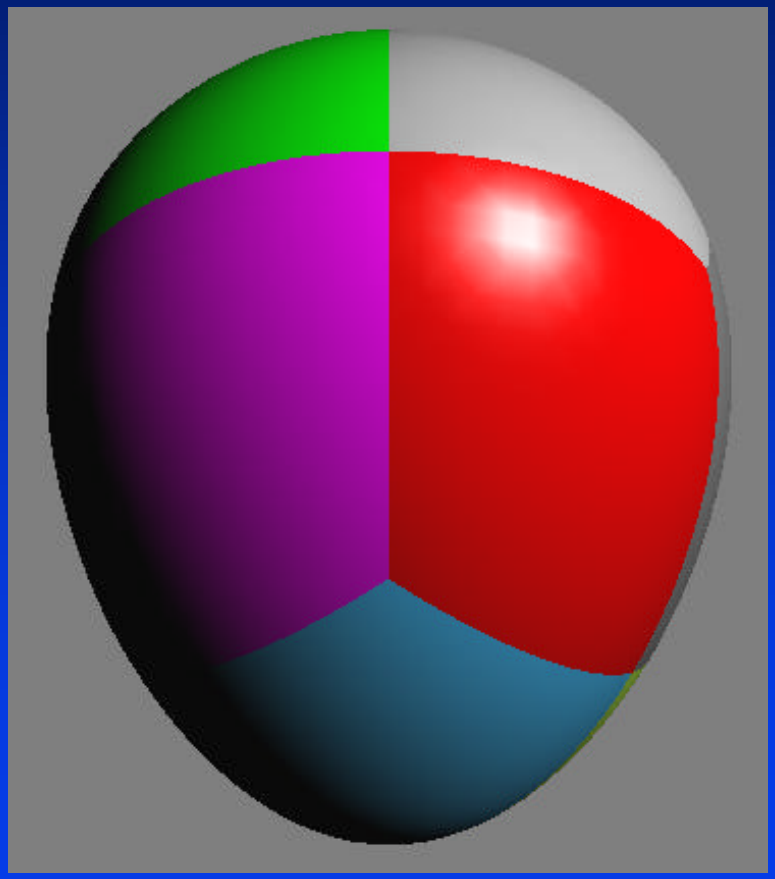


Catmull-Clark Surface Example

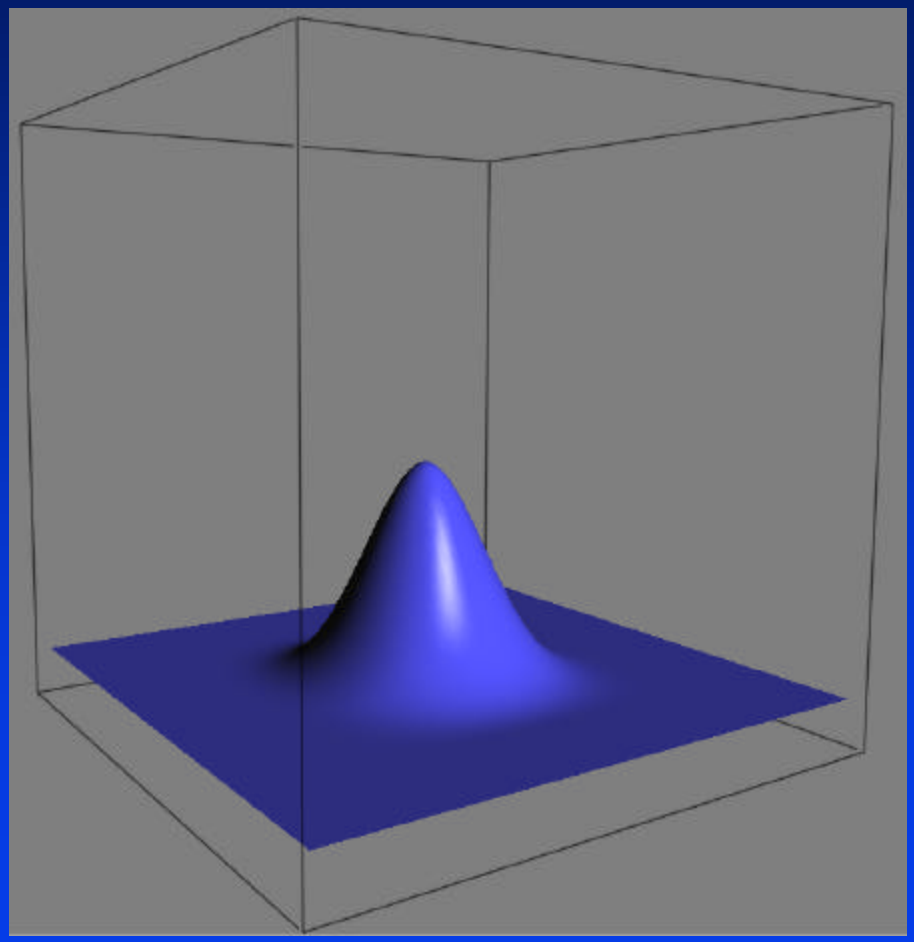


Catmull-Clark Patches

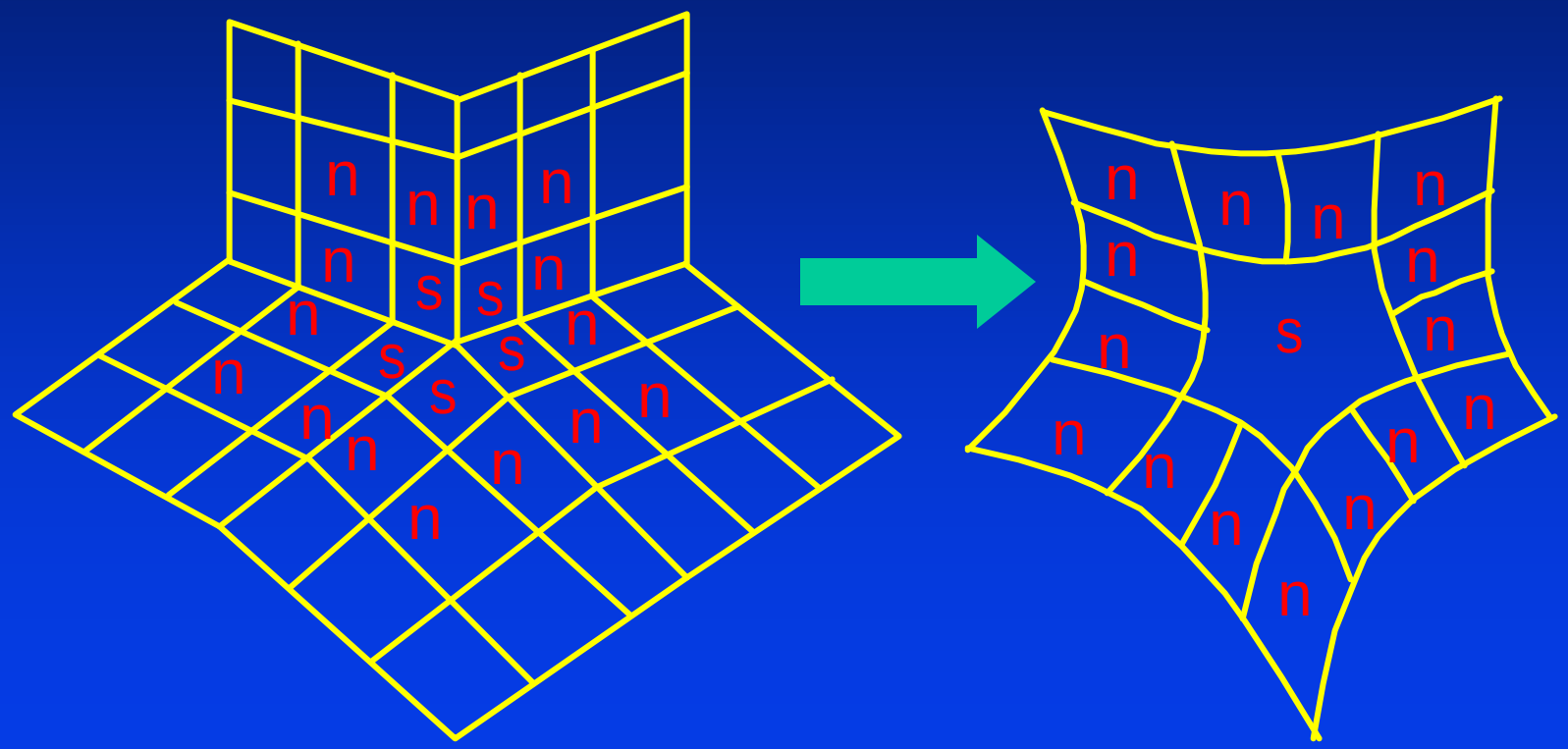




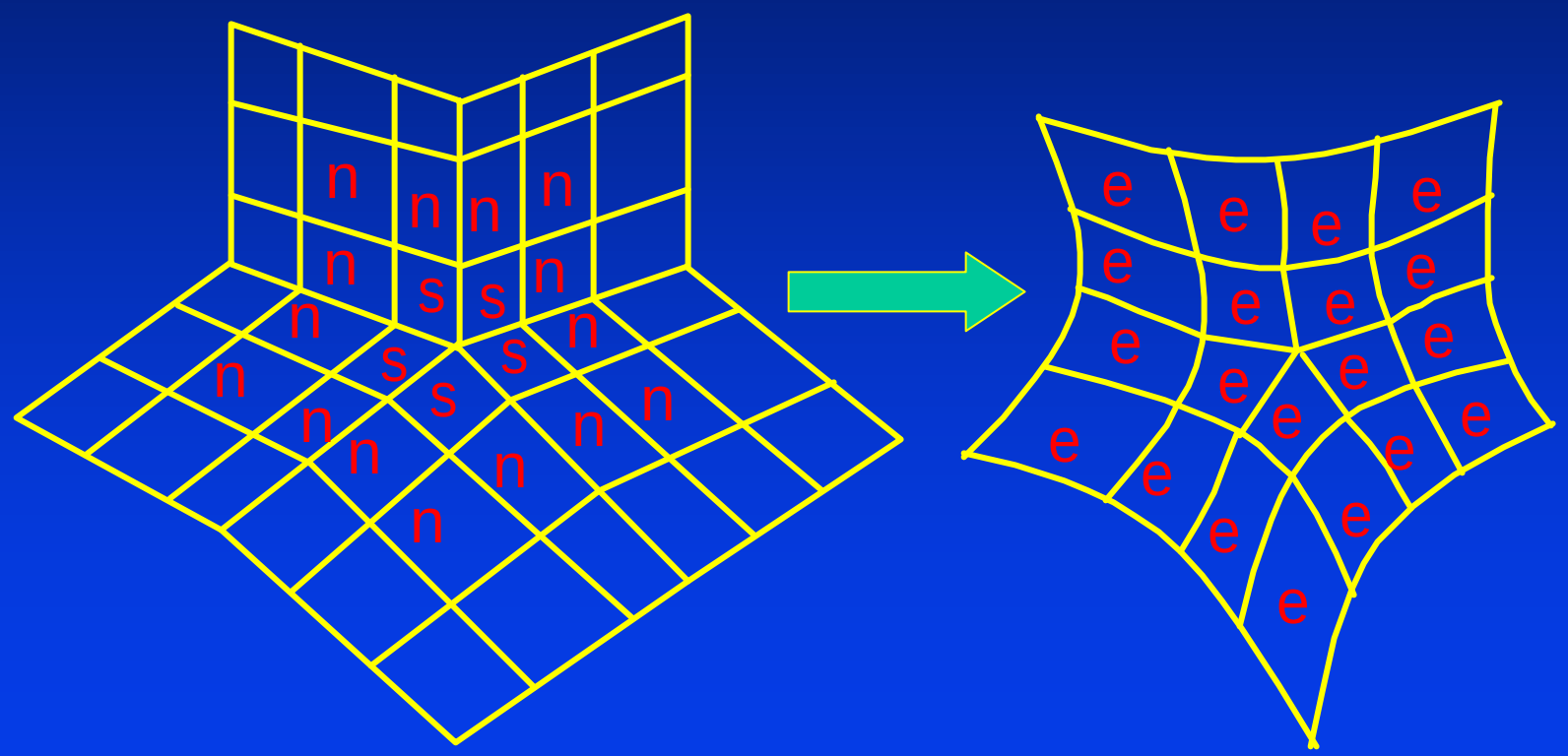
Catmull-Clark Basis Function



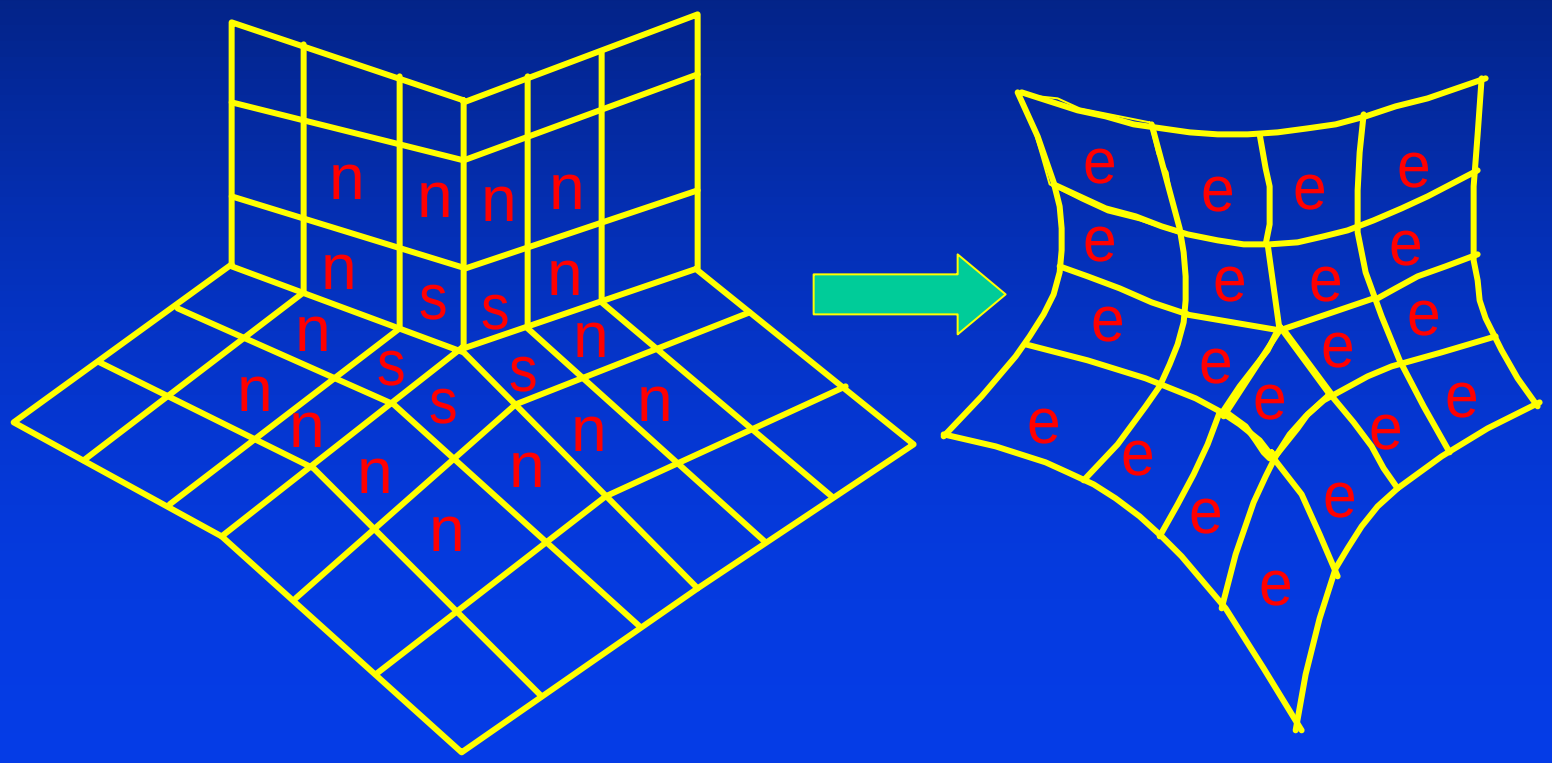
Spline Decomposition


n n n

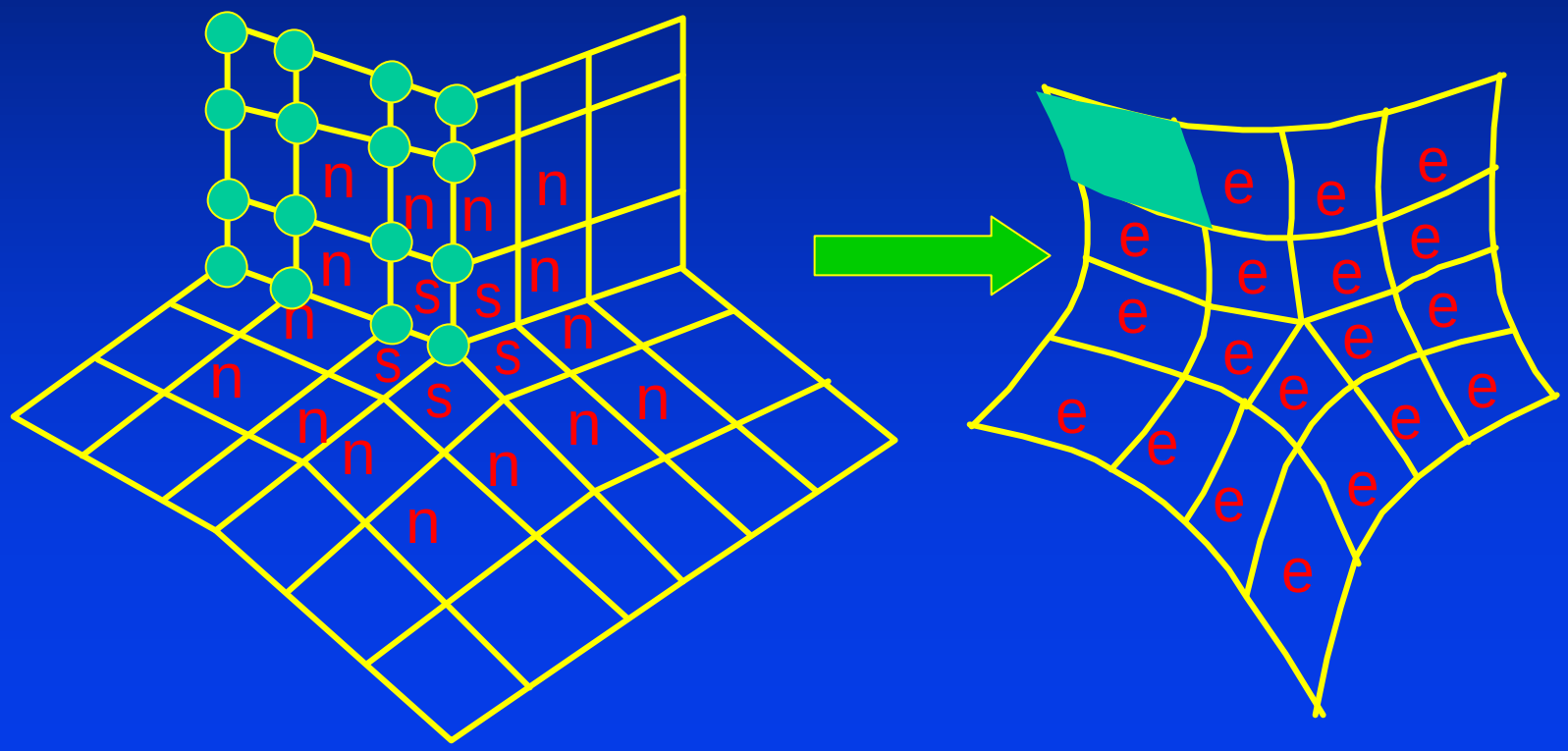
Single Type of Patch Decomposition


n n n

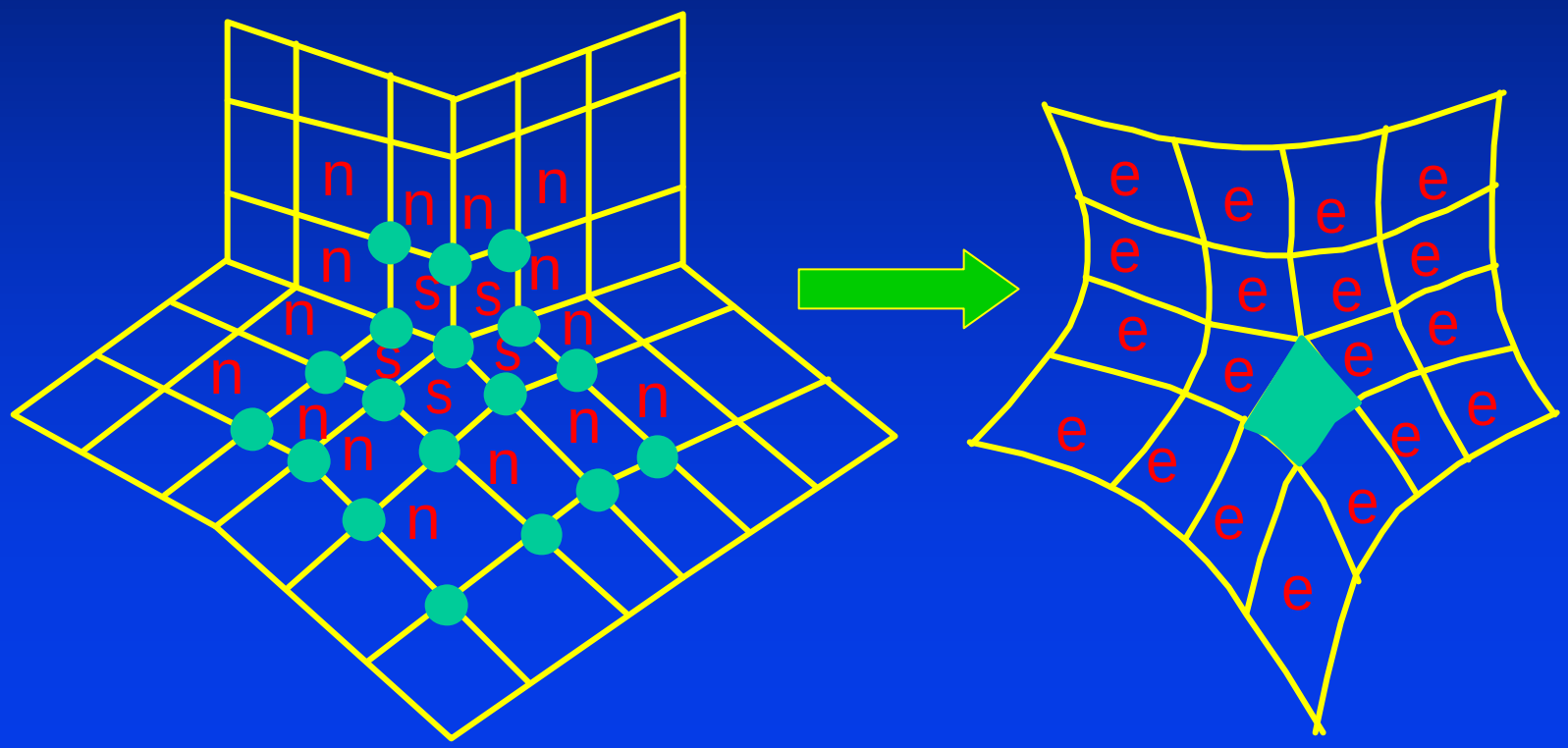
Parametric Domain



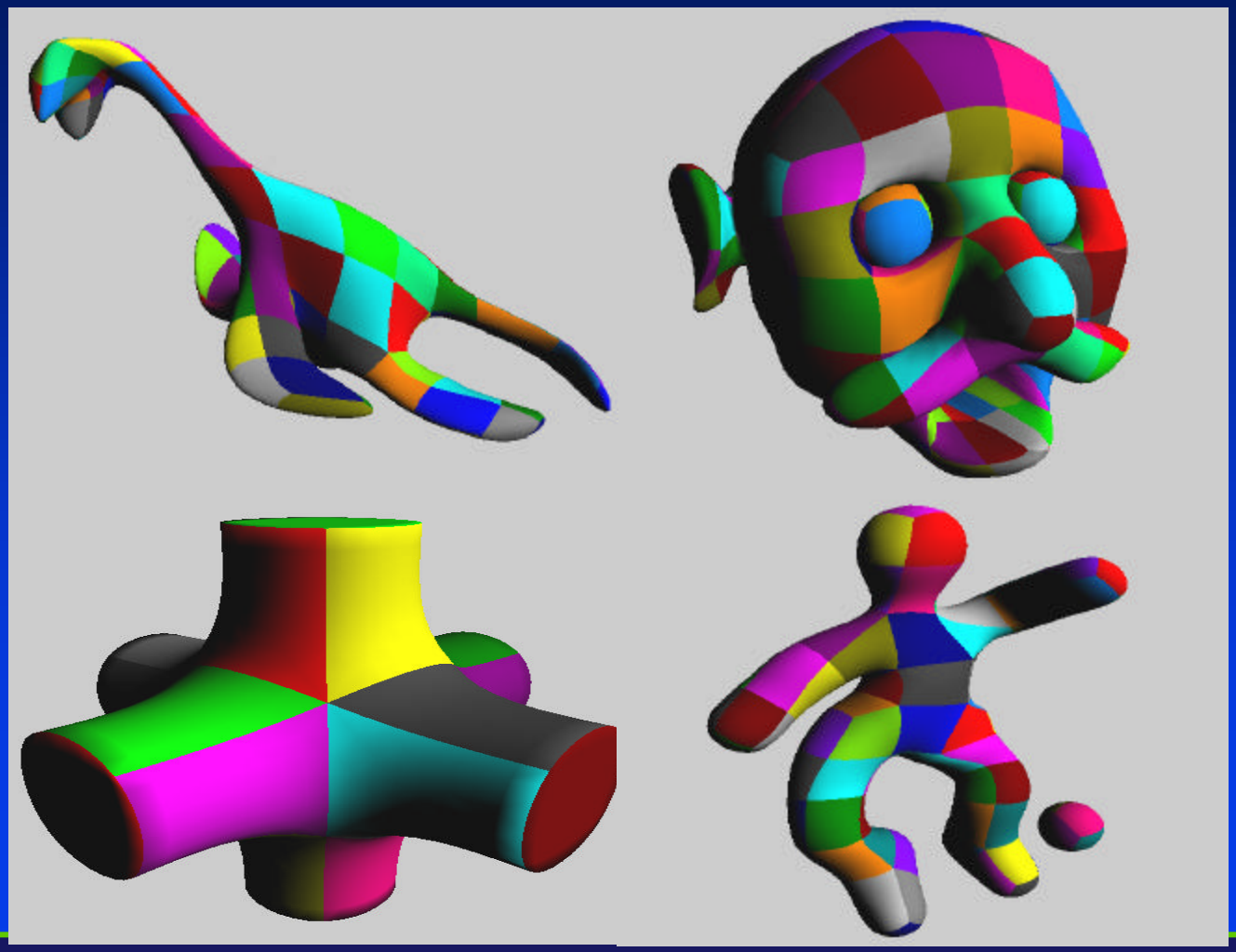
Control Vertices for Catmull-Clark Surface



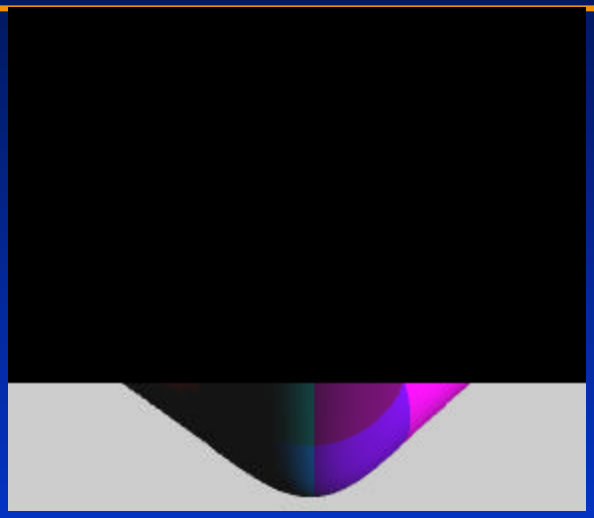
Control Vertices for Catmull-Clark Surface



Interactive Sculpting



More Examples

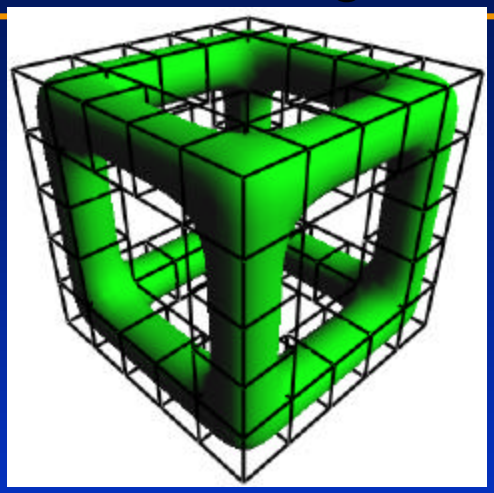


Data Structures

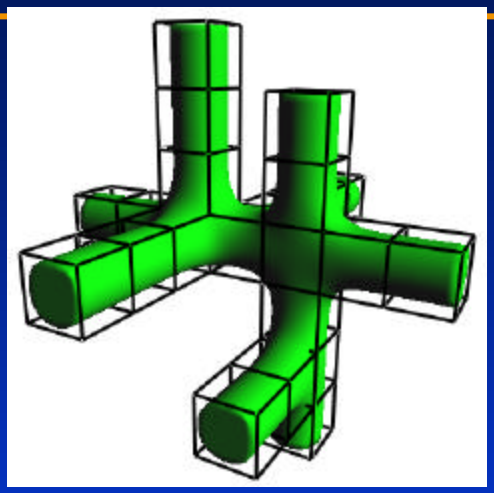
- Subdivision solids
  - Radial-edge data structure (Weiler `86)
  - Similar to winged-edge data structure
  - Stores adjacency information to accelerate queries of and changes to topology of subdivision solids
- Physical representation
  - Sparse matrices, vectors, arrays, etc.

Sculpting Tools

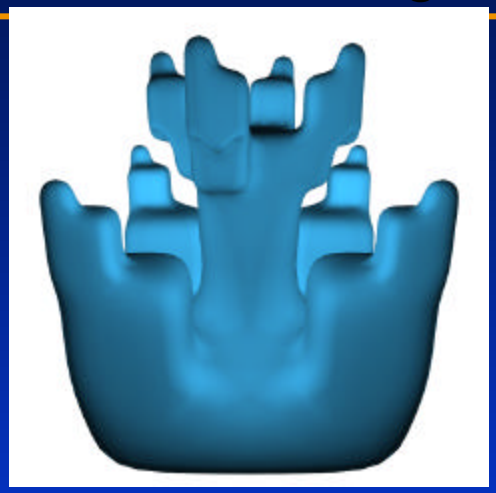
carving



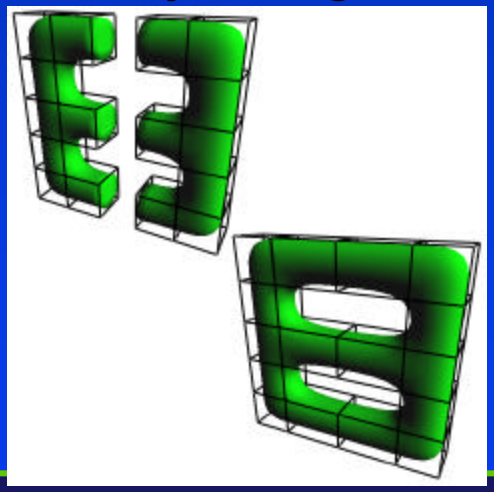
extrusion



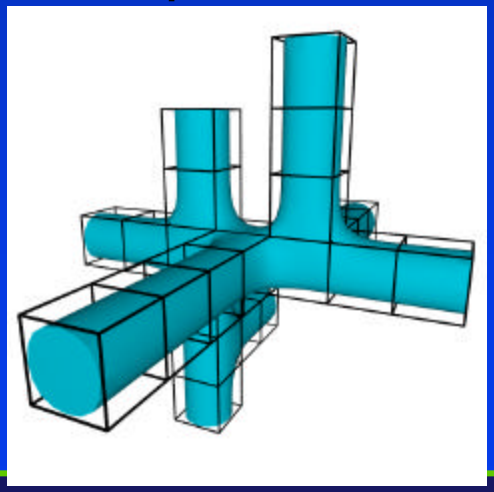
detail editing



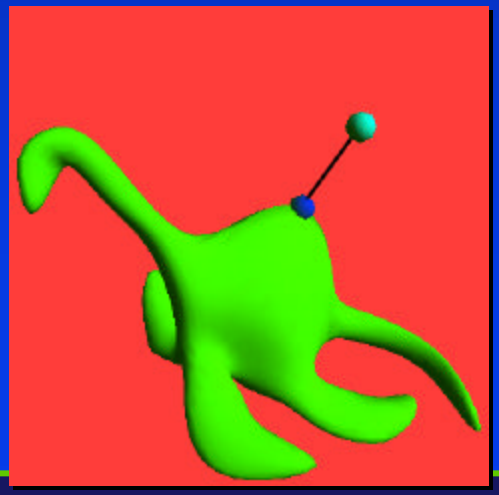
joining



sharp features



deformation

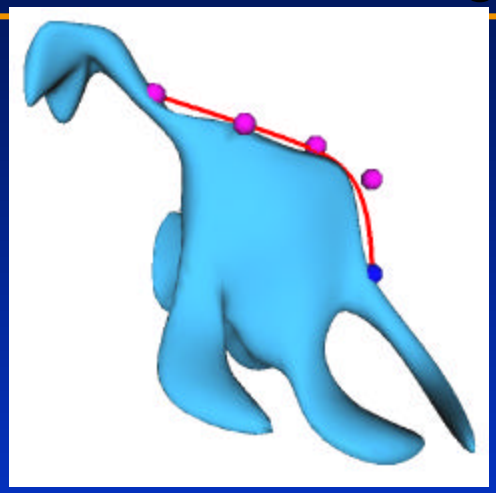


Sculpting Tools

inflation

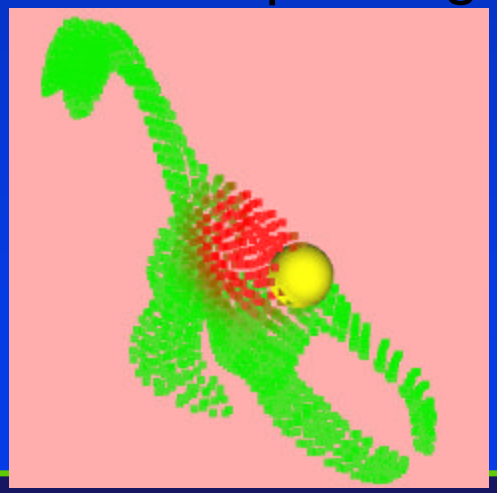
deflation

curve-based design

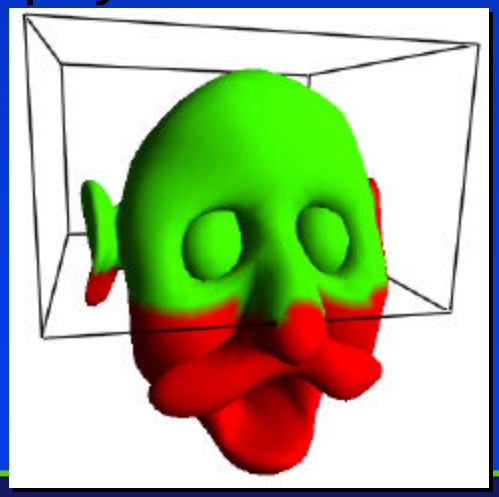


material mapping

material probing

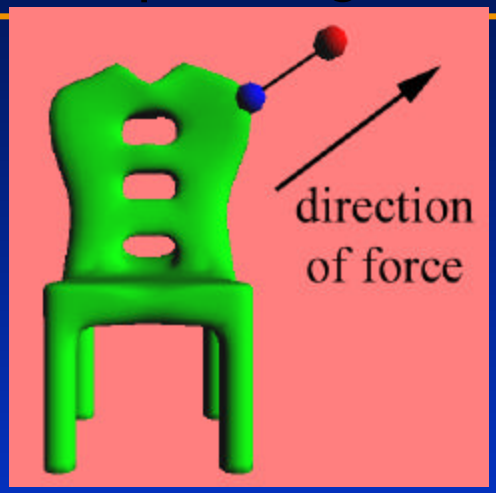


physical window

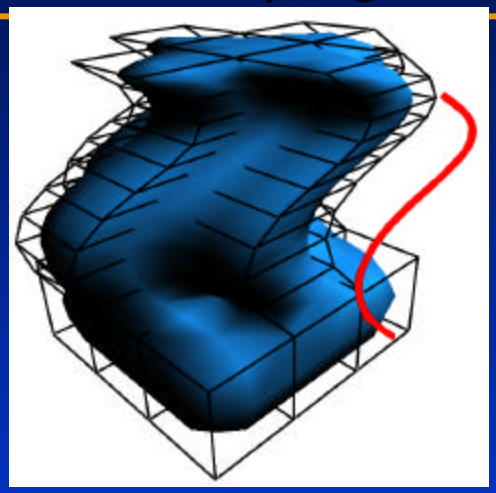


Sculpting Tools

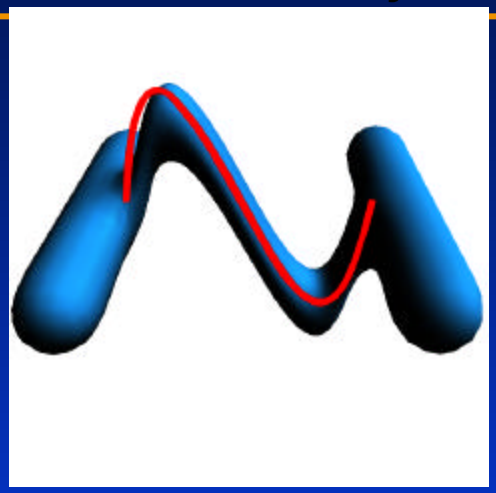
pushing



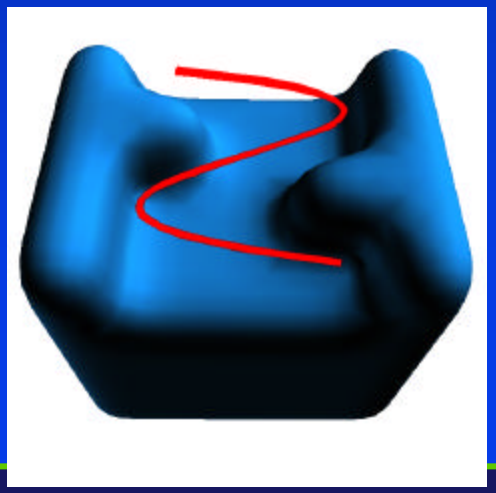
sweeping



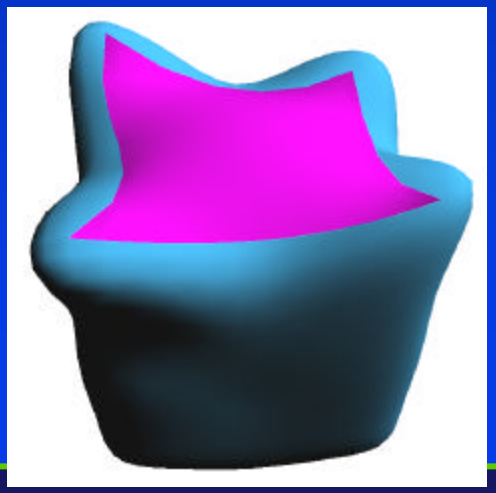
curve-based join



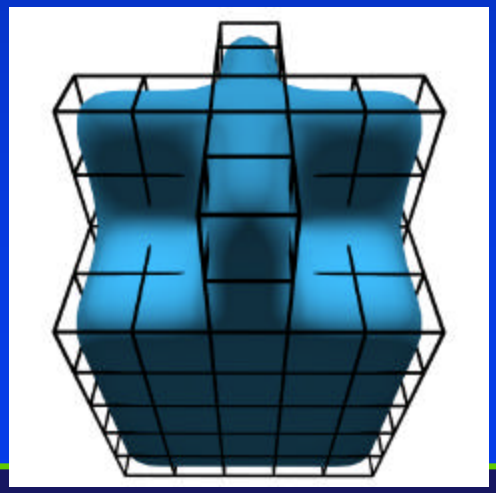
curve-based cutting



feature deformation



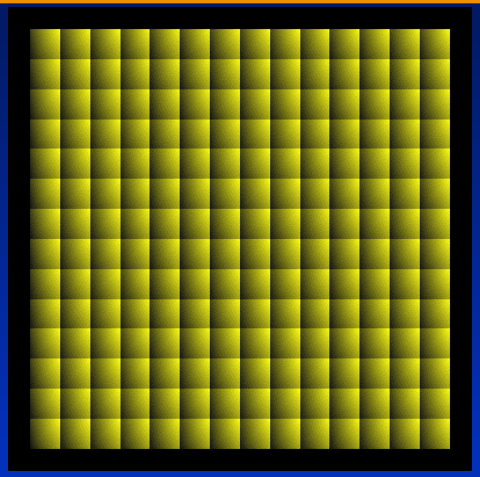
multi-face extrusion



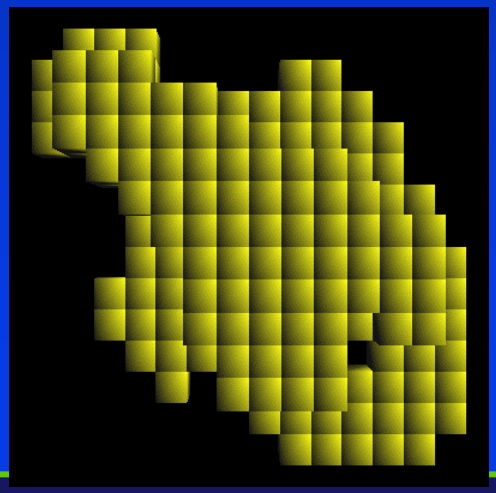
Trimmed Solids for Data Fitting

original dataset

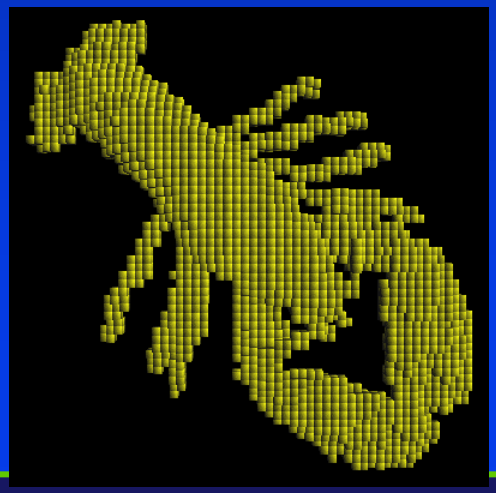
initial lattice



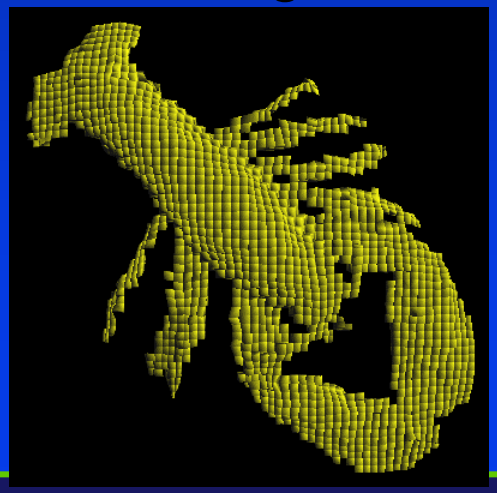
trimmed once



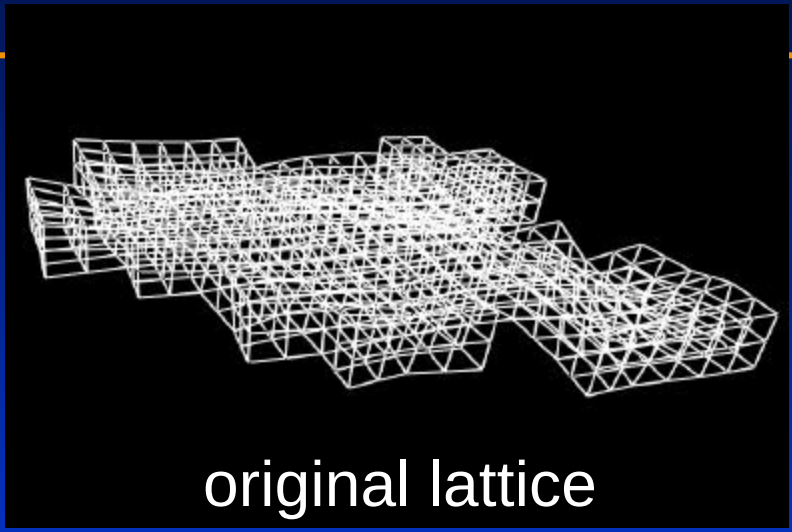
trimmed twice

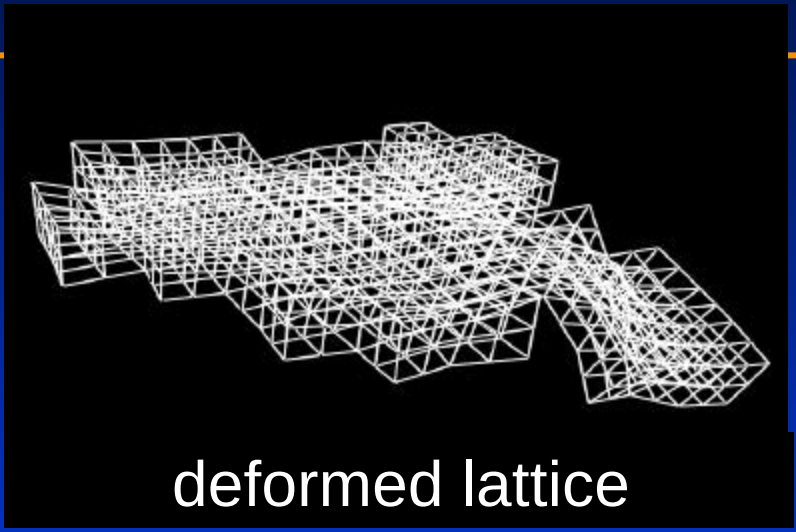


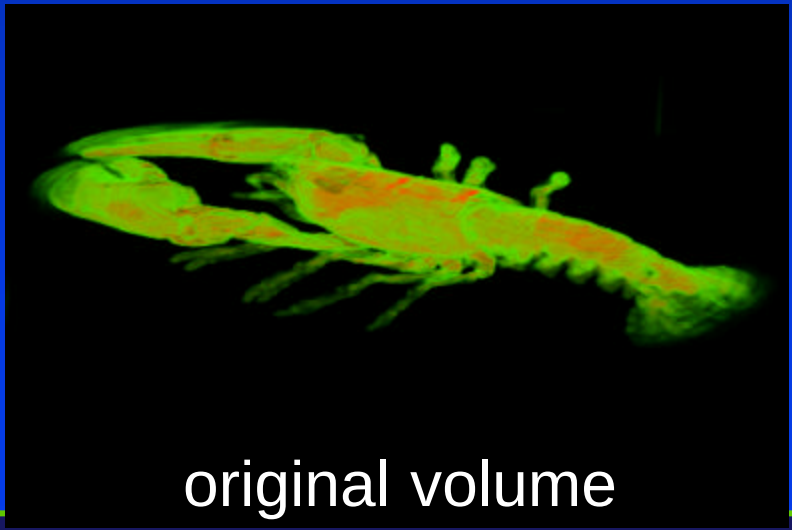
deformed geometry

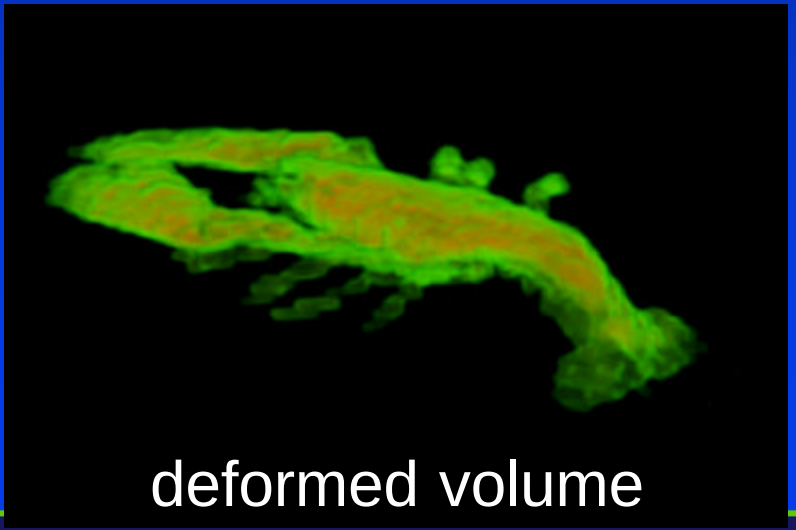


Volume Editing and Visualization

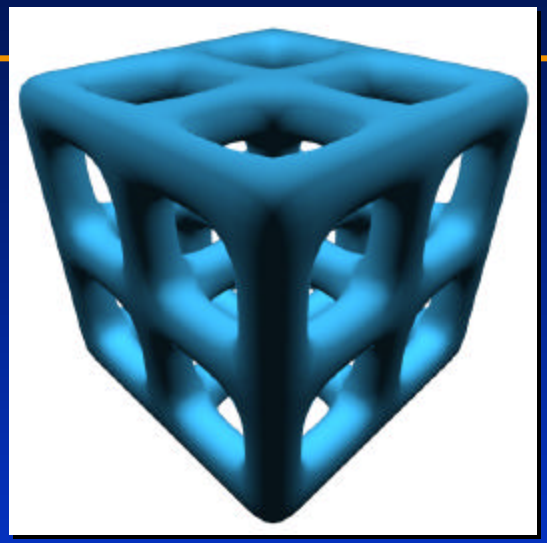
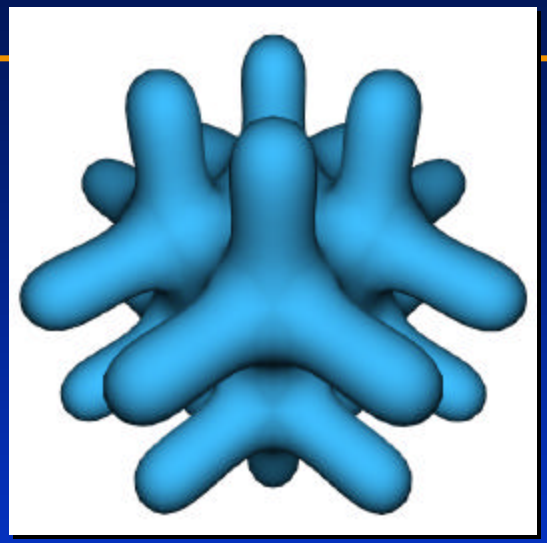
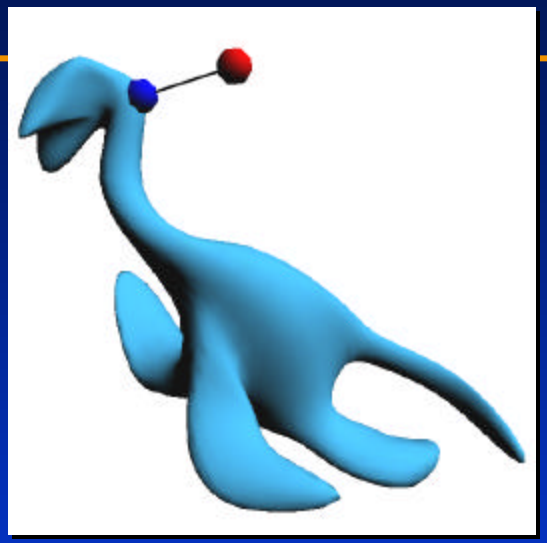


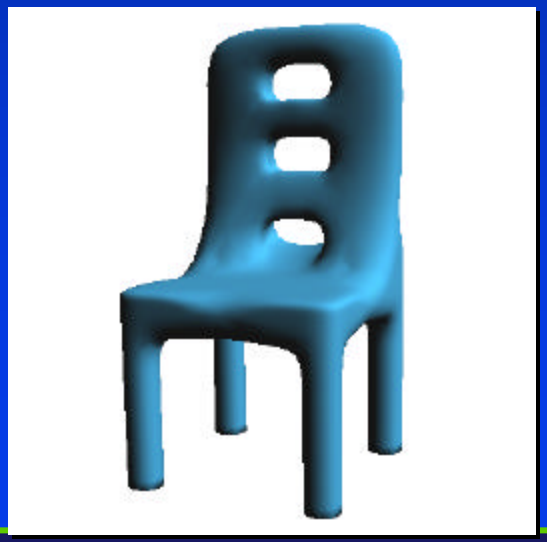
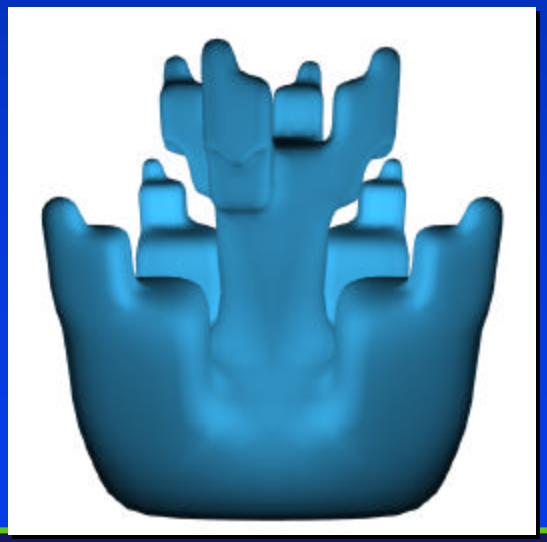




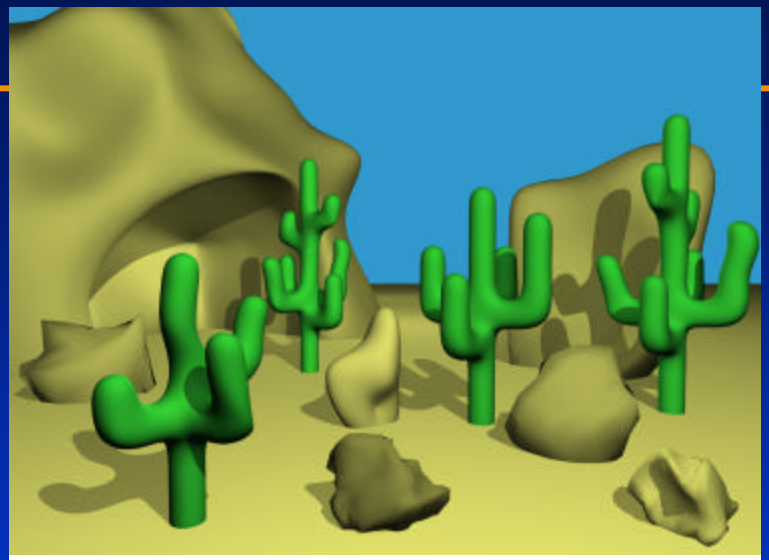


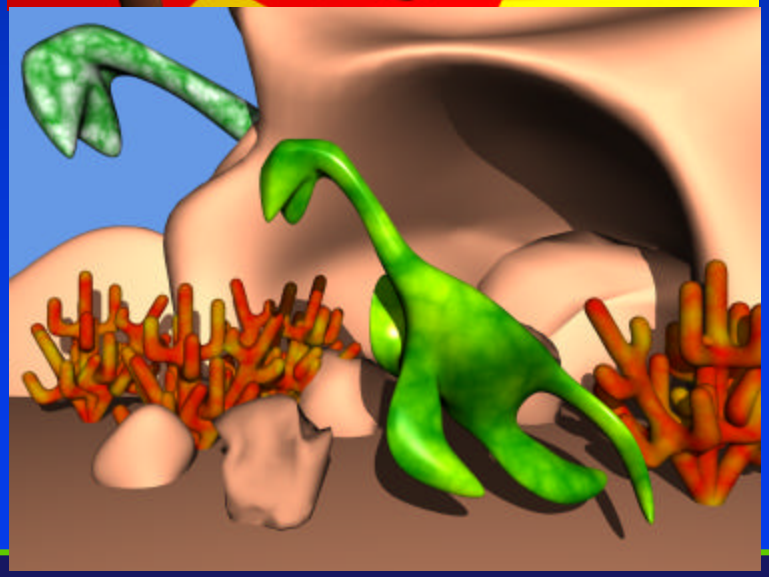
Sculpted CAD Models

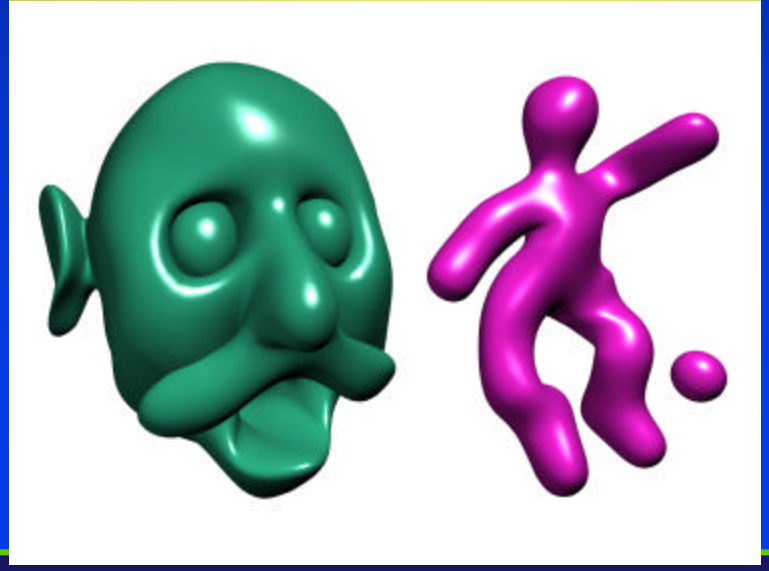




Scenes and Sculptures







Course Overview

- Fundamental idea and simple examples
- Motivation for subdivision
- Historic review on existing techniques
  - Curves, surfaces
  - Comparison
- Subdivision splines
- Current work on subdivision solids
- On-going research topics and possible future research directions (if time permits)

Subdivision for CAD/CAM

- Extensive applications in graphics and animation for entertainment industry during the past two decades
- Powerful modeling candidate for natural shapes and organic objects
- Key features
  - Complex geometry, arbitrary topology
- However, less popular for CAD-based, manufacturable models
- Current standards in CAD/CAM
  - NURBS, algebraic CSG

Current Research Overview

- Chaikin's pioneering work in 1974
- Historic perspective: subdivision as a display tool for parallel rendering
- At present, subdivision is primarily considered as a powerful modeling tool
- The unification of modeling and rendering
- Research progresses during the past three decades
  - Subdivision splines for curves
  - Surface generalization
  - Solid subdivision

Motivation for Future Research

- Ever-increasing, high expectations of
  - Improved product quality
  - Reduced product prices
  - Accelerated performance
- Challenges
  - New design theory and methodology
  - Advanced simulation methods
  - Efficient analysis tools
  - More powerful human-computer interaction
- Strategy in CAGD, FEM, CIMS
  - Subdivision-based representation, modeling, design, analysis, and manufacturing techniques for the next generation CAD/CAM system

NURBS: State of the Art

- Industry-standard in engineering design
  - Many many properties, a unified representation for free-form splines and algebraic shapes, nevertheless…….
- Limited (rectangular) parameterization
- Complicated geometry and arbitrary topology via trimming and patching
- Continuity across trimmed boundaries is far from trivial
- Surface-surface intersection must be performed, however, it is time-consuming and prone to error
- Non-linear constraints to enforce smoothness requirements
- Dynamic modeling for time-evolving scenarios is very difficult
- Extensive human intervention for patching

Subdivision Advantages

- Generalization of B-splines and NURBS
- No trimming and patching operations
- Continuity is guaranteed
- Arbitrary, irregular structure of control vertices
- Non-rectangular, local parameterization
- Feature definition via non-stationary subdivision
- Local subdivision and refinement
- Unify splines and polygonal models
- Integrate rendering and modeling tasks in a hierarchical fashion
- Polygonal approximation for fast display

More Advantages

- Unified principle for curves, surfaces, solids, and non-manifold objects
- No need for additional blending/rounding/filleting operations for solid modeling
- Hierarchical structure
- Wavelet-based multi-resolution analysis and synthesis
- Hardware implementation and acceleration based on addition and shift operations because of simplicity
- Amenable to parallel algorithms
- Numerical stable
- Diversity of subdivision schemes

Technical Challenges

- No global parameterization
- Limit shape only has theoretical significance
- Polygonal approximation in practice
- No standard for data exchange to/from NURBS
- Lack of theory/algorithm/technology in general
- A lot of potential yet to be realized

Modeling Difficulty

- A large number of irregular control vertices organized to form arbitrary lattices
- Various subdivision rules
- Shape manipulation, editing, and deformation are not easy
- Fairness criteria and functional requirements
- Both quantitative and qualitative constraints

Why Bother?

- NURBS-based algorithms and systems are no longer applicable
- Recursive subdivision procedure
- Both geometric and topological information
- Bridge the large gap between potential and reality

On-going and Future Research Efforts

- Subdivision splines as a underlying representation
- Rigorous geometric theory
- Novel modeling and design techniques
- Efficient simulation and analysis toolkits
- Computer-integrated environment

Fundamental Theory & Properties

- Paramount significant --- vital to algorithms, modeling techniques, and system integration
- Closed-form analytical formulation and properties of subdivision basis functions
- Analysis of parametric and geometric continuity
- Best numerical approximation scheme(s) for subdivision
- Robust error analysis
- Convergence performance
- Mathematical solutions to approximate standard shapes and commonly-used features

Theory

- In general, subdivision theory is under-explored
- Theory and properties lead to better algorithms and software routines
- Main foci: algebraic, geometric, differential, and integral properties
- Subdivision as a set of patches --- a linear combination of basis functions
- We shall look at basis functions

$$\mathbf{B}_i(\mathbf{x})$$

Properties

- One basis function is also a subdivision splines, consider

$$\mathbf{p}_i = \mathbf{1}, \mathbf{p}_j = \mathbf{0}, j \neq i$$

- The key is to have a better understanding of basis functions
- In comparison with NURBS basis functions
  - partition of unity
  - positivity
  - local control
  - continuity

Properties

- Convergence
- Parametric and geometric continuity
  - Necessary and sufficient conditions for Ck and Gk continuity
  - Special treatment at “extraordinary points”
- Rapid and accurate evaluation of position, tangent, normal, curvature, and other differential quantities across the entire parametric domain
- Vital to geometric design, shape quality analysis, quadrature computation, finite element assembly, dynamic interaction, etc.

Specific Topics

- Better algorithms for Catmull-Clark and Loop subdivision schemes
- Generalization to other schemes (e.g., Butterfly scheme)
- Higher-order continuity for newer, more complicated subdivision schemes

Computational Techniques

- Geometric and solid modeling toolkits
  - Interpolation, approximation, Boolean operations, boundary evaluation
- Formulation of rational subdivision through the use of weights and knots (subdivision rules)
- Optimized numerical algorithms for physics-based procedures
  - Finite-element assembly, sparse matrix calculation, the integration of FEM equations
- Complexity analysis of these algorithms
- Fundamental numerical characteristics
  - Stability, robustness, accuracy

Subdivision for Special Shapes

- Conic sections and quadratic surfaces
- Generalized cylinders
- Surfaces of revolution
- Swept surfaces, offset surfaces
- Developable surfaces
- B-reps of algebraic CSG models
- Feature representation and toolkits for feature definition, recognition, and modification
- High-level interfaces
- Practical significance

Geometric Algorithms

- In comparison with NURBS
  - Efficient and accurate evaluation of basis functions
  - Local and global differential properties
  - continuity
  - Strong convex hull
  - Variation diminishing
  - Blossom (polar forms)
  - Affine invariance
  - Shape preserving, shape predicability, shape classification
  - Linear precision
  - Knot insertion, degree-elevation
  - Unified formulation for both free-form parametric splines and commonly-used analytic shapes, and more

Typical Algorithmic Topics

- General criteria: numerically stable, efficient, and accurate
- Evaluation for position and derivatives (up to order n) at arbitrary location/region across the manifold
- The computation of both local and global differential quantities (e.g., curvature, area, etc.)
- The interpolation and/or approximation of a set of regular data points, scattered data points, normal quantities, or boundary curves
- Shape-preserving interpolation / approximation

More Topics

- The formulation and subdivision rules of subdivision splines that can either interpolation or approximate a curve network (analogous to Coon's patches)
- The principle of cross-sectional design
- Shape approximation or matching through the procedure of knot insertion, know removal, degree-elevation, degree-reduction, etc.
- Commonly-used solid modeling techniques such as surface-surface intersection, offset computation, Boolean operations, and boundary evaluation
- Constraint-based optimization of various objective functions for surface fairing and smoothing

Local Subdivision

- Global subdivision introduces more control vertices than what users can handle in practice
- Real-time simulation will be sacrificed
- Localized operation is more desirable
- Local, adaptive subdivision is non-trivial
- The fundamental requirement
  - Limit surface should be geometrically the same
  - Maintain geometric correctness
  - Ensure dynamic smoothness
  - Conservation law in physics
- How can we devise local subdivision rules?

Design Technology

- Automatic evolution and determination of topological modification
- A library of modeling, design, simulation, and analysis toolkits
- System integration
  - Theoretical results
  - Algorithmic advances
- Modeling and design environment
- Real-time, interactive performance

Technology Transfer

- Expedite the systematic evaluation of both utilities and human factors
- Verify the applicability of design and simulation toolkits
- Validate the practicality of novel engineering environments
- Commercialization

Systems

- Prototype software environment
- Interface with commercial environments
- Real-world experiments and examples
- Real-world applications

Engineering Environment

- Flexible, efficient tools and facilities
- The rapid integration with commercial design systems
- Data exchange capabilities between S-splines and NURBS are lacking
- Efficient data structure
- Software package on heterogeneous platforms
- Architecture and functionality of commercial software system
- A set of algorithms for efficient and accurate data exchange

Engineering Environment

- Additional module of data exchange to interface with all the NURBS-based routines
- Rigorous tolerance analysis to ensure transformation accuracy
- Technical issues for the new industry standard through corporate collaboration

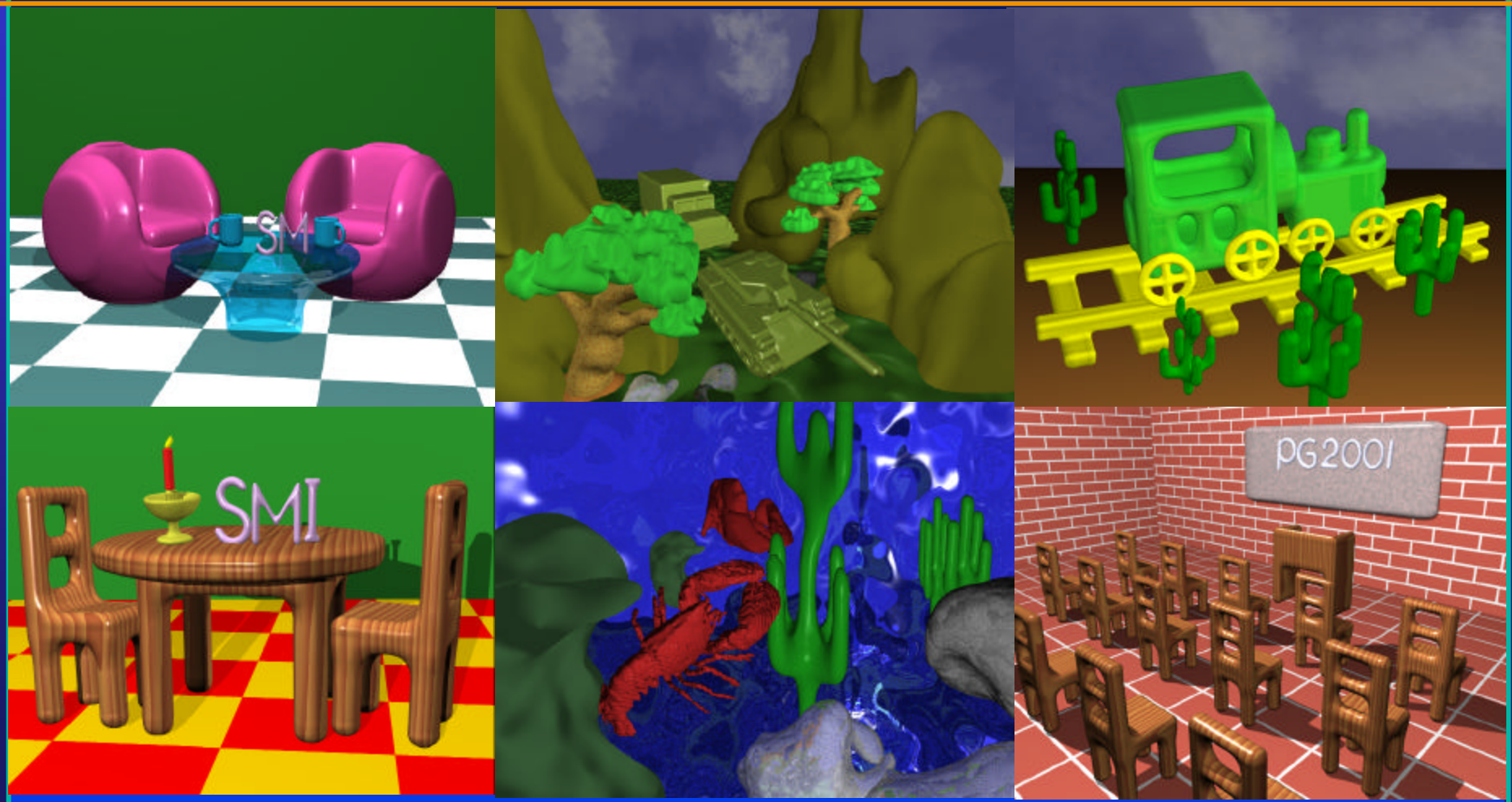
Impact

- Augment (rather than supersede) well established NURBS-based design technologies
- Generalize the newly-developed theory and methodology of physics-based modeling (e.g., D-NURBS) in industrial practice
- Contribute both to the geometric modeling and finite element analysis communities
- Promise to bridge the large gaps among interactive modeling, geometric design, finite element analysis, manufacturing
- Relevant to the entire processes of CAD/CAM/CAE

Impact

- Serve as a solid foundation for future theories and techniques which can eventually unify all aspects of modeling, design, and manufacturing
- Appeal to a spectrum of users
- Advance the state of knowledge in a wide range of fields including spline theory, computatioal geometry, finite element analysis, and interactive graphics
- The ultimate goal is virtual engineering environments
- Large-scale, complex shapes and their real-time, multi-modal interactions through the integrated, physics-based design

Acknowledgements

- U.S. National Science Foundation through the following grants: DMI-9896170, CAREER Award CCR-9896123, ITR grant IIS-0082035, IIS-0097646
- Honda America, Inc.
- Alfred P. Sloan Fellowship
- Ford Motor Company
- New York State Sensor CAT
- Brookhaven National Lab
- SUNY SPIR and Robocom International Systems
- Equipment matching funds from SUNYSB
- All of my former and current students (in particular, Chhandomay Mandal, Kevin T. McDonnell, Jing Hua, Ye Duan, Yusung Chang, Haixia Du, Hui Xie, Sumantro Ray, Robert Wlodarczyk)

Thank you