Physics-Based Graphics: Theory, Methodology, Techniques, and Modeling Environments

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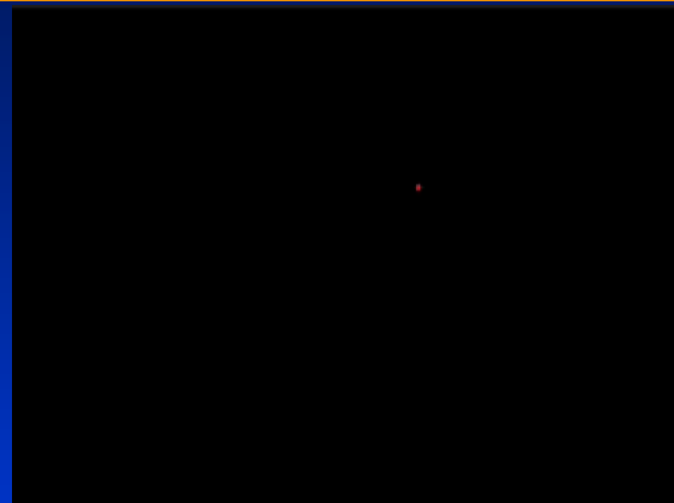
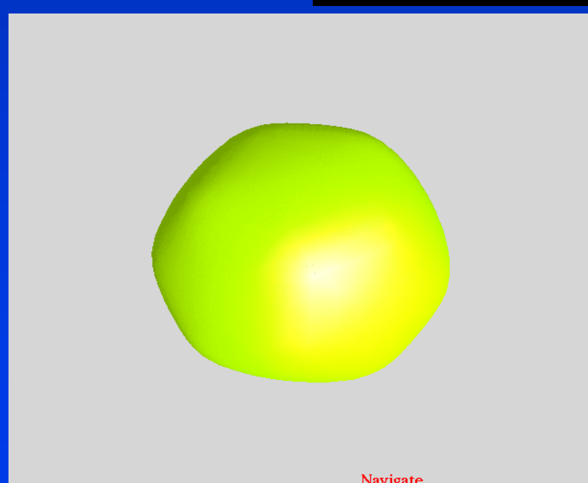
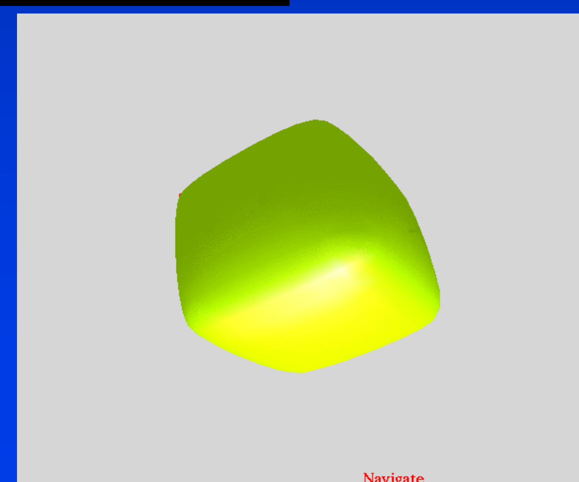
Video







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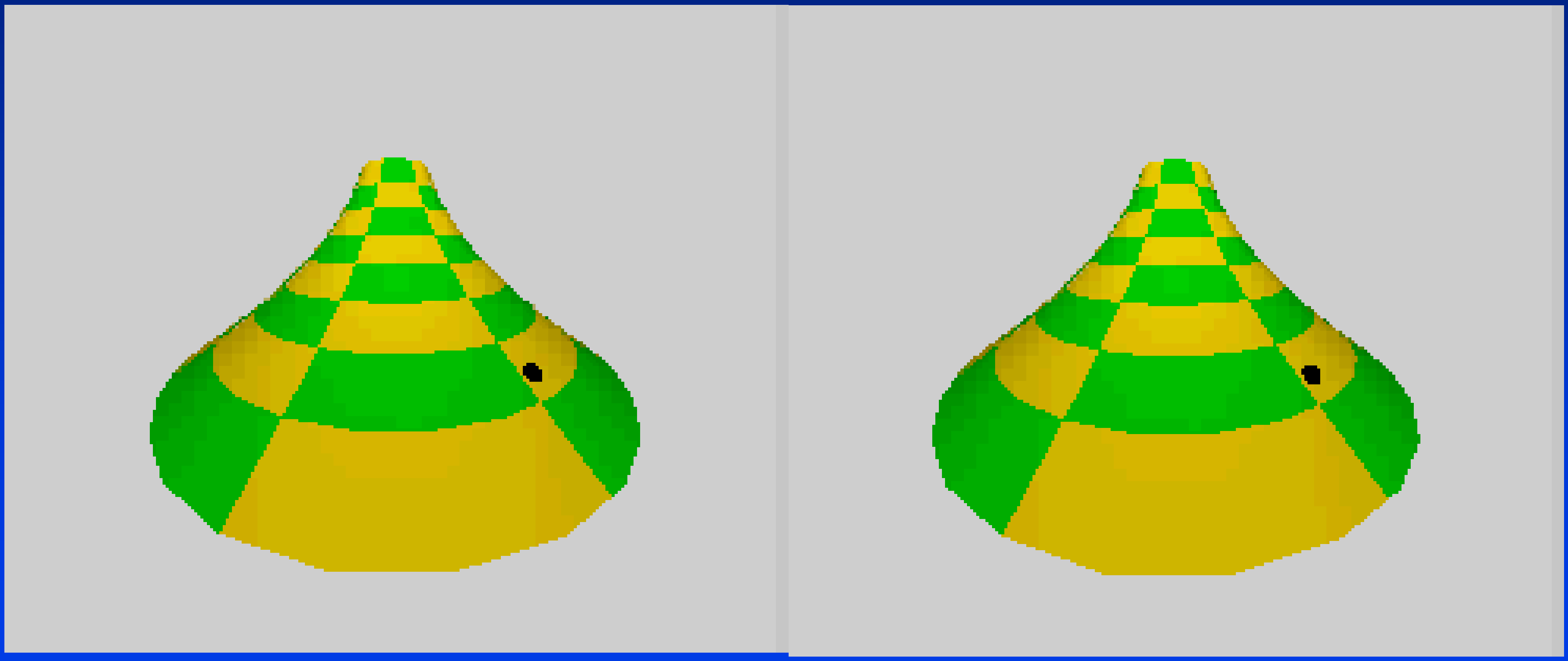


Background Knowledge and Motivations

- Overview of Graphics and its significance
- Difficulties associated traditional geometric techniques
- Physics-driven graphical modeling system with natural, intuitive haptic interaction --- We present DYNASOAR in this talk
- Brief description of some on-going research projects
- Gain a better understanding on the current state of the knowledge
- Stimulate future research interest in pursuing new research directions and undertaking more challenging research projects

Physics Basics

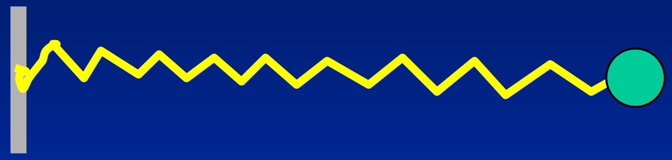
- Newton's second law

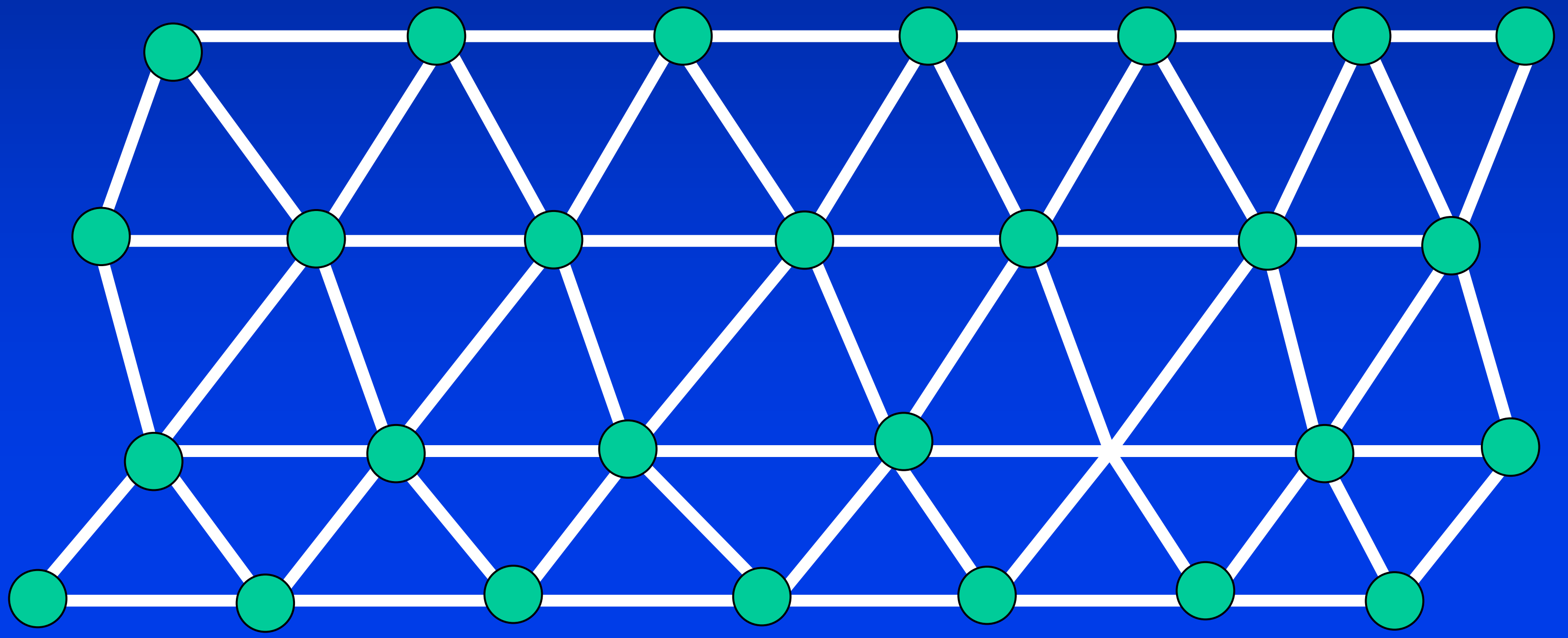
$$\mathbf{f} = m\mathbf{a}$$

- Spring energy and force:

$$E = \frac{1}{2}k(\mathbf{l} - \mathbf{l}_0) \bullet (\mathbf{l} - \mathbf{l}_0)$$
$$\mathbf{f} = k(\mathbf{l} - \mathbf{l}_0)$$

Mass-spring System





Mass-spring System

- One mass point

$$m\,\mathbf{a} = \mathbf{f}$$

$$\mathbf{a} = \frac{d\,\mathbf{v}}{dt}$$

$$\mathbf{v} = \frac{d\,\mathbf{p}}{dt}$$

$$\frac{dE}{d\,\mathbf{p}} = \mathbf{f} = m\,\mathbf{a}$$

- Particle (mass) system

$$\mathbf{a} = (\ldots\ldots + \mathbf{f}^{i} + \ldots\ldots)\,/\,m$$

$$\mathbf{a} = ((\ldots + \mathbf{f}_{e}^{i} + \ldots) - (\ldots + \mathbf{f}_{i}^{j} + \ldots))\,/\,m$$

$$m\frac{d^{2}\mathbf{p}}{dt^{2}} + c\frac{d\mathbf{p}}{dt} + \sum_{i} f_{\text{int}}^{i} = \sum_{i} f_{ext}^{i}$$

Mass-spring System

- Mass-spring system

$$\mathbf{M}\frac{d^2\mathbf{p}}{dt^2}+\mathbf{C}\frac{d\mathbf{p}}{dt}+\mathbf{K}\mathbf{p}=\mathbf{f}$$

- Numerical simulation

$$\mathbf{a}=\frac{\mathbf{v}^{t}-\mathbf{v}^{t-\delta t}}{\delta t}$$

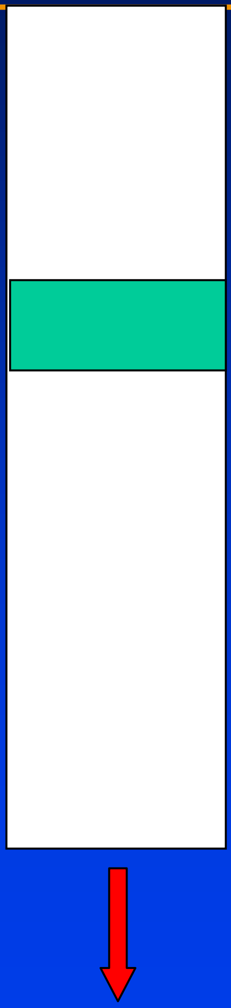
$$\mathbf{v}=\frac{\mathbf{p}^{t}-\mathbf{p}^{t-\delta t}}{\delta t}$$

From Matrix Algebra to Differential Equations

- The transition from the discrete model to the continuous model
- The central idea is equilibrium!!!
- For a discrete model such as the mass-spring system, we arrive at solving a linear equation and making use of matrix algebra
- For a continuous model, in fact we are getting differential equations
- Let us examine one simple example next

Example: an Elastic Bar

- Basic concepts
- Displacement
- Material properties
- Forces
- Boundary conditions



$$\left(c\frac{du}{dx}\right)_{x+\Delta x} - \left(c\frac{du}{dx}\right)_{x} + f\Delta x = 0$$

$$-\frac{d}{dx}\left(c\frac{du}{dx}\right) = f$$

From Rod to Beam

- Horizontal force (2nd order equations)

$$\frac{d}{dx}(c\frac{du}{dx}) = f(x)$$

- Vertical load (4th order equations)

$$\frac{d^2}{dx^2}(c\frac{d^2u}{dx^2}) = f(x)$$

From Continuous to Discrete

- How do we solve the previous differential equation?
- In general, analytical formulation is impossible
- Numerical algorithms must be sought
- The discretization of the continuous model leads to the linear algebra again!!!
- Once again, we are considering equilibrium as a general principle

Function Optimization

- Minimization or maximization
- Consider a single variable function f(x)
- Minimize f(x) (equivalently, maximize –f(x))
- This, in general, leads to a non-linear equation

$$g(x) = \frac{d}{dx}\left(f(x)\right) = 0$$

- One example for a quadric function

$$f(x) = \frac{1}{2}ax^2 - bx + c$$
$$g(x) = \frac{d}{dx}\left(f(x)\right) = ax - b = 0$$
$$x = \frac{b}{a}$$

Optimization

- Commonly-used numerical techniques
- Generic form (extend to n-component vector): to minimize $f(x_1, x_2, \ldots, x_n)$
- Solution for (multi-variate) optimization
- Necessary condition --- first-order derivative

$$g_i(x) = \frac{\partial f}{\partial x_i} = 0$$

- A set of equations, oftentimes solve n-variable non-linear equations

Optimization

- If P is a quadratic function of x (a special case)

$$\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T$$

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{x}^T\mathbf{b} + c$$

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{b} = \mathbf{0}$$

- Linear equations
  - Direct method, iterative method
- Additional constraints
- Non-linear equations
- Complicated cases --- no derivatives

Calculus of Variations

- Assume x(u) is not a function defined over [0,1] (the unknown is now a function)
- The cost function is an integral!
- Minimize

$$G(x) = \int_0^1 f(x(u))\,du$$

$$\frac{\partial}{\partial x}\big(G(x)\big) = 0$$

- Taylor expansion

$$\int_0^1 f(x(u)+y(u))du = \int_0^1 f(x(u))du + \int_0^1 y(u)\frac{\partial}{\partial x}\big(f(x(u))\big)du + \int_0^1 O(y(u)^2)du$$

First Variation

- To minimize the above functional, we need

$$\frac{\partial G(x(u))}{\partial x(u)} = 0$$

- The derivative is the first variation!
- Euler equation (strong form)

$$\frac{\partial f(x(u))}{\partial x(u)} = 0$$

One Dimensional Example

- Generic form $G(x(u)) = \int_0^1 f(x(u), x_u(u))du$
- Taylor expansion to compute the first variation

$$\int_0^1 f(x(u) + y(u), x_u(u) + y_u(u))du = \int_0^1 f(x(u), x_u(u))du +$$

$$\int_0^1 (y(u)\frac{\partial}{\partial x}\left(f(x(u))\right) + y_u(u)\frac{\partial}{\partial x_u(u)}(f(x(u), x_u(u))))du + \ldots\ldots$$

- Detailed derivation

$$\int_0^1 (y\frac{\partial f}{\partial x})du + \int_0^1 (\frac{\partial f}{\partial x_u})dy =$$

$$\int_0^1 (y\frac{\partial f}{\partial x} - y\frac{d}{du}(\frac{\partial f}{\partial x_u}))du + (\frac{\partial f}{\partial x_u}y(1) - \frac{\partial f}{\partial x_u}y(0))$$

One Dimensional Example

- For any y (Euler equation)

$$\frac{\partial f}{\partial x} - \frac{d}{du}\left(\frac{\partial f}{\partial x_u}\right) = 0$$

- More complicated examples and the first variation

$$G(x) = \int_0^1 f(x, x_u, x_{uu}, \ldots\ldots)\, du$$

$$\frac{\partial G(x)}{\partial x} = 0$$

- The Euler equation is

$$\frac{\partial f}{\partial x} - \frac{d}{du}\left(\frac{\partial f}{\partial x_u}\right) + \frac{d^2}{du^2}\left(\frac{\partial f}{\partial x_{uu}}\right) - \frac{d^3}{du^3}\left(\frac{\partial f}{\partial x_{uuu}}\right) + \frac{d^4}{du^4}\left(\frac{\partial f}{\partial x_{uuuu}}\right) + \ldots = 0$$

Two Dimensional Case

- Generic form $$P(x(u,v)) = \int F(x(u,v), x_u(u,v), x_v(u,v))dudv$$
- Euler equation $$\frac{\partial F}{\partial x} - \frac{\partial}{\partial u}\left(\frac{\partial F}{\partial x_u}\right) - \frac{\partial}{\partial v}\left(\frac{\partial F}{\partial x_v}\right) = 0$$
- Higher-order derivatives are involved

$$P(x(u,v)) = \int F(x(u,v), x_u(u,v), x_v(u,v), x_{uu}(u,v), x_{uv}(u,v), x_{vv}(u,v), \ldots\ldots)$$

$$\frac{\partial F}{\partial x} - \frac{\partial}{\partial u}\left(\frac{\partial F}{\partial x_u}\right) - \frac{\partial}{\partial v}\left(\frac{\partial F}{\partial x_v}\right) + \frac{\partial^2}{\partial u^2}\left(\frac{\partial F}{\partial x_{uu}}\right) + \frac{\partial^2}{\partial u \partial v}\left(\frac{\partial F}{\partial x_{uv}}\right) + \frac{\partial^2}{\partial v^2}\left(\frac{\partial F}{\partial x_{vv}}\right) + \ldots\ldots = 0$$

Dynamics and Least Motion

- Time-varying behavior due to temporal variable t
- The system is dynamic (not static)
- The motion equation is within the variational framework
- Newton's laws

$$\mathbf{f} = m\,\mathbf{a}$$

- Least motion principle and Euler equation based on variational analysis

Dynamics and Least Motion

$$A = \int (K(x_t(t)) - P(x(t)))\, dt$$

$$K(x_t(t)) = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2$$

$$P(x(t)) = mgx$$

$$-\frac{d}{dt}\left(m \frac{dx}{dt} \right) - mg = 0$$

Lagrange Mechanics

- Lagrangian equation of motion (Lagrangian mechanics in a discrete form)

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{p}_i}\right) - \frac{\partial T}{\partial p_i} + \frac{\partial F}{\partial \dot{p}_i} + \frac{\partial U}{\partial p_i} = f_i$$

- Kinetic energy (continuous form and discretized form)

$$T(x(u,v,t), x_t(u,v,t))$$

$$T(p_i, \dot{p}_i)$$

Lagrange Mechanics

- Damping energy (continuous form and discretized form)

$$F(x_t(u,v,t))$$
$$F(\dot{p}_i)$$

- Potential energy (continuous form and discretized form)

$$U(x(u,v,t),x_u(u,v,t),x_v(u,v,t),\ldots)$$
$$U(\dot{p}_i)$$

- The action integral is minimized if the trajectory is governed by Mechanics

Classical and Modern Physics

- Wave equation
- Heat equation
- Classical mechanics
- Quantum mechanics
- Relativity

(Partial) Differential Equations

- PDEs are employed to describe physical phenomena
- Serve as a foundation for mathematical modeling
- Ordinary (single variable) differential equations
- Partial (multiple variable) differential equations
- Analytic solution is rare
- Numerical computation is necessary for approximated solution

A PDE Formulation

- PDE (Partial Differential Equation)

$$\sum_{n=0}^{r} \sum_{\substack{l+m=n \\ l,m \geq 0}} \alpha_{l,m}(u,v) \frac{\partial^n}{\partial u^l \partial v^m} f(u,v) = g(u,v)$$

  - Order r
  - $\alpha_{l,m}(u,v)$, $g(u,v)$: control functions
  - $f(u,v)$: unknown function of u,v

PDE Techniques for Graphics

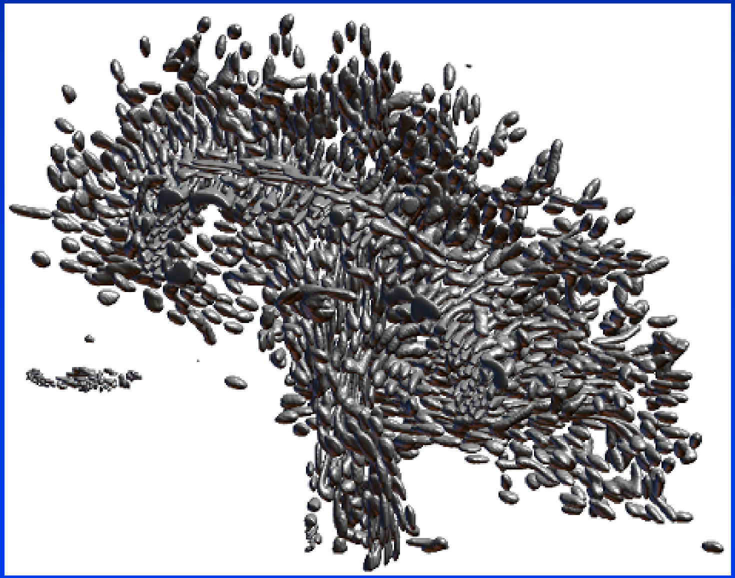
- Image processing

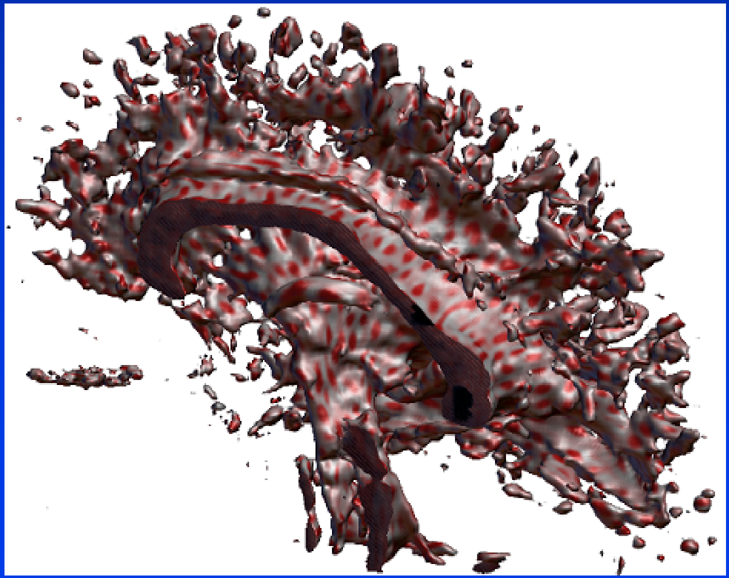
PDE Techniques for Graphics

- Smoke simulation

PDE Techniques for Graphics

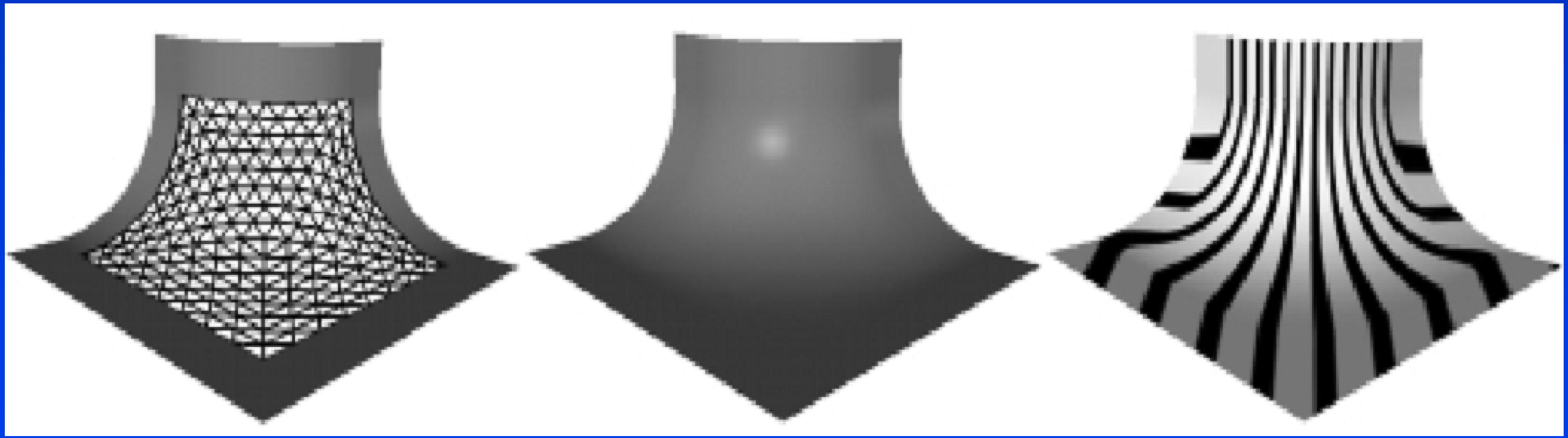
- Tensor Visualization





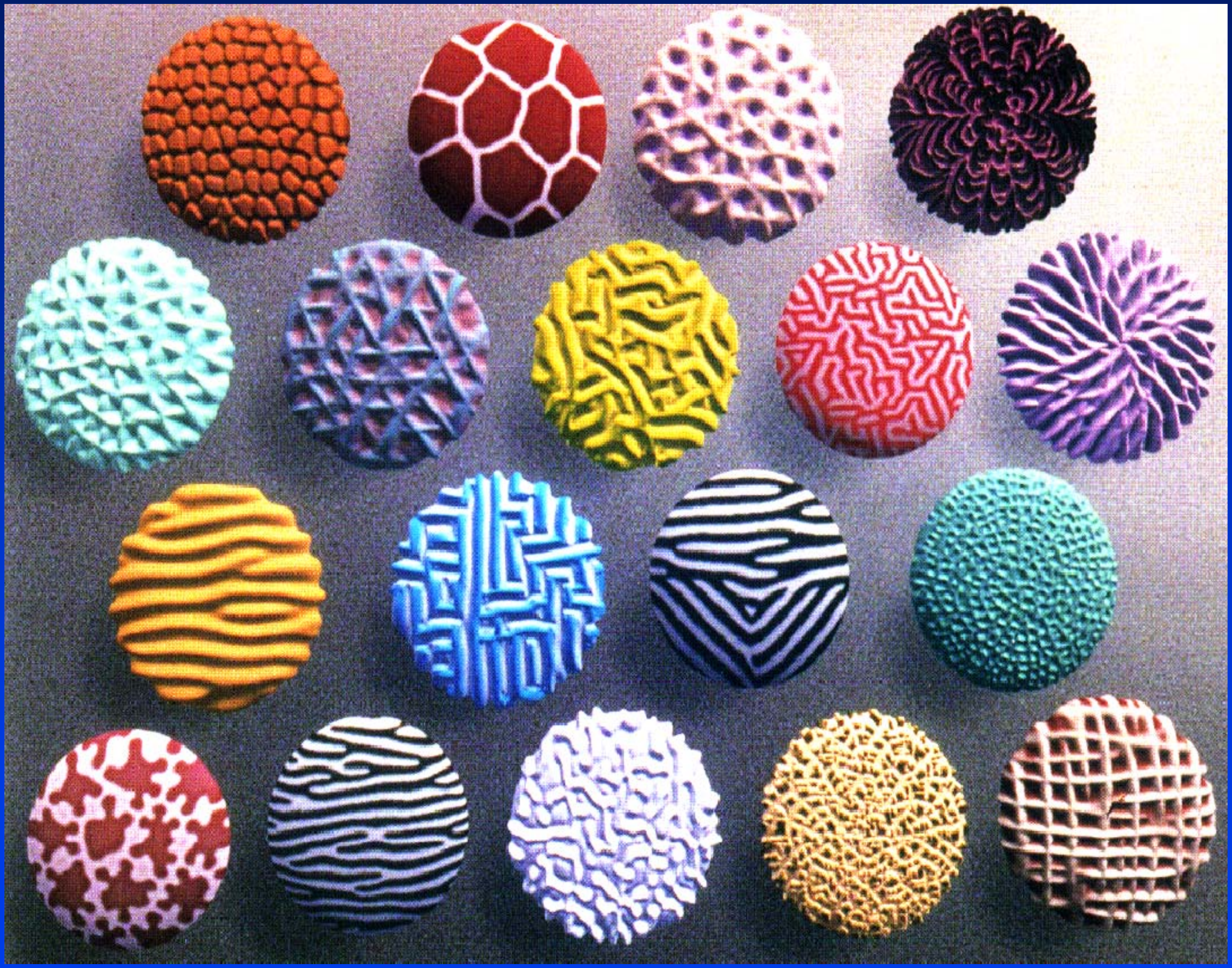
PDE Techniques for Graphics

- Surface fairing for shape modeling



[Schneider and Kobbelt 00]

Texture Synthesis



Numerics

- Numerical discretization
  - Finite difference
  - Finite element
- Boundary constraints
  - Boundary condition
  - Initial value condition
- Numerical characteristics
  - Convergence
  - Stability
  - Efficiency
  - Parallelism

Computer Graphics Overview

- Algorithm, software, and hardware techniques for image synthesis of computer-generated graphical models --- modeling + rendering
- Fundamental methodology and technology to other visual computing areas including visualization, vision, animation, virtual reality, HCI, CAD/CAM, biomedical applications, etc.
- My current focus is on graphics modeling
- Modeling techniques are founded upon geometric representation and computation

Geometric Modeling Overview

- Point, point cloud
- Line, poly-line, curve, curve network
- Plane, triangle, rectangle, polygon
- Bivariate parametric surfaces, free-form splines, surfaces defined by implicit functions (e.g., polynomials and other well-known functions)
- Solid models: CSG, B-rep, cell decomposition (tetrahedra, voxel cubes, prisms, cross-sectional slices), trivariate parametric super-patches
- Subdivision-based curves, surfaces, and solids as well as other procedural modeling techniques
- PDE-based models

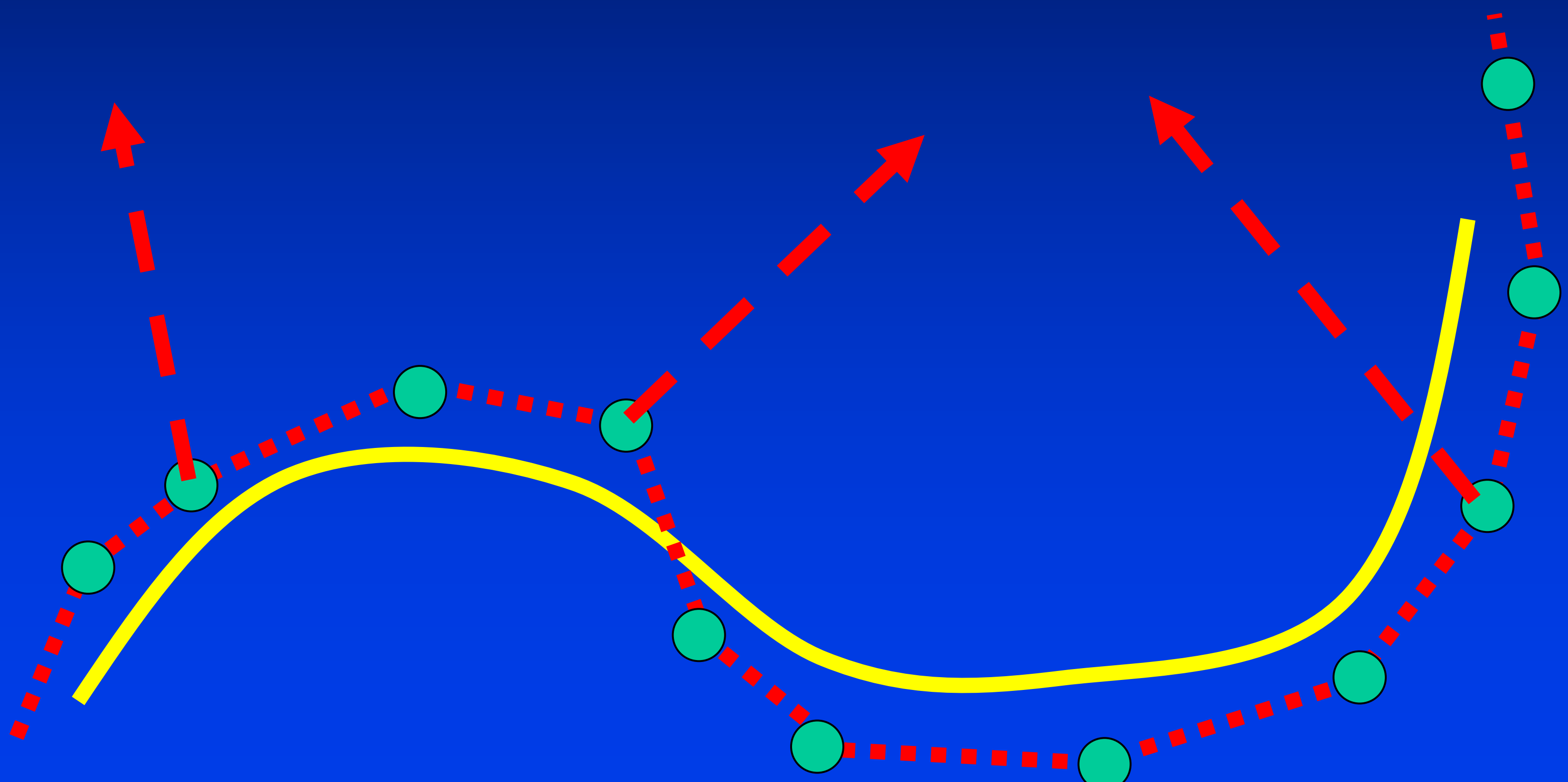
Geometric Modeling

- Shape representation
  - Parametric polynomial
  - Piecewise rational spline
  - Recursive subdivision form
  - Implicit function
- Design paradigms
  - Interpolation/approximation
  - Optimization
  - Cross-sectional design
  - Blend and offset
  - Solid modeling

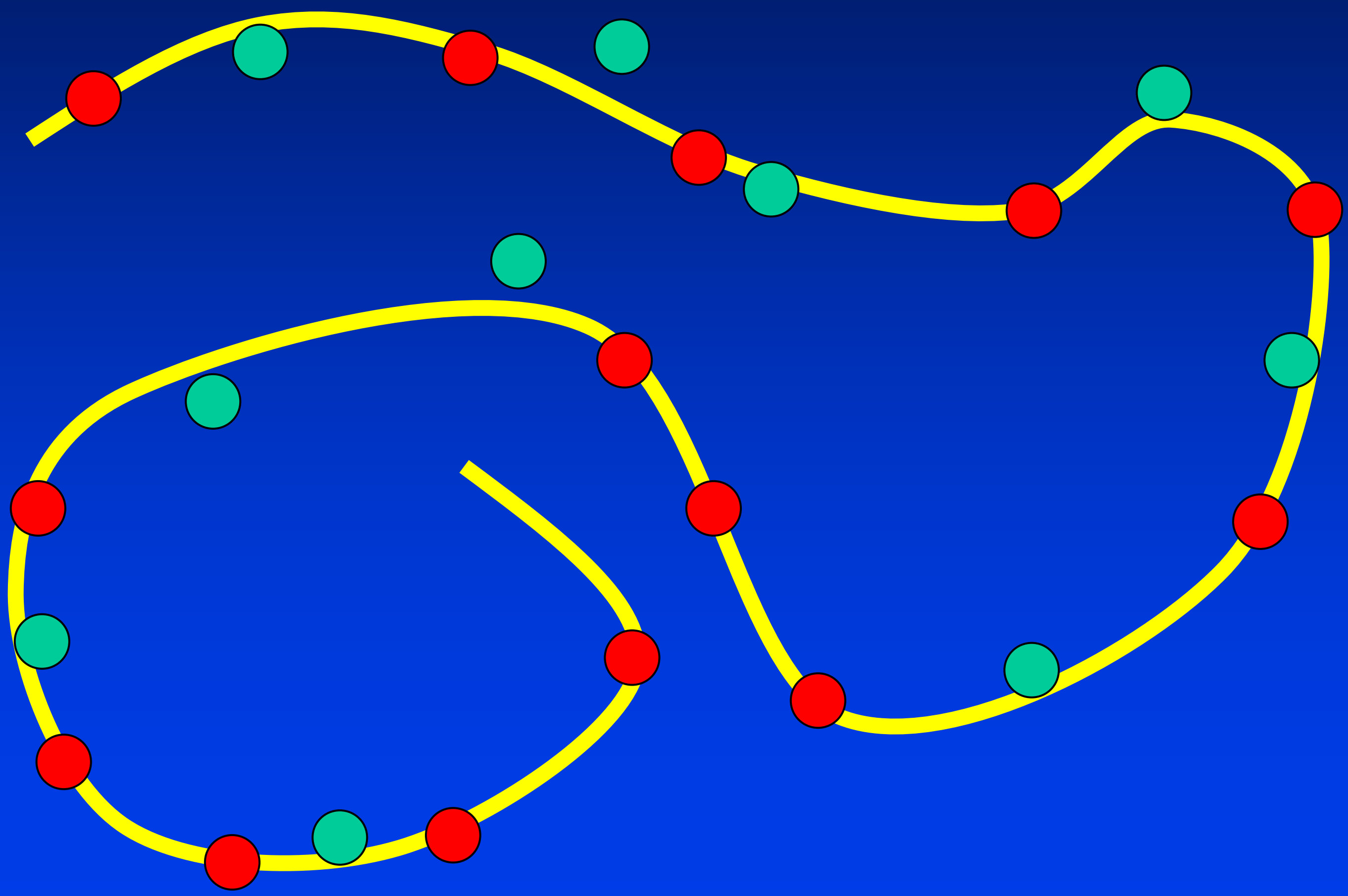
Geometric Modeling Tools

- Intuitive DOF (degree of freedom) manipulation
- Interpolation/approximation
- Cross-sectional design: curve network creation and manipulation
- Reverse engineering from clay models or CAD data
- Constraint-based iterative optimization
- Conventional approaches can be difficult
- New design techniques and tools are necessary

Control Point Manipulation

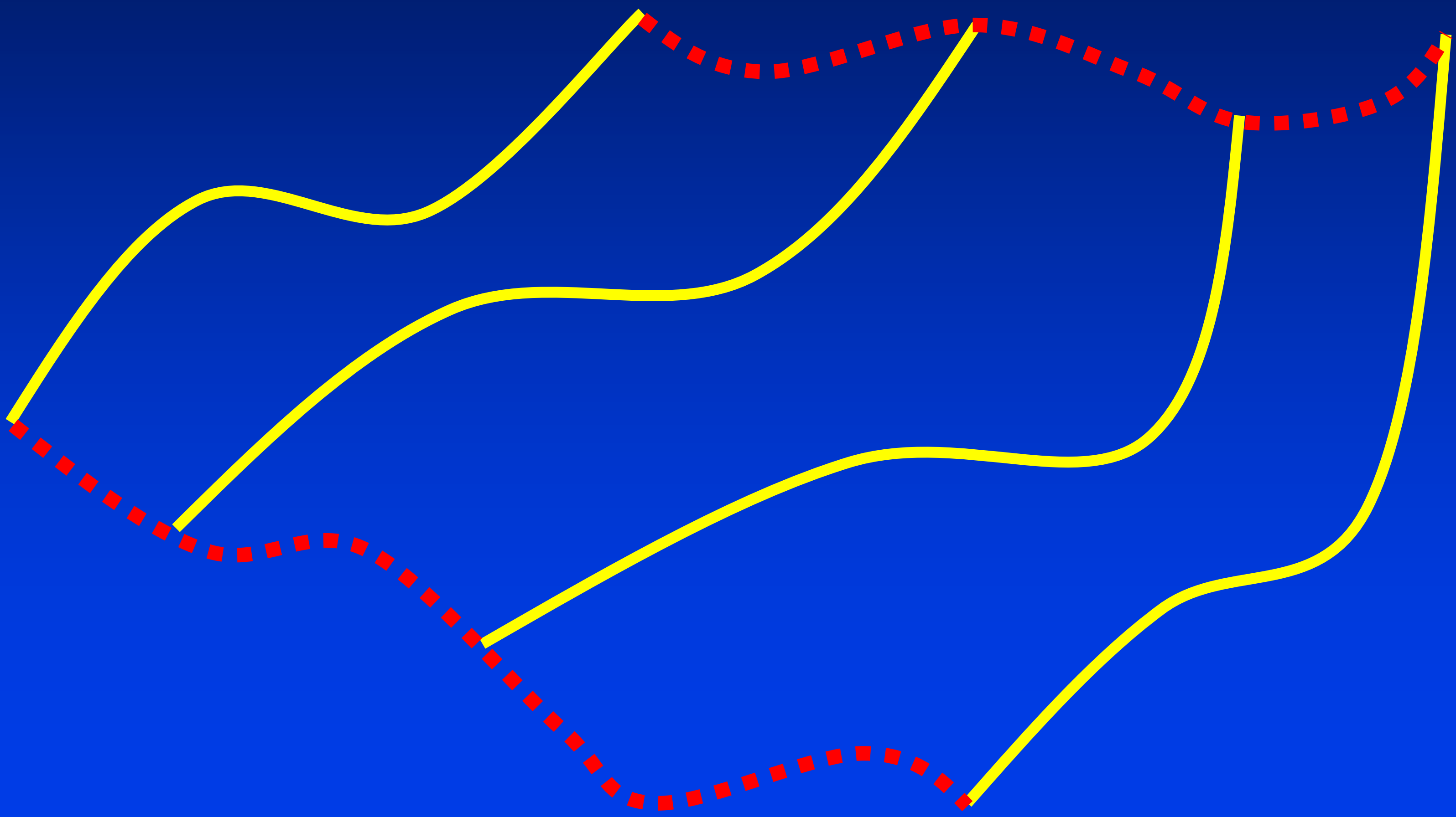


Interpolation / Approximation

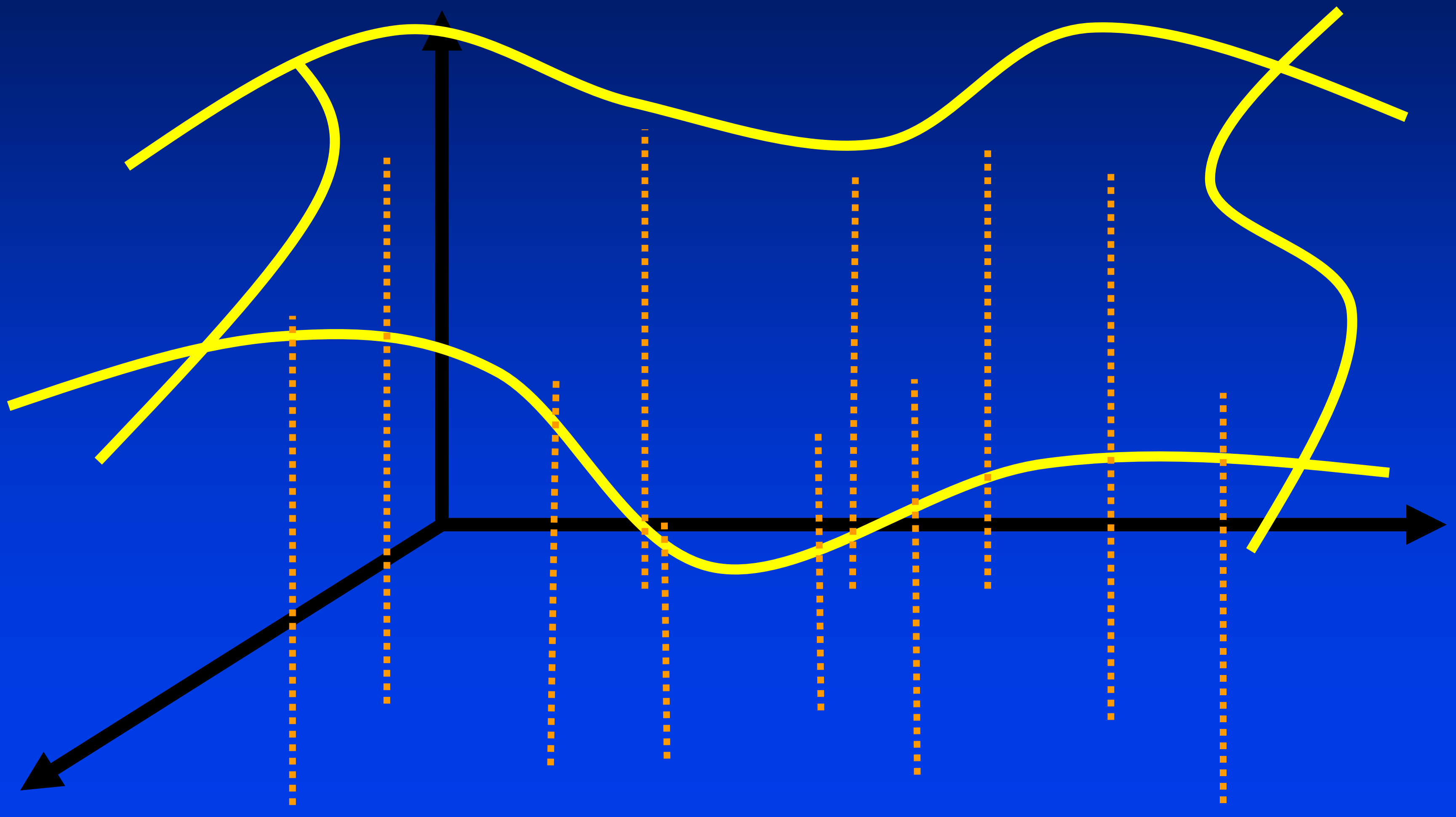
- 

Cross-Sectional Design

Cross-Sectional Design



Scattered Data Interpolation



Modeling Difficulties for Traditional Schemes

- The geometry is abstract, rigid, and complex
- Users must have sophisticated mathematics in order to manipulate a large number of underlying geometric parameters to create, edit, instantiate, control, interact, and understand CAD datasets
- Lack of effective, interactive sculpting toolkits for the natural and intuitive manipulation of geometric objects
- More difficult to handle solid objects, no tools for kinematic & dynamic analysis of physical solids
- Primarily focus geometry, cannot handle topology modification easily

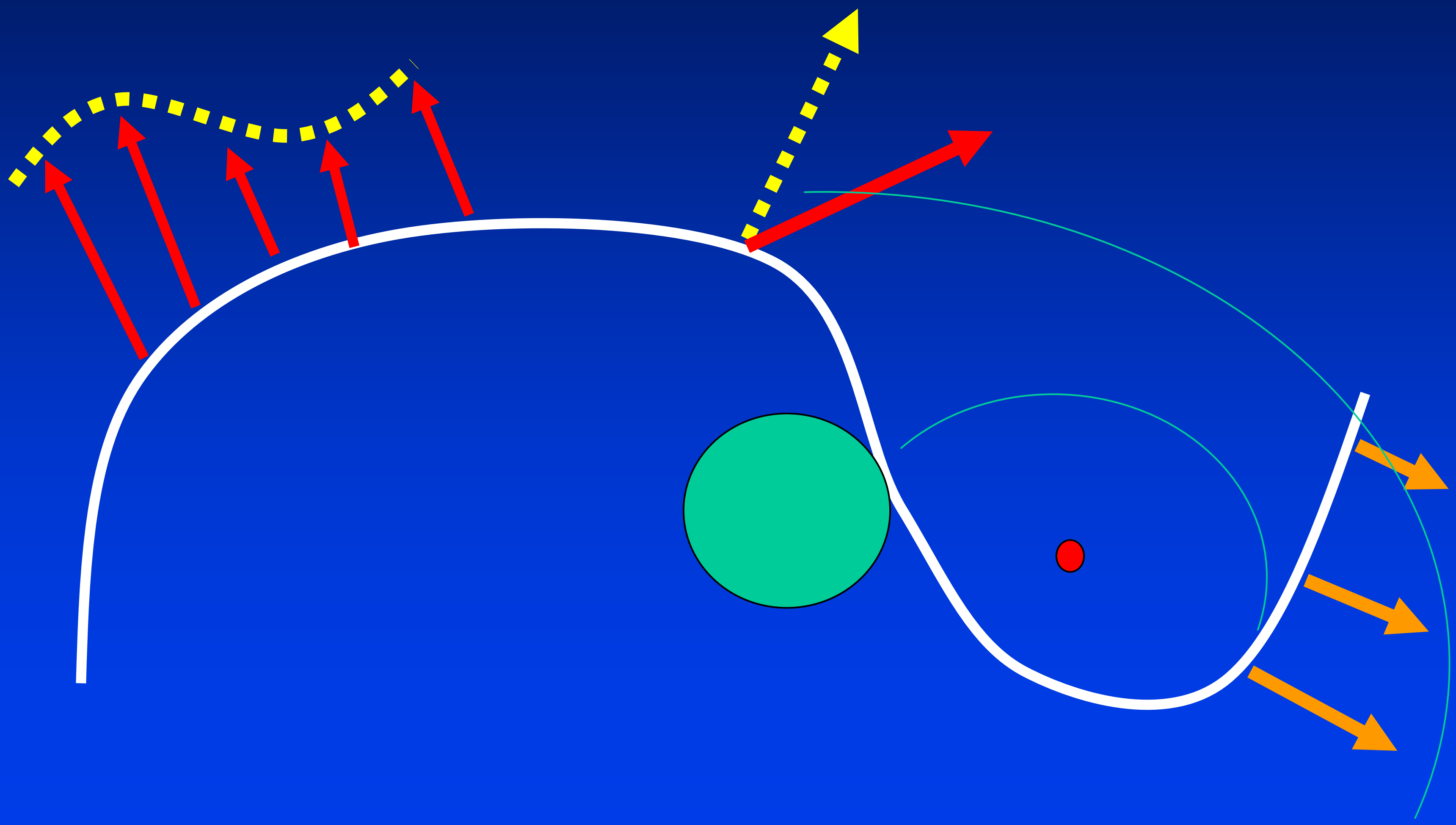
Engineering Design

- CAD/CAM
  – Conceptual design, analysis, evaluation, prototyping, manufacturing, assembly, production, etc.
- Iterative and innovative procedure
- Critical for other downstream CAD/CAM activities
  – Design decisions affect final products in terms of quality, feasibility, cost, time, etc.
- Primary objective: define product geometry
- Techniques and tools
  – Advanced graphics interface
  – Efficient algorithm and software
  – Specialized CAD hardware system

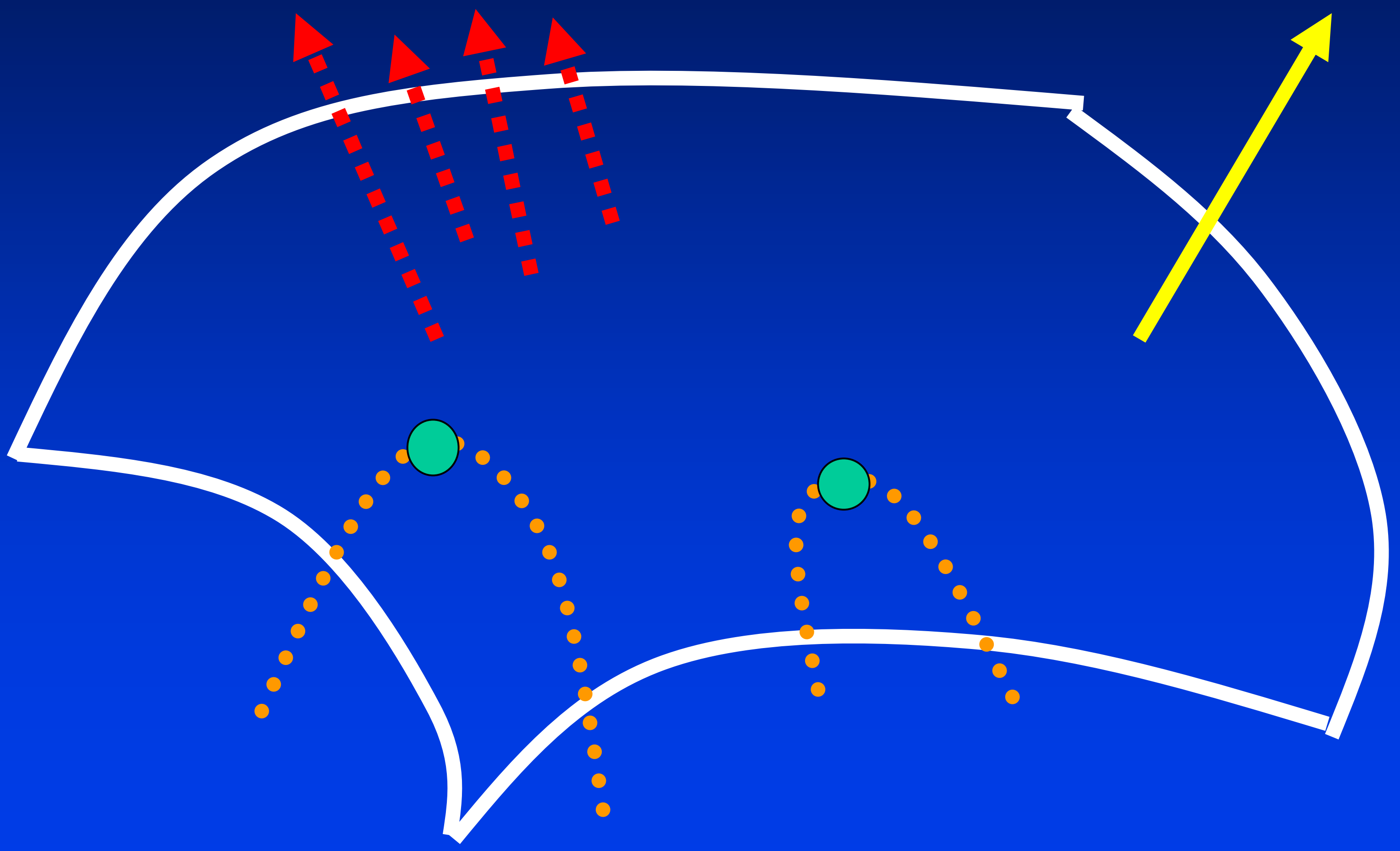
Physics-based Design

- Long-term objectives
  - New interactive design environment for CAD/CAM
  - Variety of new force-based design tools
- New approach
  - Physics-based geometric modeling and design
- Rationales
  - Difficulties with conventional approaches
  - Integration of geometry with physics
    - Improve interactive design, support intuitive interaction via forces
- D-NURBS theory and practice
- Future research topics

Sculpting Tools



Surface-based Tools



Physics-based Design

- Geometric models + physical laws = dynamic models
- Integration of static geometry with dynamic behavior
- Energies express global "fairness" criteria
- Forces support direct manipulation, interactive sculpting, and intuitive interaction
- Constraints permit functional design
- Shape optimization via evolution to equilibrium
- Dynamics allow time-varying shape design and control
- Automatic DOF selection

Physics-based CAGD as a New Theory and Methodology

- A novel graphical modeling and geometric design technique, the integration of geometric objects, material properties, and their physical and dynamic behaviors
- The geometry is governed by physical laws (e.g., Lagrangian equation of motion in classical physics, partial differential equations in mathematics, etc.), the large number of geometric control parameters (e.g., B-spline control points) are determined by physics
- The deformable motion is natural subject to energy optimization with geometric constraints, users can interact with geometric models via forces
- Can be easily accessed by a wide spectrum of users, ranging from CS professionals and engineering designers to naïve users or even computer illiterates, a unified framework for modeling, design, analysis, simulation, test, prototyping, and manufacturing

DYNASOAR:
DYNAmic Solid Objects of ARbitrary topology

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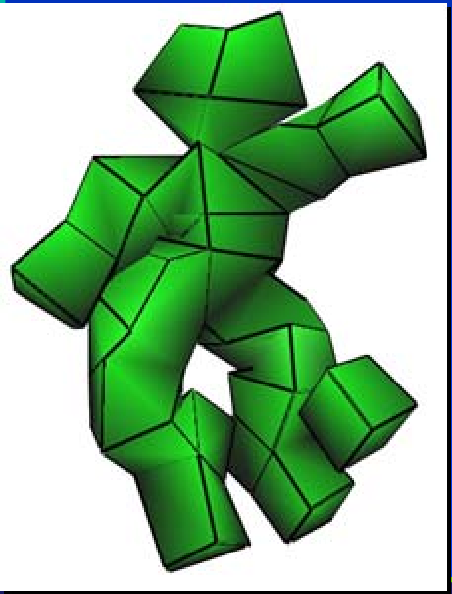
Presentation Overview

- DYNASOAR --- a novel, dynamic solid modeling system for objects of complicated geometry and arbitrary topology
- New technologies
  - subdivision-based solid geometry
  - physics-based design paradigm
  - haptics-based manipulation and interface
  - multi-thread, parallel simulation algorithm
  - powerful design and sculpting toolkits
- Versatile, various applications
  - virtual sculpting & prototyping, FEM analysis & simulation, data fitting and segmentation, visualization, etc.

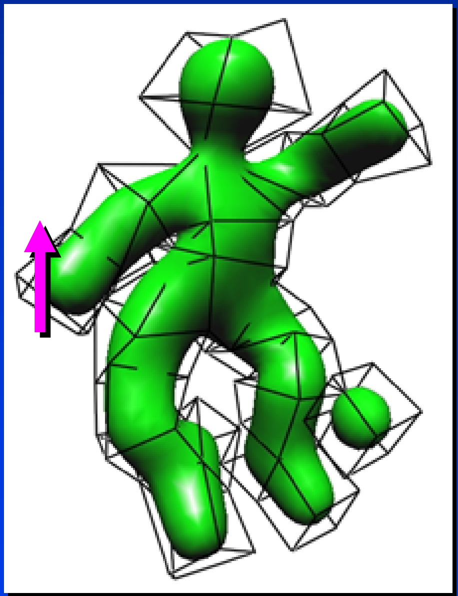
Our Ideas

- Physics-based sculpting and design for real-world objects
- Virtual Clay: various users can employ CAD tools to interact with, deform and topologically modify virtual solid objects

control lattice (geometry)



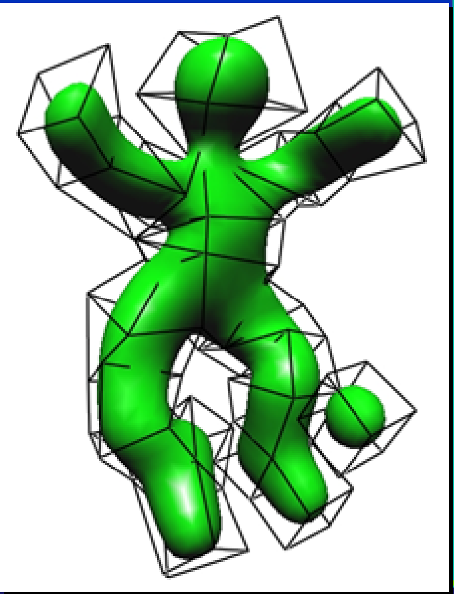
subdivide



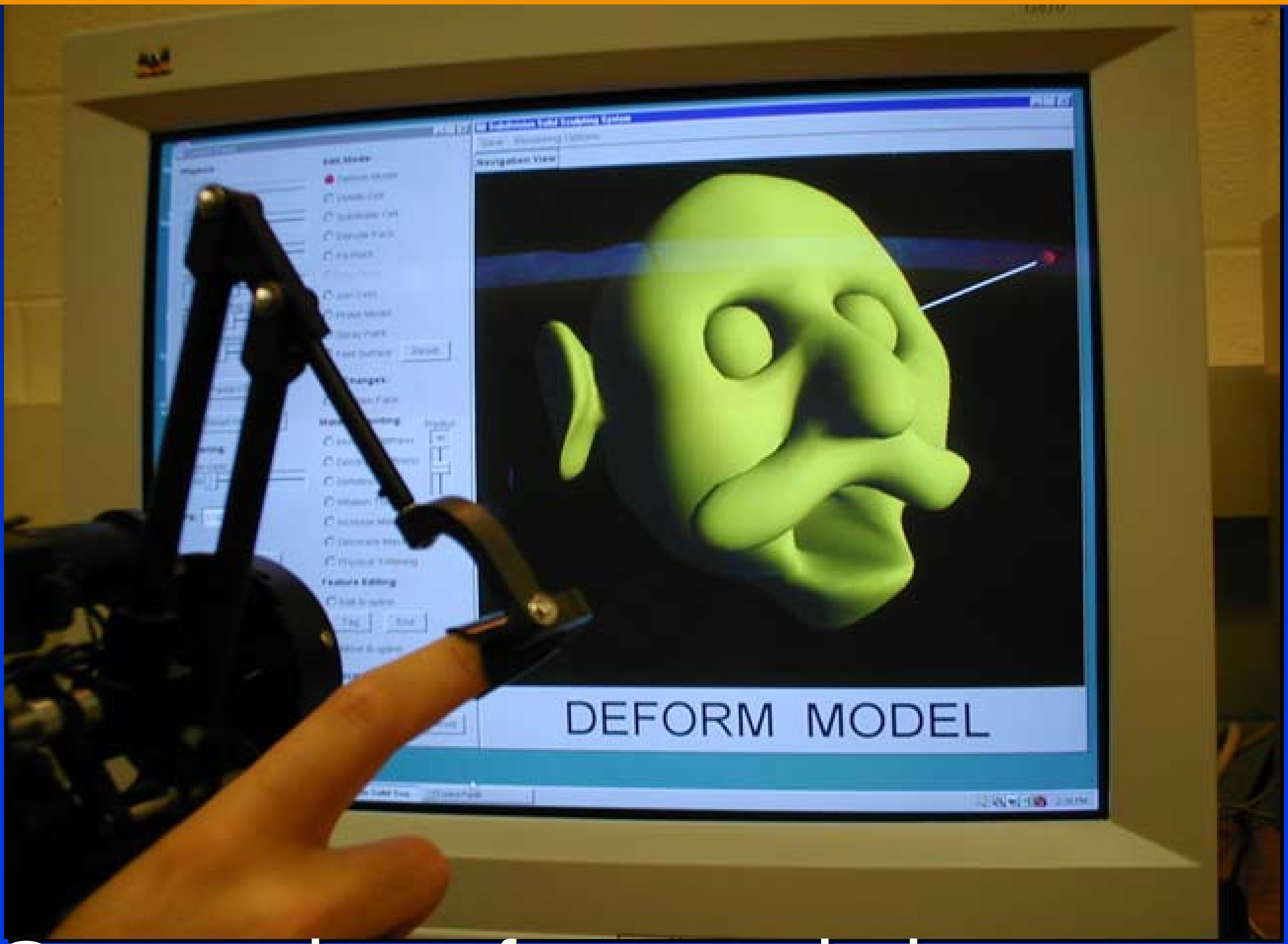
Subdivided solid

add physics; sculpt

geometry & physics



Haptic Manipulation



Screenshot of our sculpting system

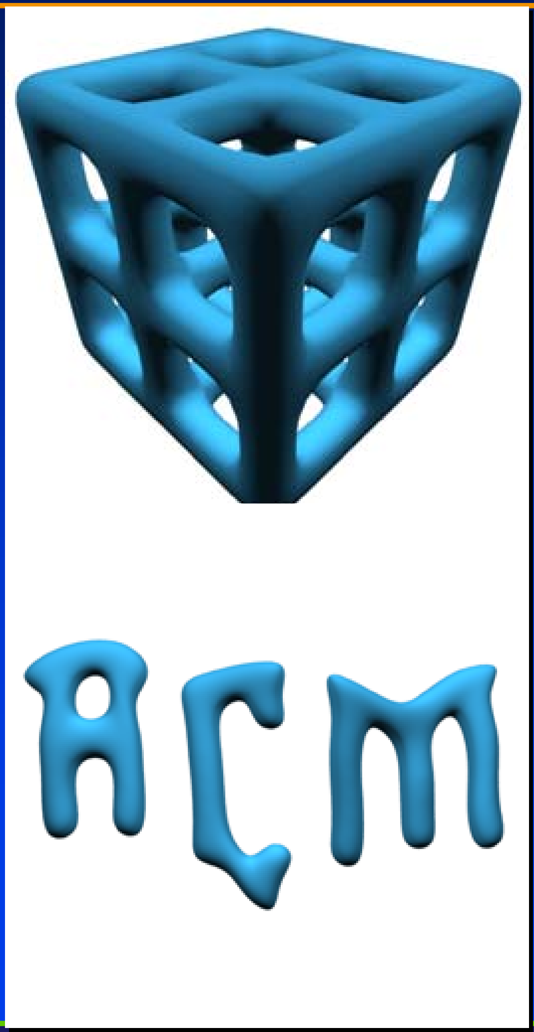
Proposed Solution

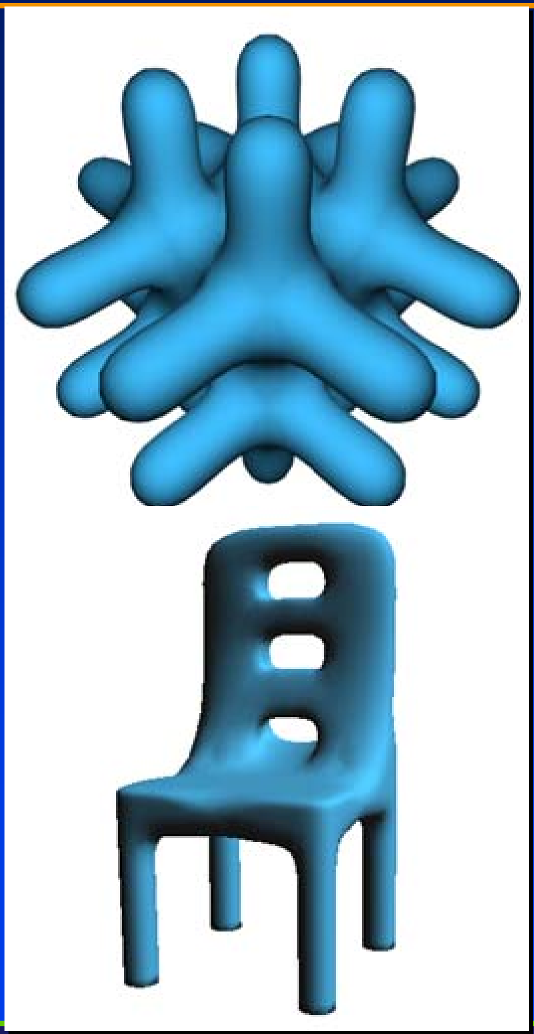
- Combines subdivision solids with physics-based modeling and haptic sculpting interface
- Subdivision solids offer geometric foundation
- Finite Element Method (FEM) and its numerical algorithm employed to represent material properties, simulate dynamic behaviors, and conduct material analysis tasks
- Supports realistic, direct manipulation of sculpted objects
- Offers users a spectrum of powerful sculpting tools
- Provides a novel framework for design and analysis applications

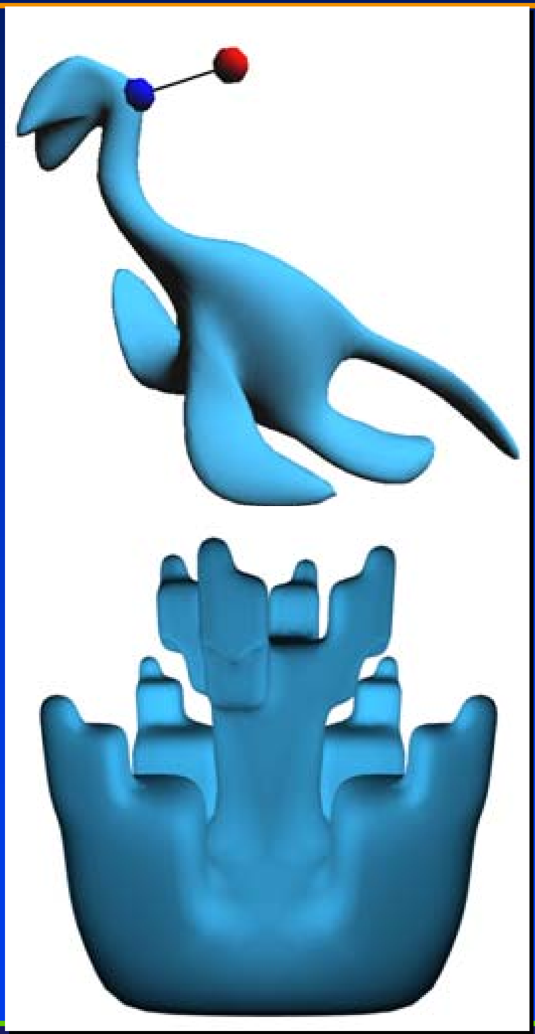
Haptics Interface

- Much more natural than conventional 2-D interface media such as keyboard and mouse, closer to real-world scenarios
- Realize the full potential of physics-driven modeling methodology
- Broaden the computer accessibility by a wider range of users including vision-impaired users and younger generations
- Stimulate knowledge advancements in algorithm design, software, hardware, HCI
- Serve as a foundation for next-generation, multi-modal interface that can integrate acoustic, haptic, visual channels

Sculpted CAD Models

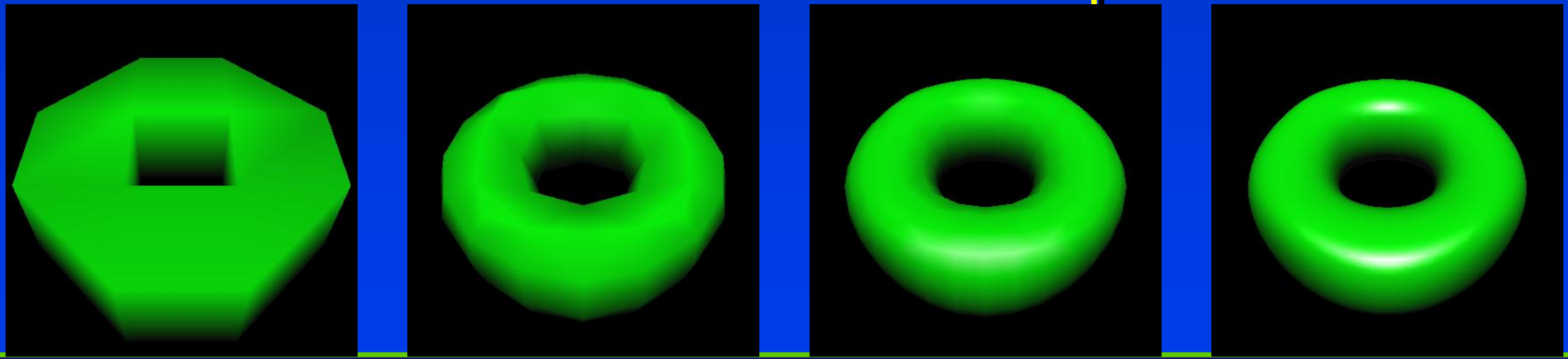






Subdivision Concepts

- “Simple” recursive algorithms
- Subdivision curves and surfaces popular and well-researched in CAD and interactive graphics
- Simple subdivision rules generate mathematically smooth splines in the limit
- Can handle arbitrary topology objects with ease
- Can round off corners and smooth sharp features



Prior Work on Subdivision

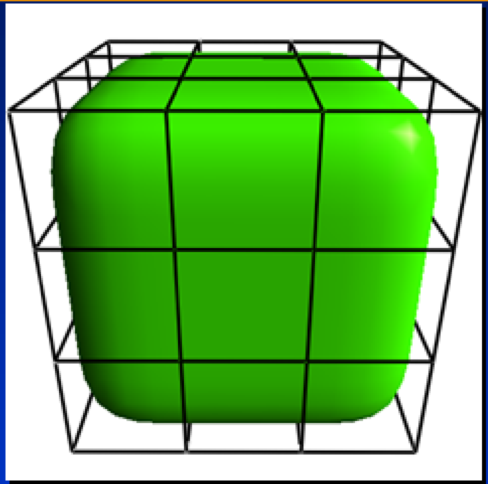
- Curves
  Chaikin ’74; Dyn et al. ’86, ’87, ’88
- Surfaces
  Catmull and Clark ’78; Doo and Sabin ’78;
  Loop ’87; Dyn ’90; Kobbelt ’96; Lounsbery ’94;
  Welch and Witkin ’92; Zorin ’96; DeRose ’98;
  Sederberg et al ‘98; Stam ’98; Levin ’99
- Solids
  MacCracken and Joy ’96 (*but*, for free-form deformation!)

and many more!

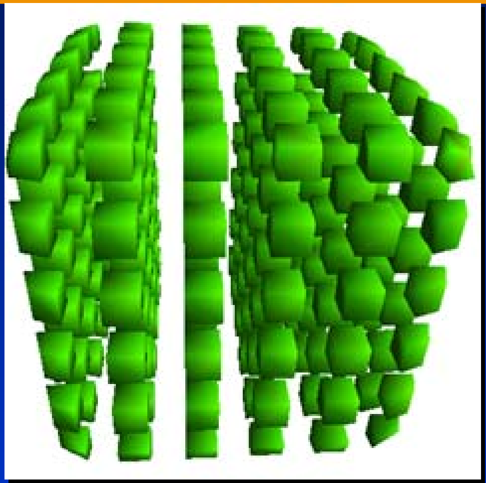
Subdivision Solids

- Little published research on subdivision solids
- Invented by MacCracken and Joy `96
- Developed as a novel FFD technique
- We propose to use such solids as a new solid modeling technique for a novel dynamic sculpting environment
- Generalization of Catmull-Clark surfaces to solids
- Start with a control lattice and subdivide until desired smoothness is attained
- Motivations: heterogeneous material distributions, arbitrary topologies, volumetric sculpting

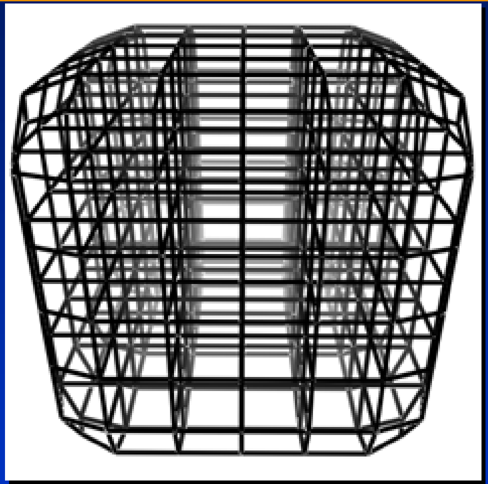
Examples: Solid vs. Surface



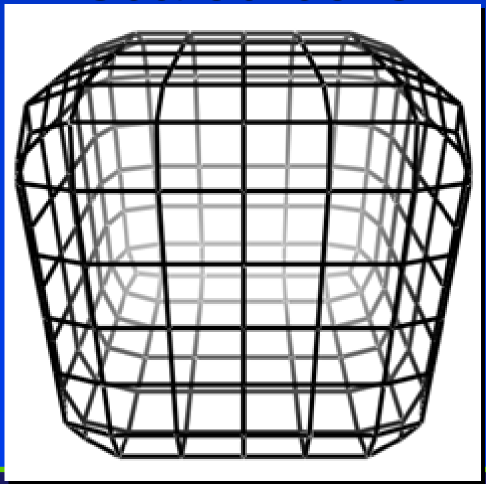
control lattice &
boundary surface



scaled cells

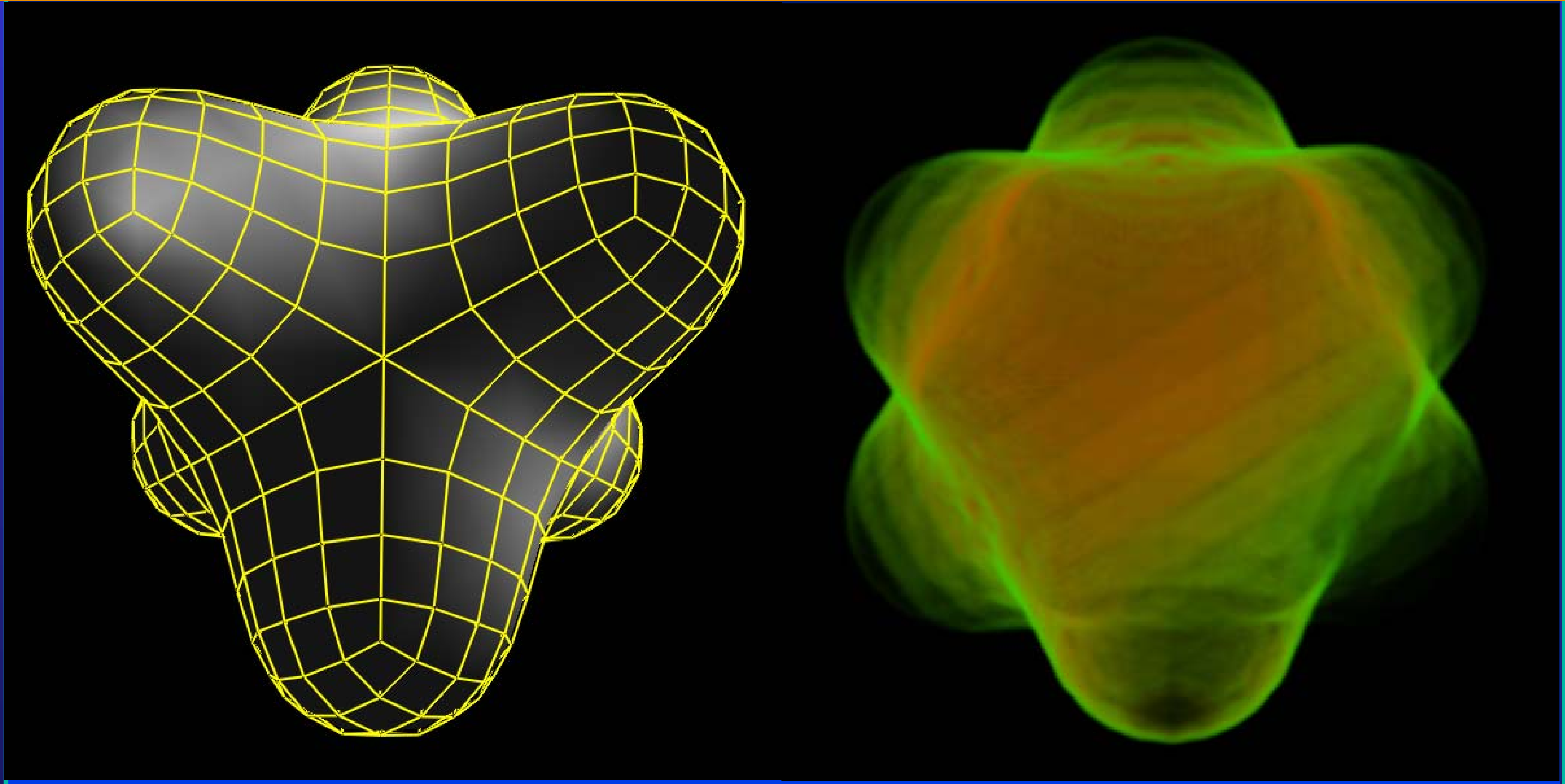


solid wireframe



boundary surface
wireframe

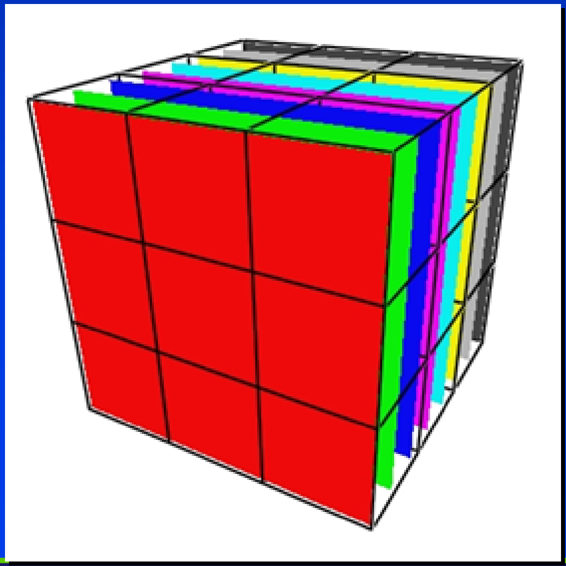
Heterogeneous Material

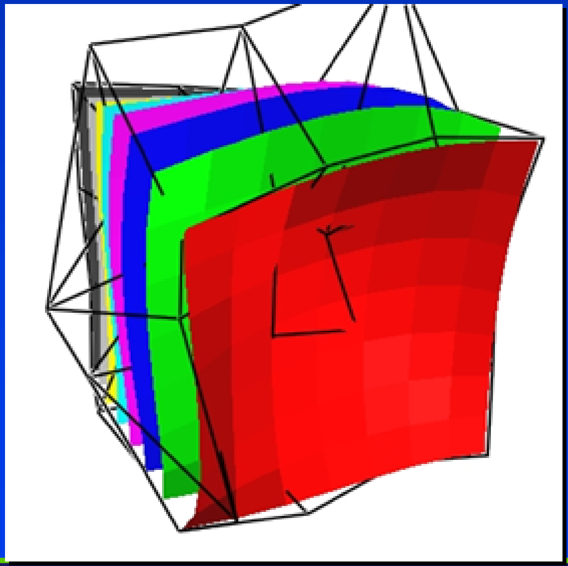


Spline Mathematics

MacCracken-Joy subdivision solids are in fact a generalization of tri-cubic B-spline solids:

$$\mathbf{s}(u,v,w)=\sum_{\mathrm{i=0}}^{\mathrm{n}}\sum_{\mathrm{j=0}}^{\mathrm{m}}\sum_{\mathrm{k=0}}^{\mathrm{l}}\mathbf{p}_{\mathrm{i,j,k}}B_{\mathrm{i},4}(u)B_{\mathrm{j},4}(v)B_{\mathrm{k},4}(w)$$





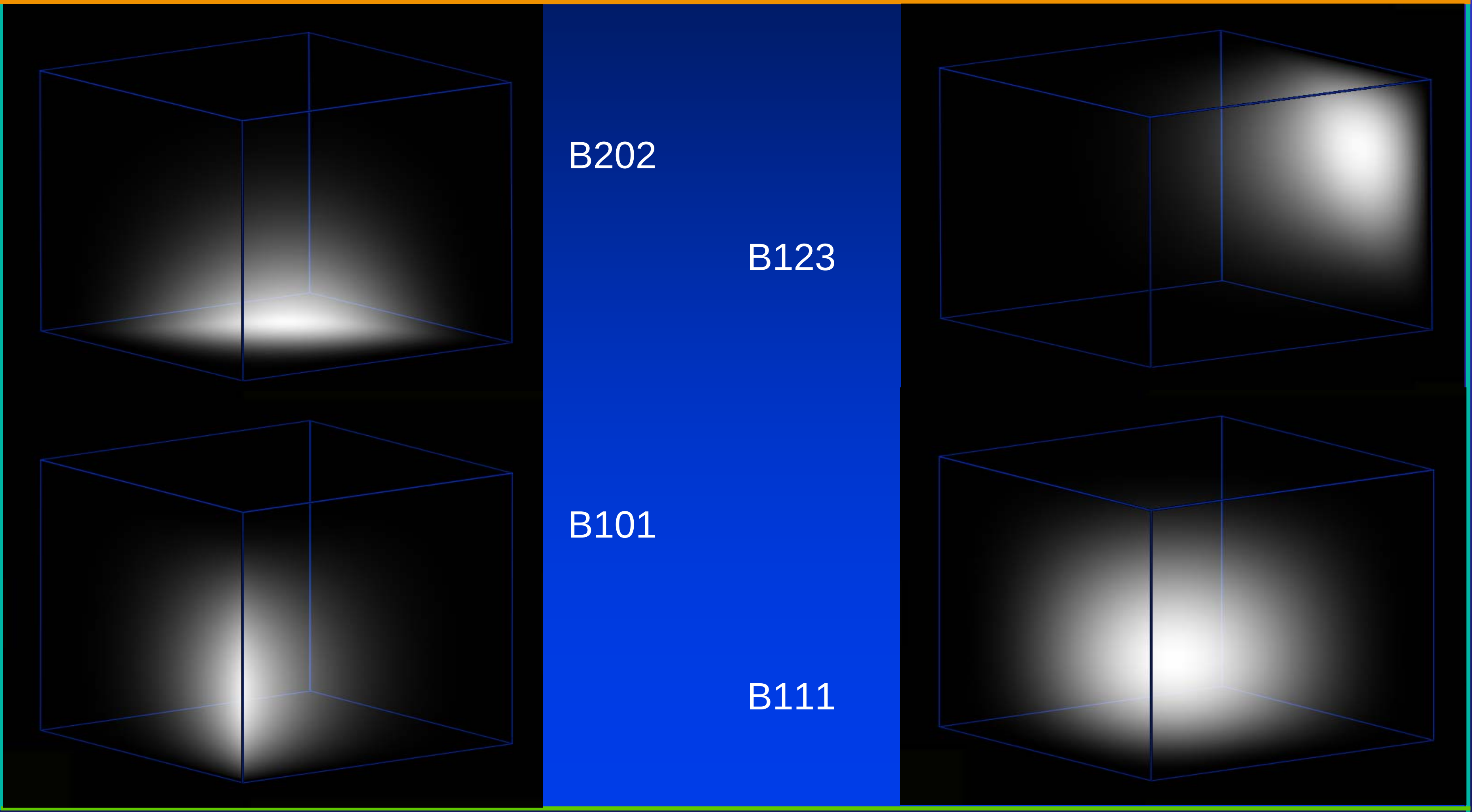
Subdivision Mathematics

No known closed-form expression exists for the basis function of a subdivision solid:

$$\mathbf{s}(\mathbf{x}) = \sum_{i=0}^{n} \mathbf{p}_i \hat{B}(\mathbf{x}) \qquad \hat{B}\,?$$

We must therefore rely on the use of subdivision rules to define the solid…

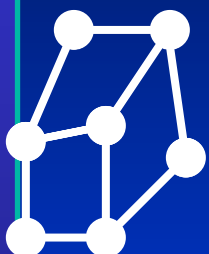
B-Spline Basis Functions



Subdivision Solids

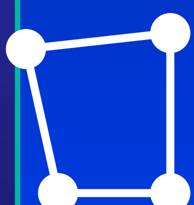
- Control lattice assembled from cells, faces, edges, and vertices
- Vertices → edges → faces → cells
- Like procedural subdivision surfaces:
  - one subdivision rule for each type of geometric "entity" (+ cell rule)
  - each geometric entity contributes a new vertex during the subdivision process
  - assemble new finer subdivision solid after computing new vertices

Subdivision Solid Rules



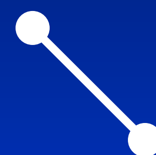
cell - point :

take centroid



face - point :

$$f = \frac{c_0 + 2a + c_1}{4}$$



edge - point :

$$e = \frac{c_{avg} + 2a_{avg} + (n-3)m}{n}$$

vertex - point :

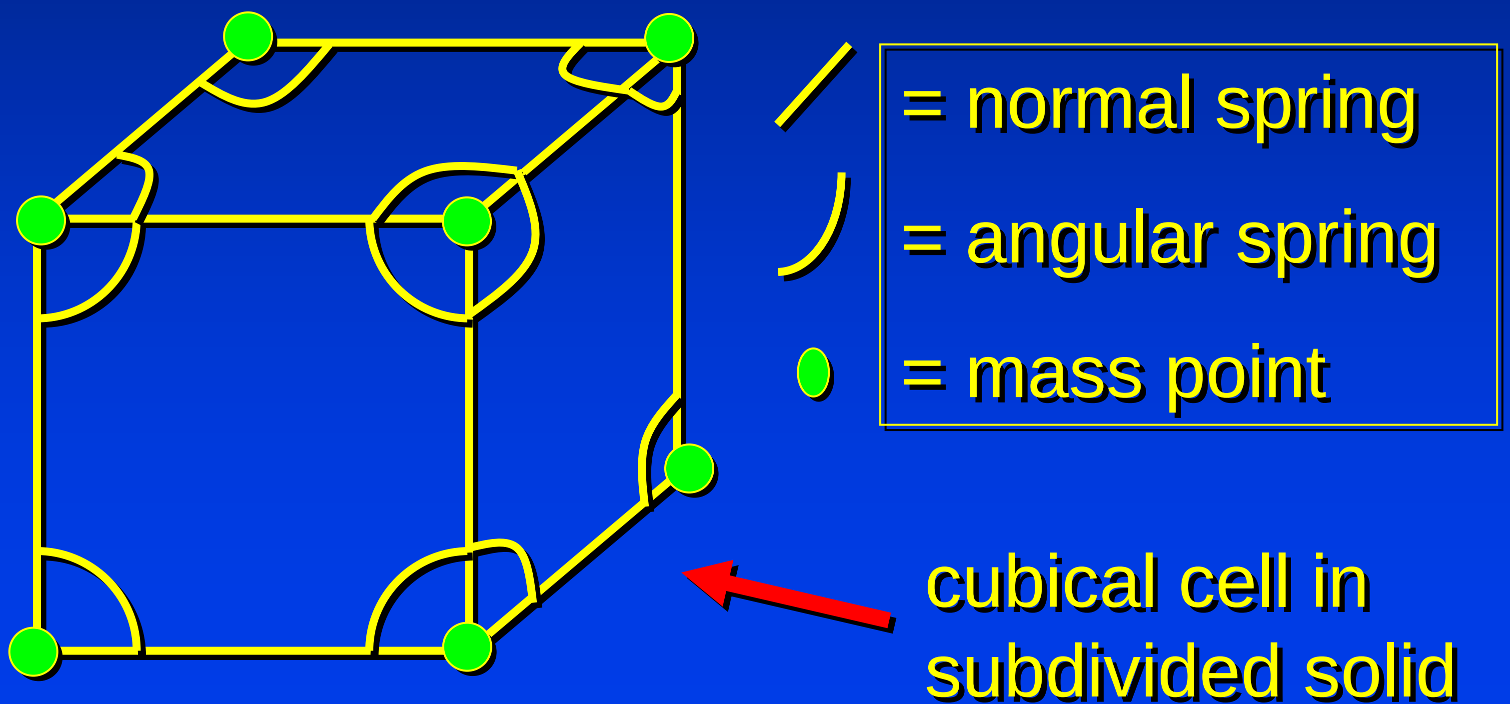
$$v = \frac{c_{avg} + 3a_{avg} + 3m_{avg} + p}{8}$$

DYNASOAR

- Combine subdivision solid model with physics-based modeling
  - – assign mass, damping and stiffness to subdivided solid
- Provide user with geometric-, haptics- and force-based sculpting tools
- Geometry of subdivision solid object evolves in tandem with physical simulation
- New approach to virtual solid sculpting

Mass-Spring System

Augmented mass-spring lattice



Equation for Mass-Spring System

We use a discrete version of the Lagrangian equation of motion:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{D}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{f}_{\mathbf{d}}$$

where

$\mathbf{M}$ = mass matrix

$\mathbf{D}$ = damping matrix

$\mathbf{K}$ = stiffness matrix

$\mathbf{d}$ = discrete material distribution

$\mathbf{f}$ = external user-applied forces

Discrete Time Derivatives

Discrete derivatives are computed as follows:

$$\ddot{\mathbf{p}}_{i+1} = \frac{(\mathbf{p}_{i+1} - 2\mathbf{p}_i + \mathbf{p}_{i-1})}{\Delta t^2}$$

$$\dot{\mathbf{p}}_{i+1} = \frac{(\mathbf{p}_{i+1} - \mathbf{p}_{i-1})}{2\Delta t}$$

Discretized Equation for Simulation

Given the previous equations we derive the implicit time integration formula:

$$(2\mathbf{M}_{\mathbf{p}} + \Delta t\mathbf{D}_{\mathbf{p}} + 2\Delta t^2\mathbf{K}_{\mathbf{p}})\mathbf{p}_{i+1} =$$

$$2\Delta t^2\mathbf{f}_{\mathbf{p}} + 4\mathbf{M}_{\mathbf{p}}\mathbf{p}_i - (2\mathbf{M}_{\mathbf{p}} - \Delta t\mathbf{D}_{\mathbf{p}})\mathbf{p}_{i-1}$$

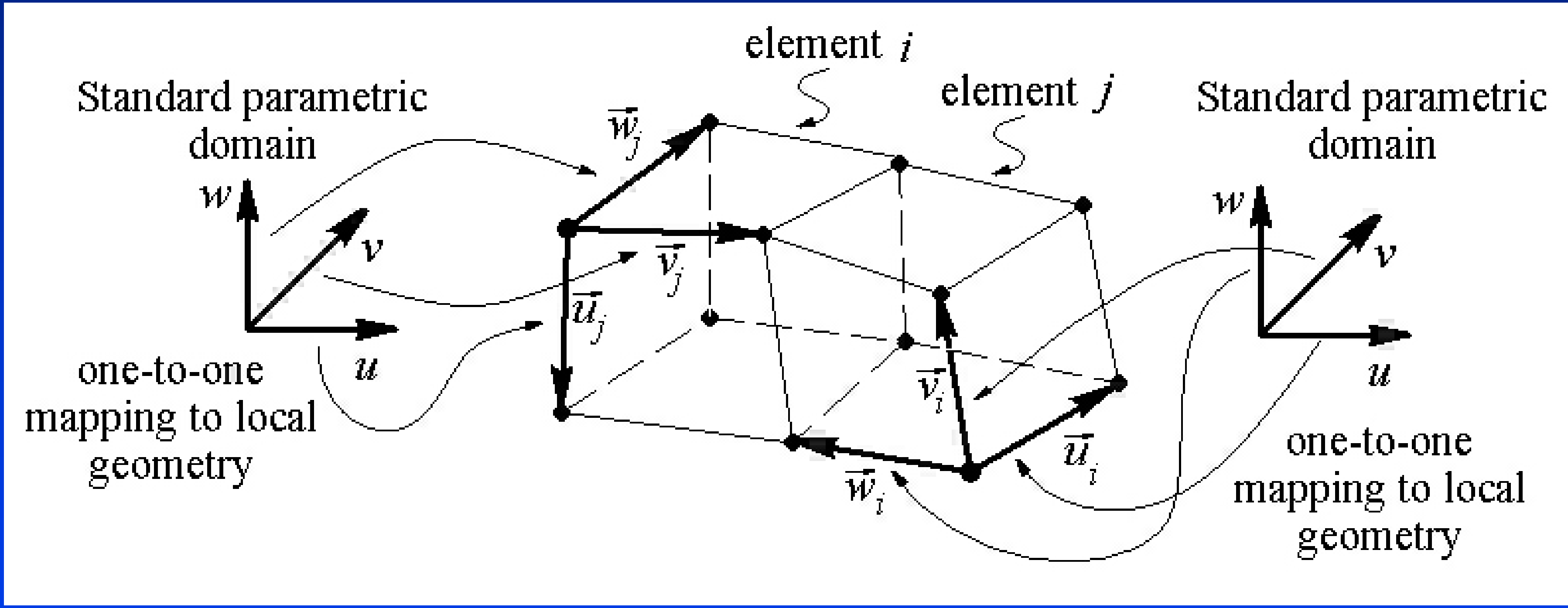
where

$$\mathbf{M}_{\mathbf{p}} = \mathbf{A}^{\mathrm{T}}\mathbf{MA}$$

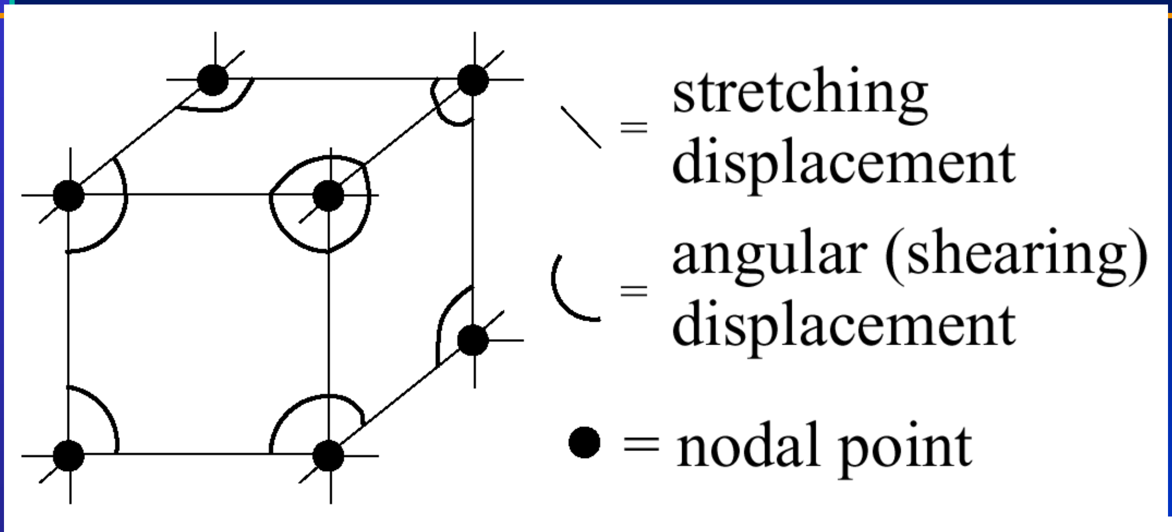
$$\mathbf{D}_{\mathbf{p}} = \mathbf{A}^{\mathrm{T}}\mathbf{DA}$$

$$\mathbf{K}_{\mathbf{p}} = \mathbf{A}^{\mathrm{T}}\mathbf{KA}$$

Element Parameterization

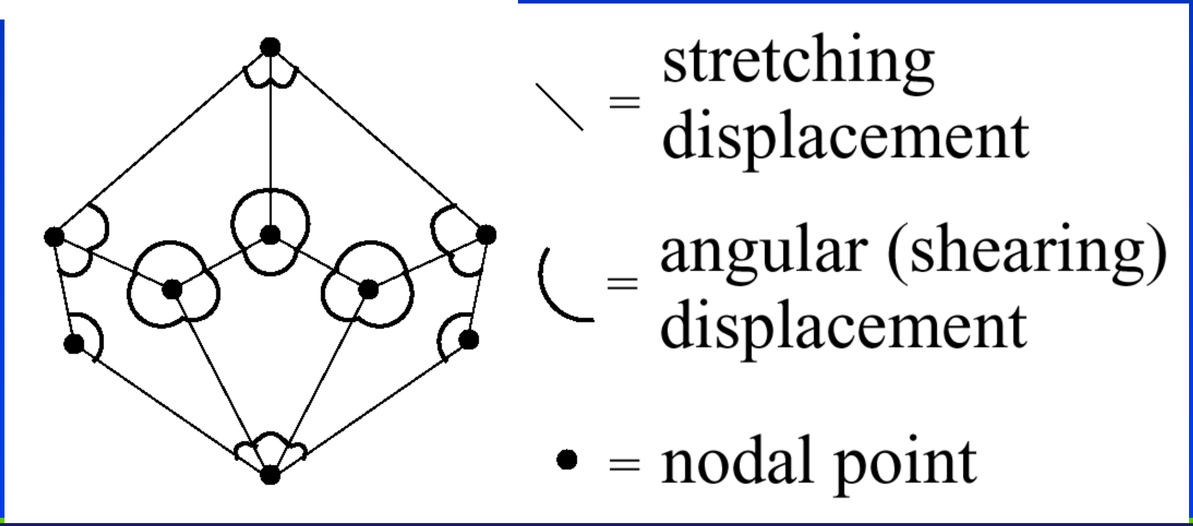


Finite Elements



normal cell

special cell



Dynamics Equation for FEM

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{D}\dot{\mathbf{d}} + \mathbf{K}\delta_{\mathbf{d}} = \mathbf{f}_{\mathbf{d}}$$

Equation of motion drives physical simulation:

- $\mathbf{M}$ = mass matrix
- $\mathbf{D}$ = damping matrix
- $\mathbf{K}$ = stiffness matrix
- $\mathbf{d}$ = discrete material distribution
- $\delta_{\mathbf{d}}$ = displacement (*e.g.*, from rest shape)
- $\mathbf{f}$ = external forces

Hybrid Equation of Motion

$$(2\mathbf{M_p} + \Delta t\mathbf{D_p} + 2\Delta t^2\mathbf{K_p})\mathbf{p}_{i+1} =$$

$$2\Delta t^2\mathbf{f_p} + 4\mathbf{M_p}\mathbf{p}_i - (2\mathbf{M_p} - \Delta t\mathbf{D_p})\mathbf{p}_{i-1}$$

$$\mathbf{M_p} = \mathbf{A}^{\mathrm{T}}\mathbf{MA} \qquad \mathbf{D_p} = \mathbf{A}^{\mathrm{T}}\mathbf{DA}$$

$$\mathbf{K_p} = \mathbf{A}^{\mathrm{T}}\mathbf{KBA} \qquad \mathbf{f_p} = \mathbf{A}^{\mathrm{T}}\mathbf{f_d} - \mathbf{A}^{\mathrm{T}}\mathbf{KCAp_0}$$

$$\mathbf{B} = \text{stress due to displacement}$$

$$\mathbf{C} = \text{stretching and bending energy}$$

Element Matrices

What are M, D and K?

$$\mathbf{M} = \iiint \mu \mathbf{J}^{\mathbf{T}} \mathbf{J} \, du \, dv \, dw$$

where

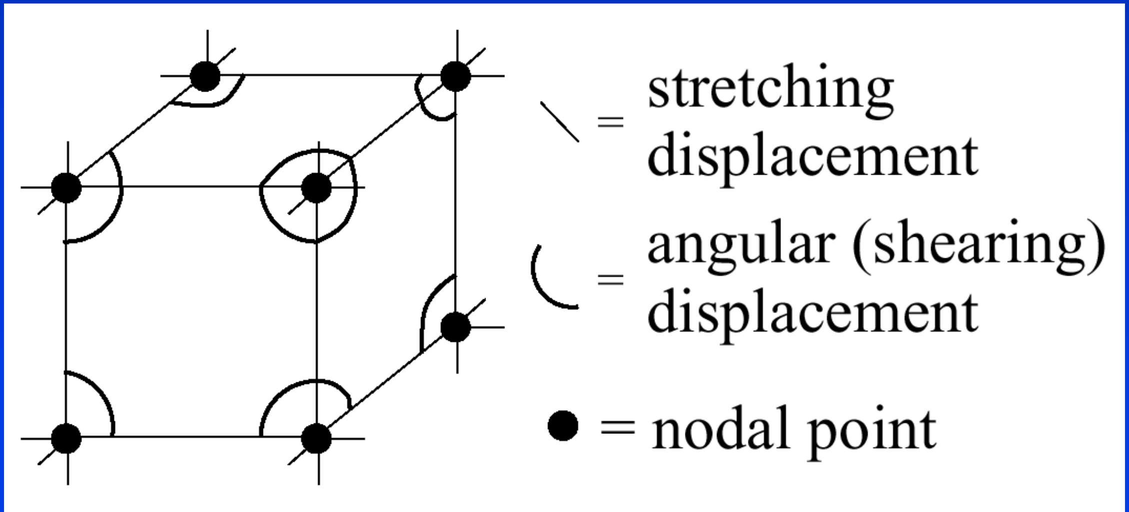
μ is a continuous mass distribution

$\mathbf{J} = [B_0 \cdots B_7]$

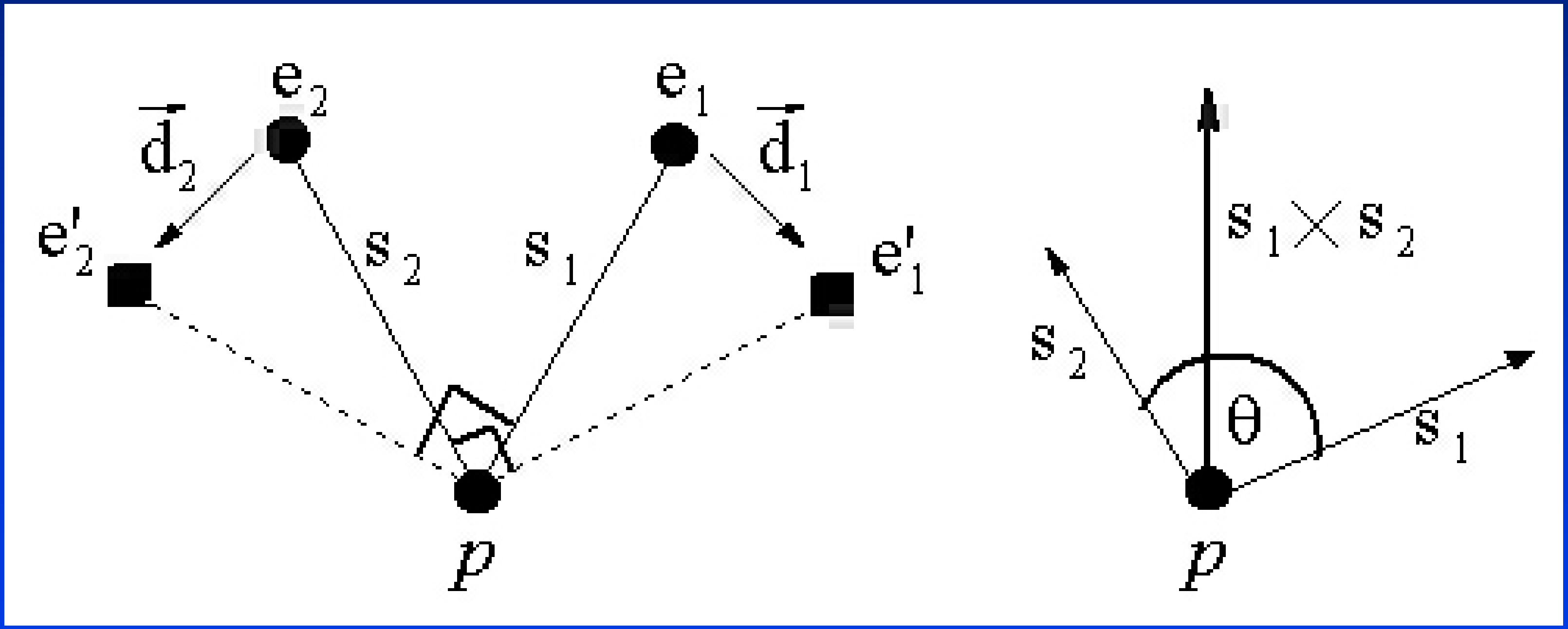
and B_i is the ith FEM shape function

Element Matrices

- D has a similar definition
- K has application-specific definitions
  - for small deformations
  - for large deformations



Stiffness Formulation



Gaussian Quadrature

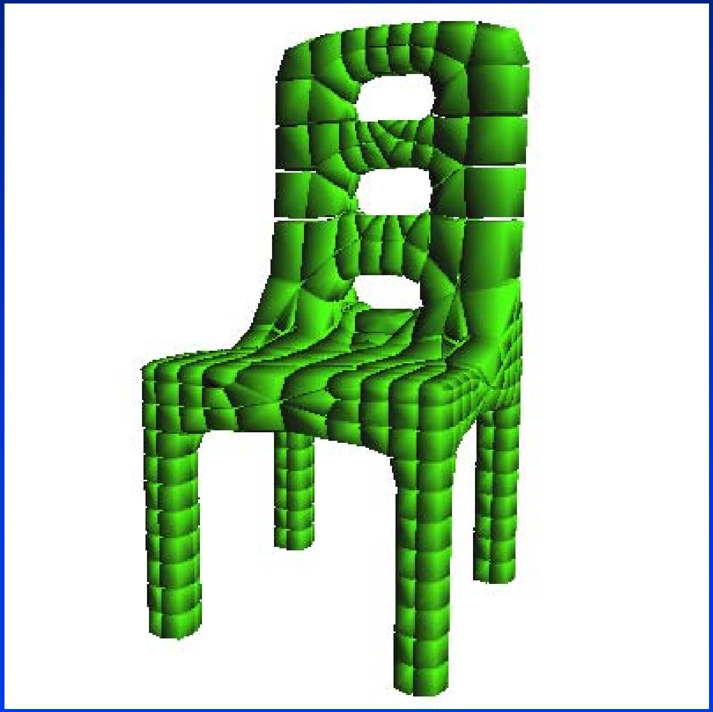
- How are the integrals evaluated?
- Technique used is *Gaussian Quadrature*
- GQ evaluates

$$\int_{w_0}^{w_1}\int_{v_0}^{v_1}\int_{u_0}^{u_1} g(u,v,w)\,du\,dv\,dw$$

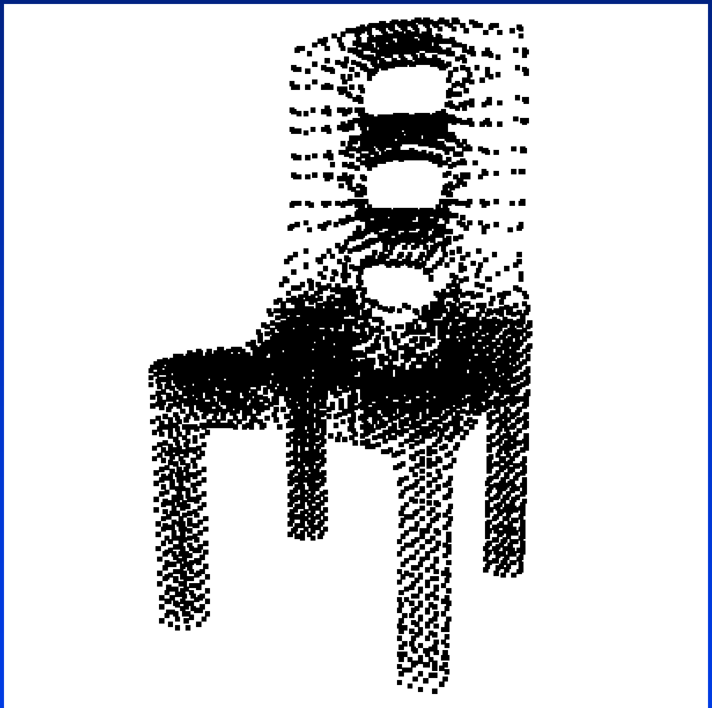
as

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} q_k^u q_j^v q_i^w\, g(u_k, v_j, w_i)$$

Gaussian Quadrature



finite elements

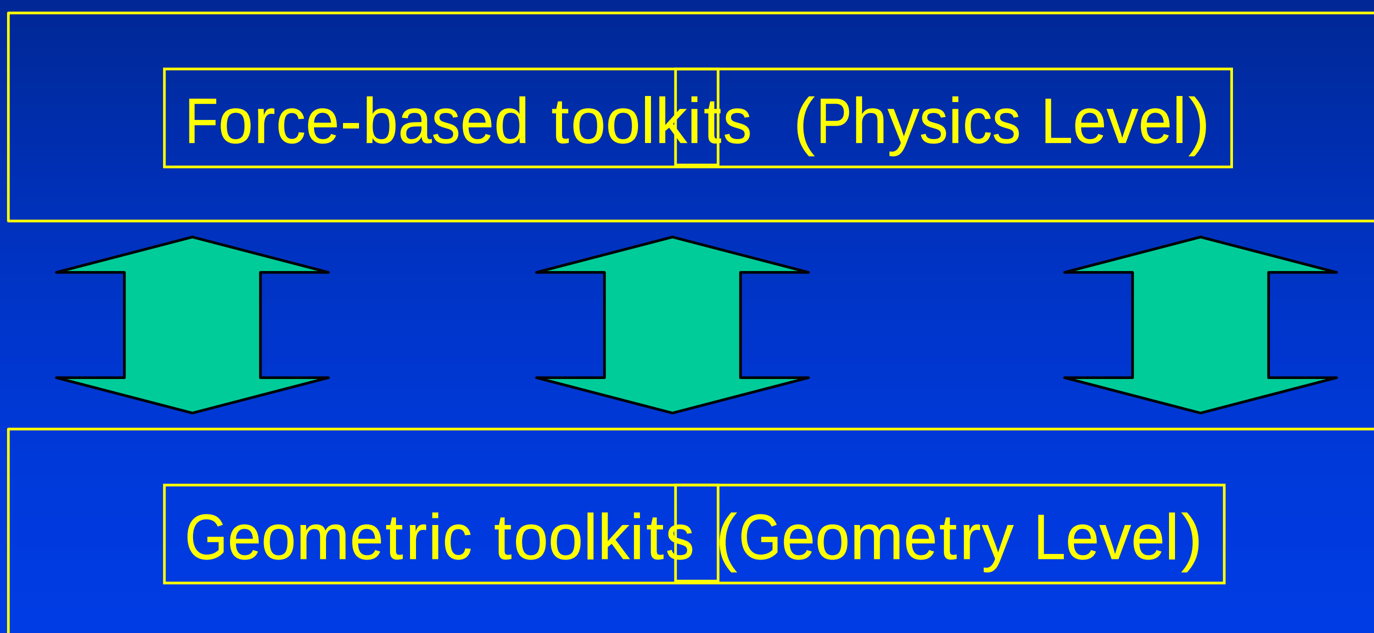


quadrature points

Physics-Based Shape Design

- Two-level approach

Physics-Based Design Framework



Physics-Based Geometric Design

- Generalization of geometric design process
- Standard geometric toolkits still usable
- Two-level design framework
- Additional physics-based toolkits
  - Sculpting forces, elastic energies, linear and non-linear constraints
- Integration of traditional design principles

Physics-Based Geometric Design

- Enhance geometric design with additional advantages
  - Automatic determination of geometric unknowns
  - Complicated geometry transparent to designers
  - Intuitive shape variation governed by physical properties
  - Valuable for non-expert users and engineers
  - Relevant to the entire CAD/CAM processes

Numerical Implementation

- Finite element analysis approach
- New subdivision surface finite element
  - Normal elements, special elements
- Gaussian quadrature to assemble element matrices
- Numerical time integration of motion equation
- Efficient parallel algorithm
- Force applications
- Hierarchical model

Finite Element Data Structure

FEM Data Structure

Element matrices

Physical quantities

Local geometric data

Element DOFs

Element matrices

Physical quantities

Local geometric data

Element DOFs

Global degrees of freedom array of subdivision surfaces

Global geometric information of subdivision surfaces

Physics-Based Geometric Design

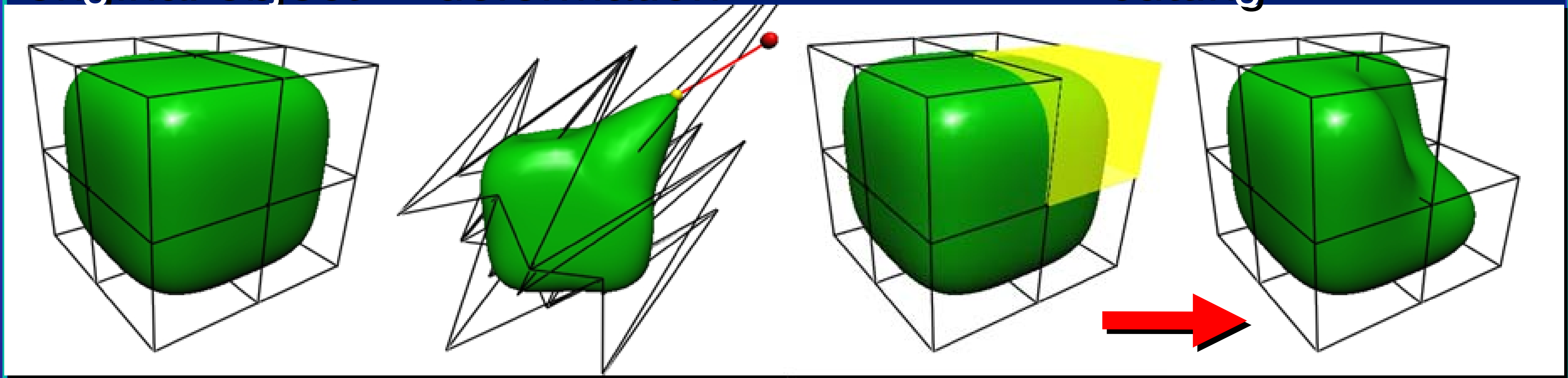
- Generalization of geometric design process
- Standard geometric toolkits still usable
- Additional physics-based toolkits
  - Sculpting forces, elastic energies
  - Linear and non-linear constraints
- Enhance geometric design with new advantages
  - Complicated geometry transparent to designers
  - Intuitive shape variation
  - Valuable for non-expert users and engineers
  - Relevant to the entire CAD/CAM process

Applications

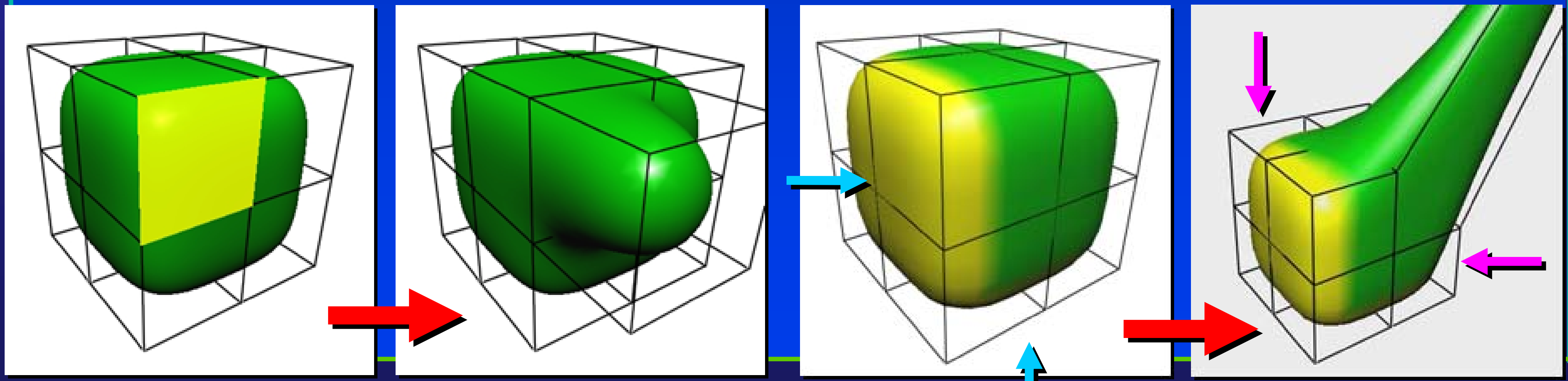
- Geometric modeling and shape design
- Virtual sculpting
- Rapid prototyping
- Physical simulation and animation
- Finite element analysis
- Material and dynamics evaluation
- Data fitting and segmentation
- Volume visualization

Simple Sculpting Examples

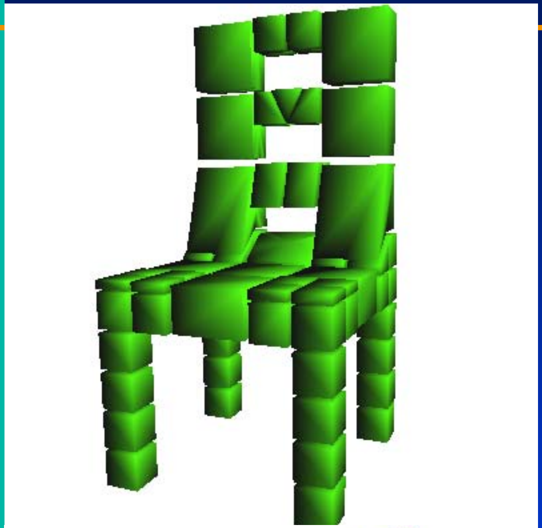
original object deformation cutting



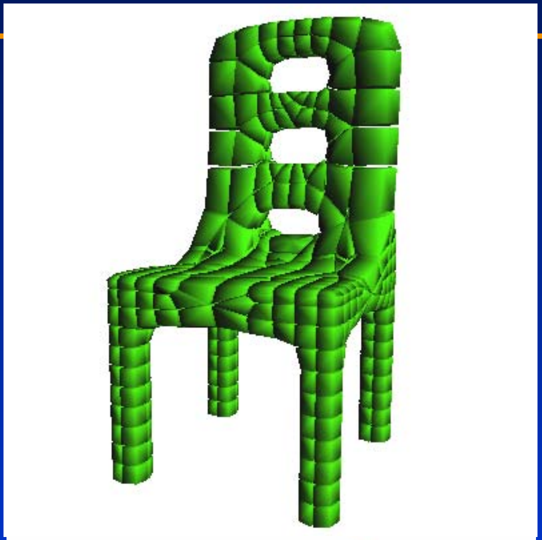
extrusion fixed regions



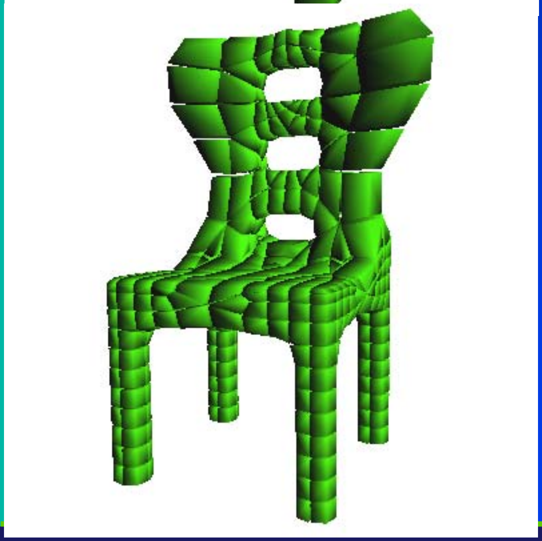
FEM Simulation



control lattice



finite elements



deformed object

photo-realistic rendering

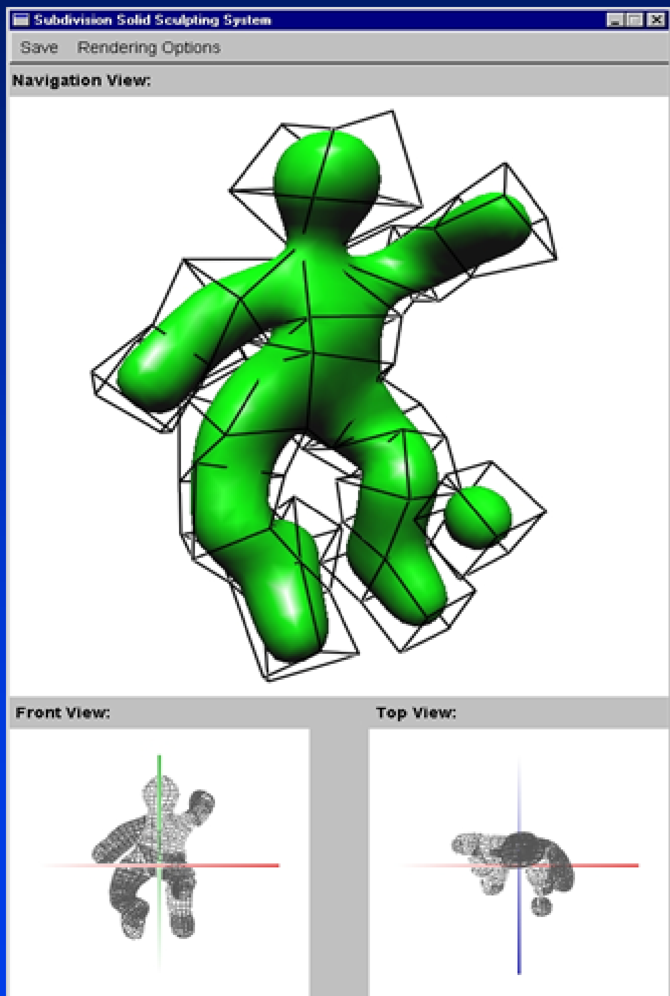
Data Structures

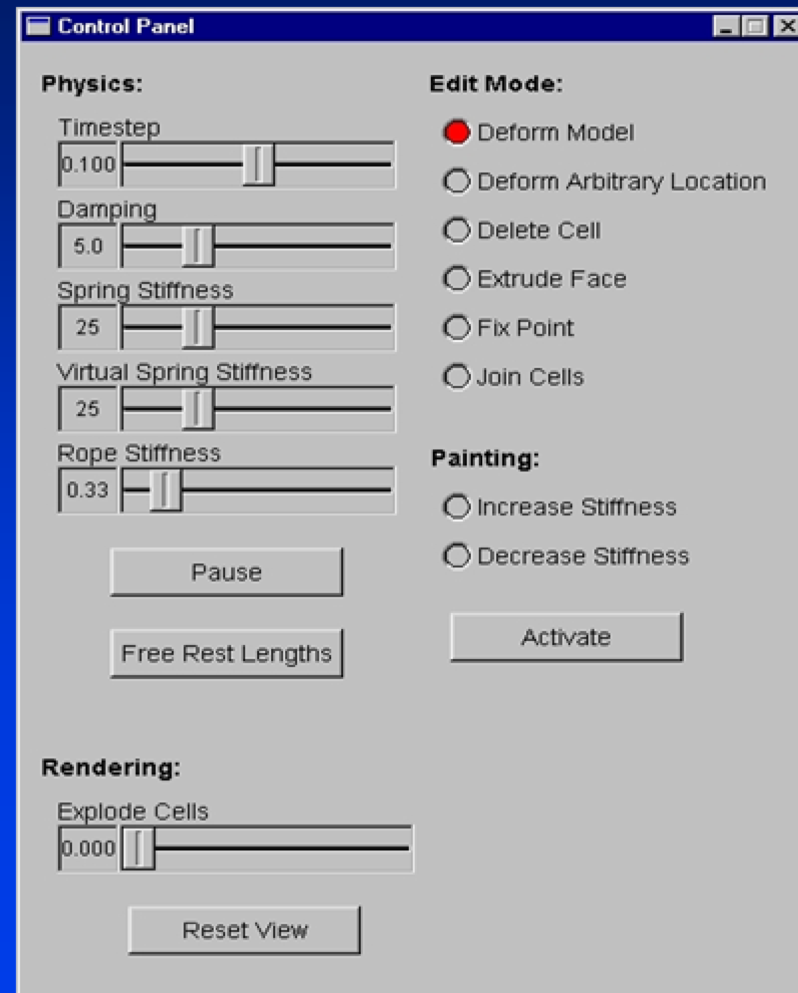
- Subdivision solids
  - radial-edge data structure (Weiler `86)
  - similar to winged-edge data structure
  - stores adjacency information to accelerate queries of and changes to topology of subdivision solids
- Physical representation
  - sparse matrices, vectors, arrays, etc.

Virtual Sculpting Environment

- Suite of extensible virtual sculpting tools
  - haptic: stretch, probe, …
  - geometric and topological: cut, extrude, join, …
  - physical: change material, inflate, …
- On-screen GUI controls
- Sensable Technologies PHANToM haptic I/O device
- Runs on 550 MHz PC, 512 MB RAM

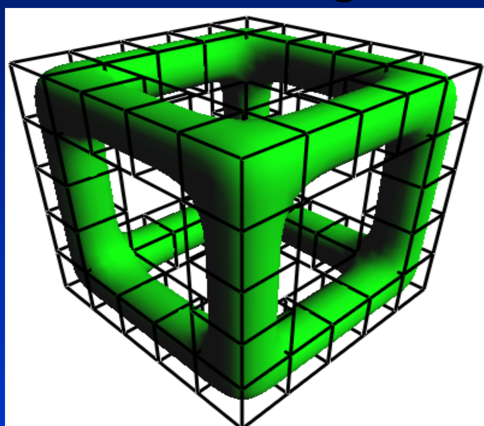
Graphics-based Interface



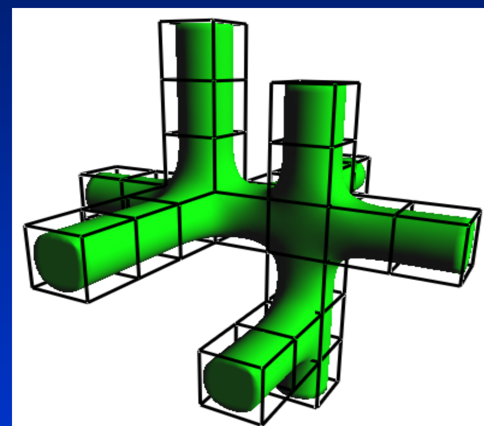


Sculpting Tools

carving



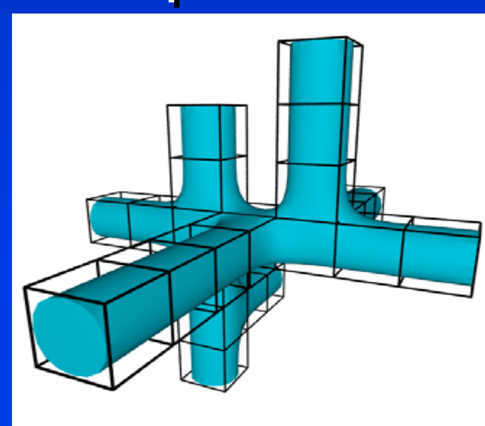
extrusion



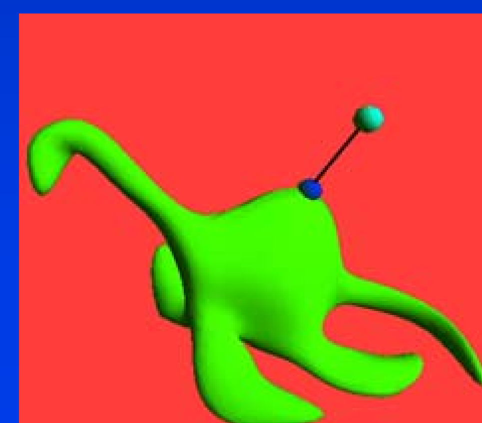
detail editing

joining

sharp features



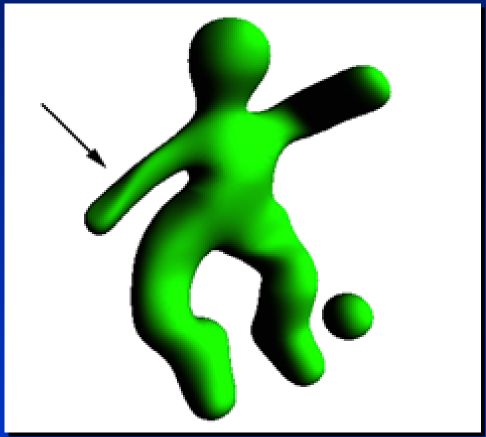
deformation



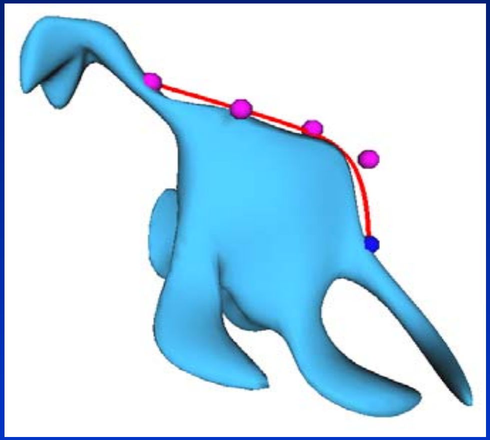
Sculpting Tools

inflation

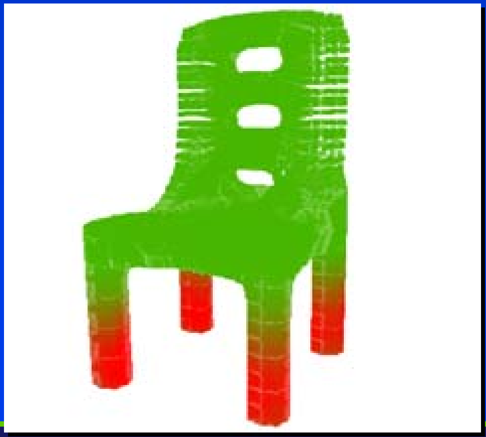
deflation



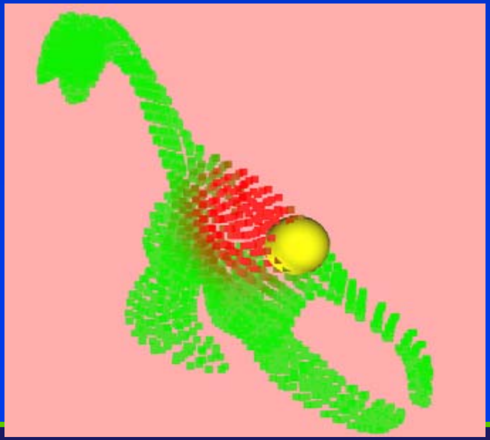
curve-based design



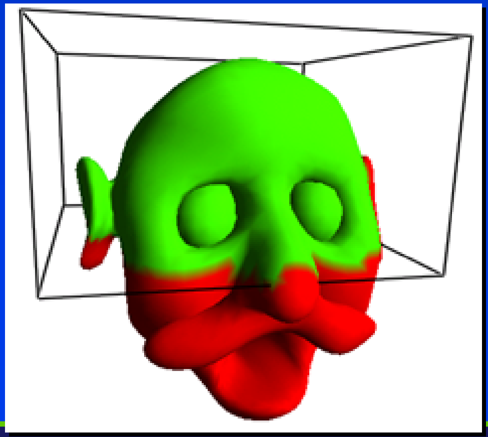
material mapping



material probing

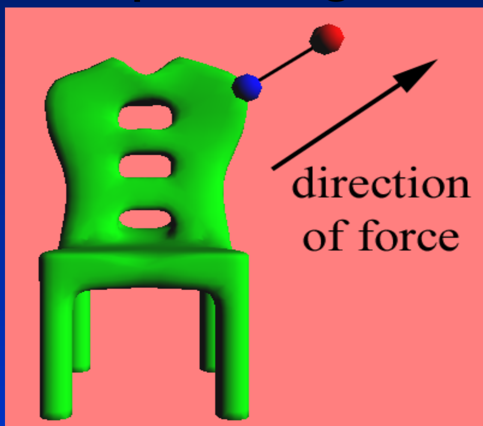


physical window

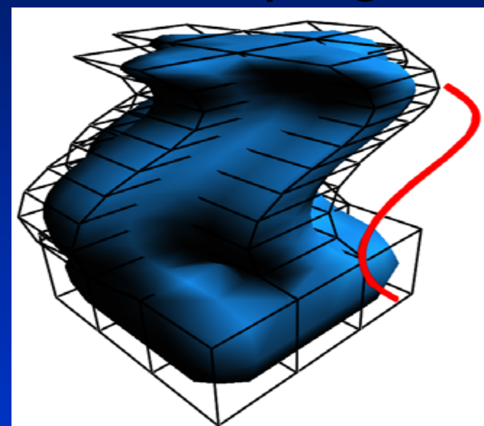


Sculpting Tools

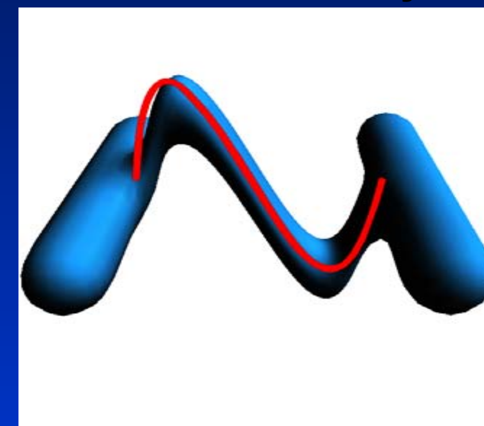
pushing



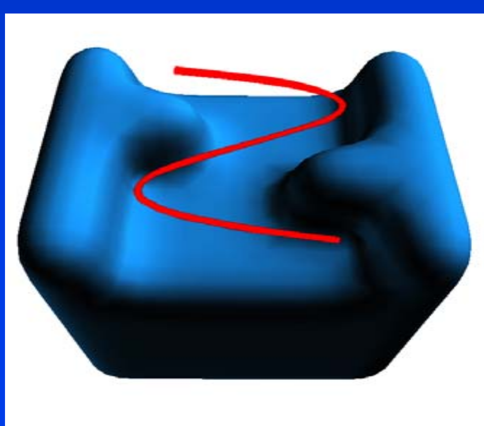
sweeping



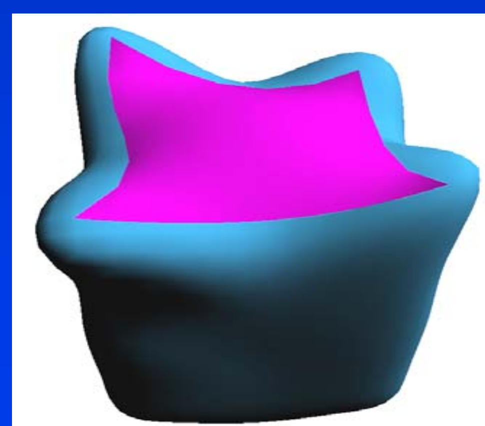
curve-based join



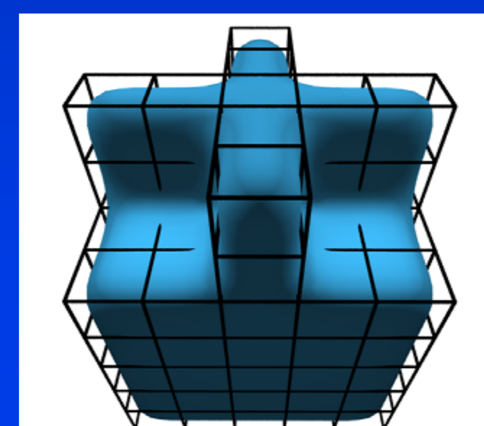
curve-based cutting



feature deformation



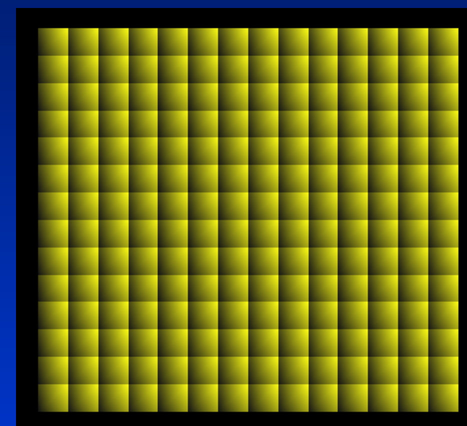
multi-face extrusion



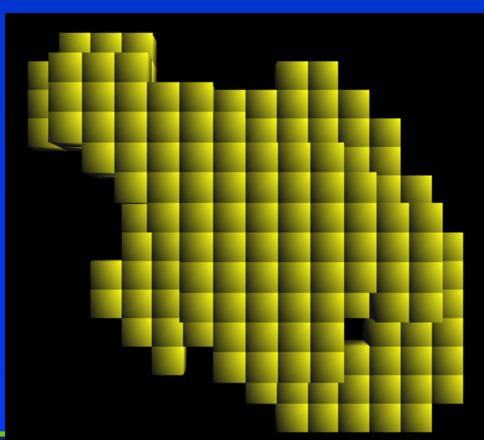
Trimmed Solids for Data Fitting

original dataset

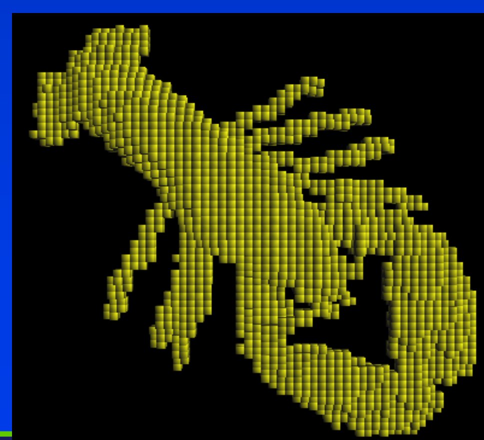
initial lattice



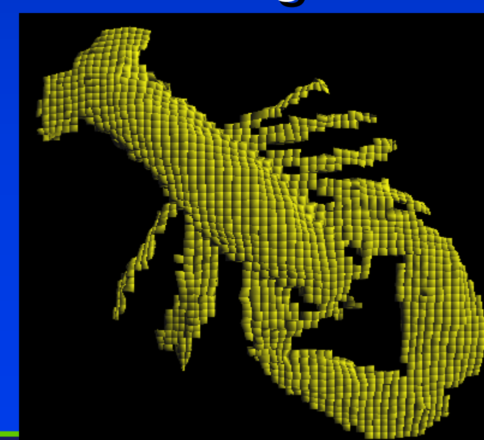
trimmed once



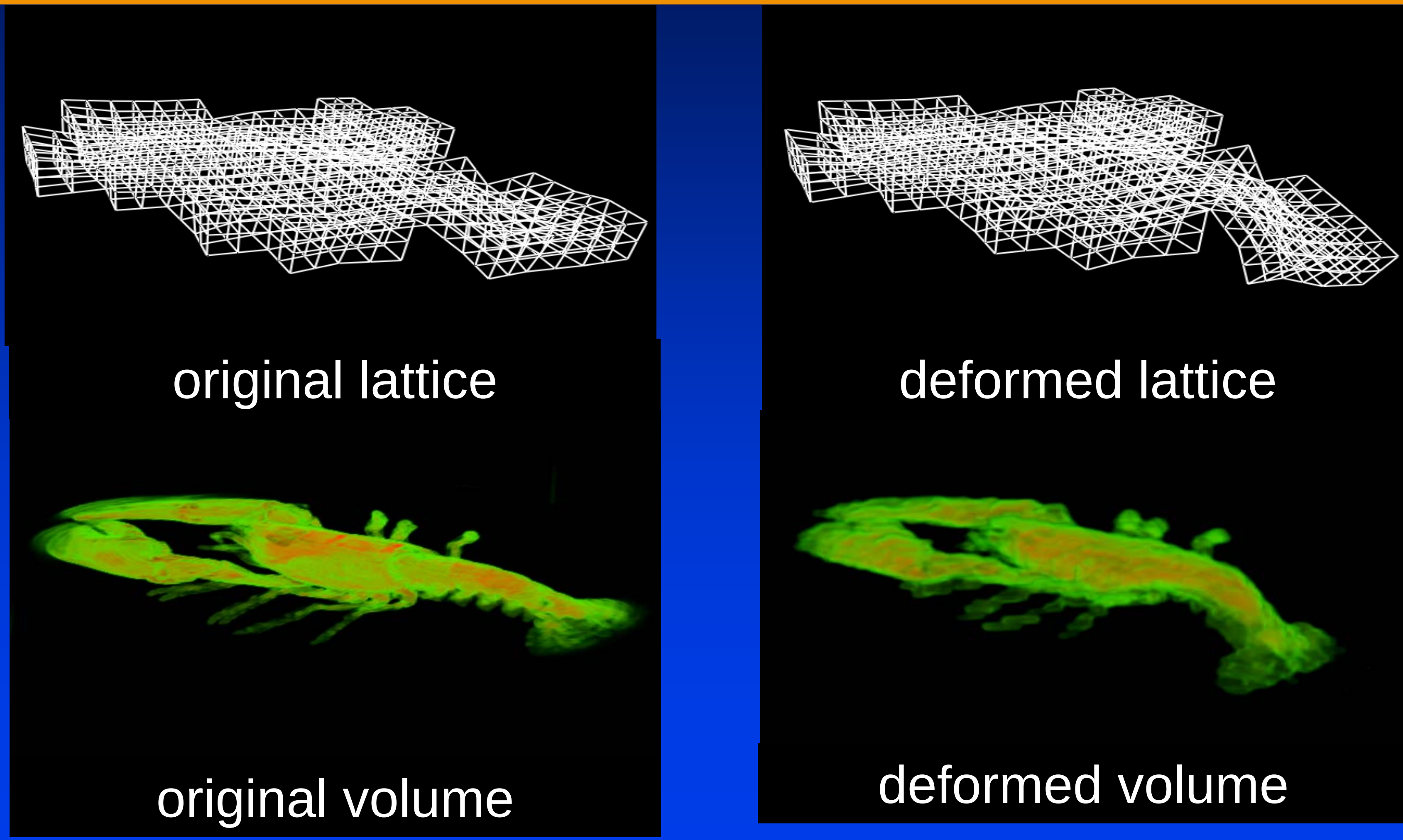
trimmed twice



deformed geometry



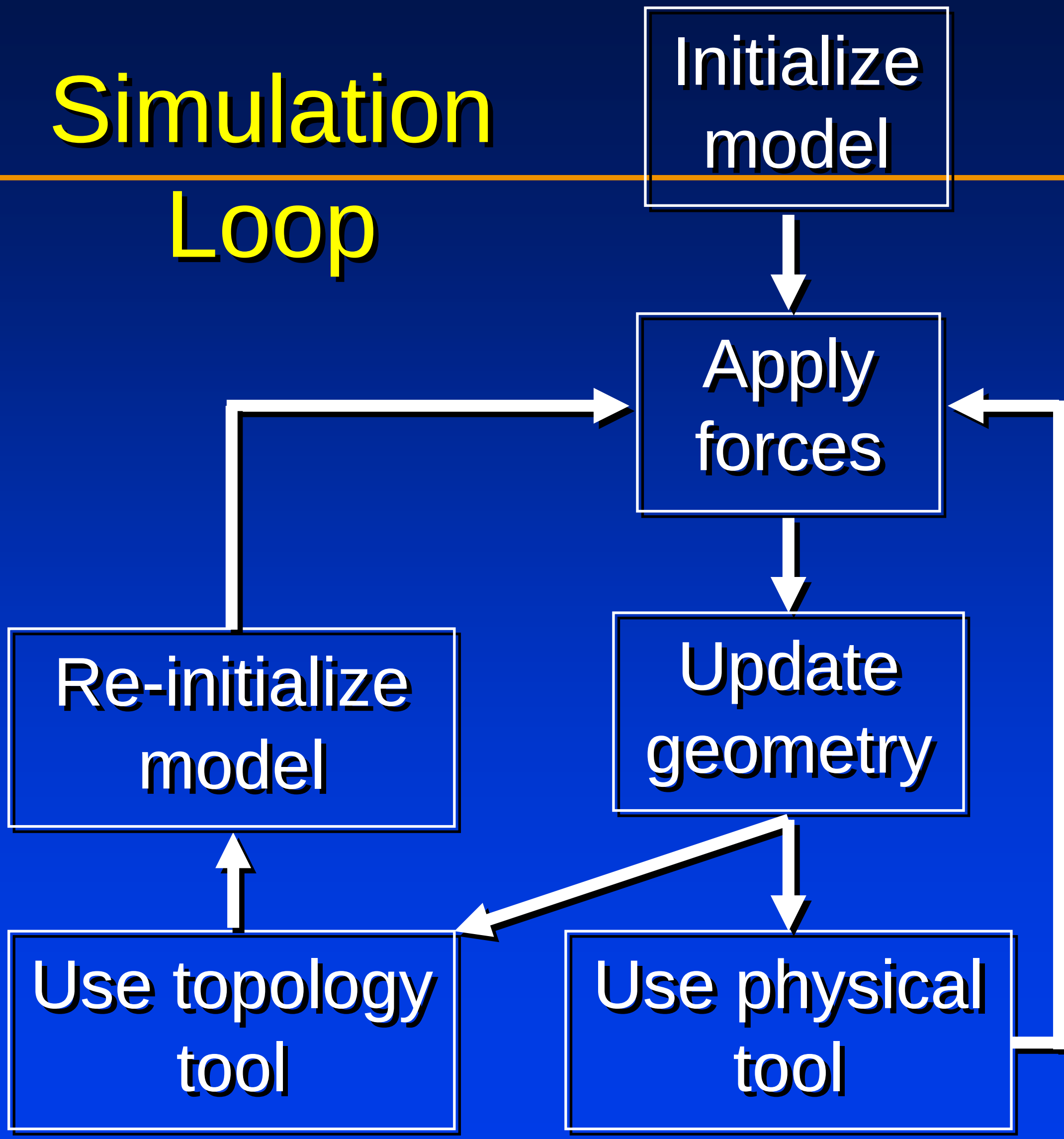
Volume Editing and Visualization



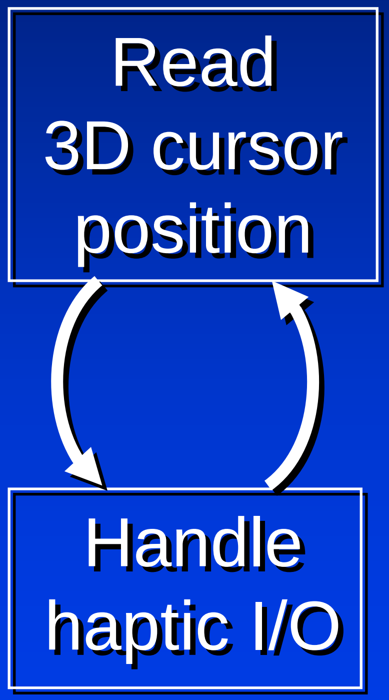
Run-time Interaction

- System de-coupled into Simulation and Haptics loops
- Haptic interface runs in separate loop to guarantee real-time update rates
- Equation of motion solved at each time-step in Simulation loop
- Physical simulation guides deformation of geometry

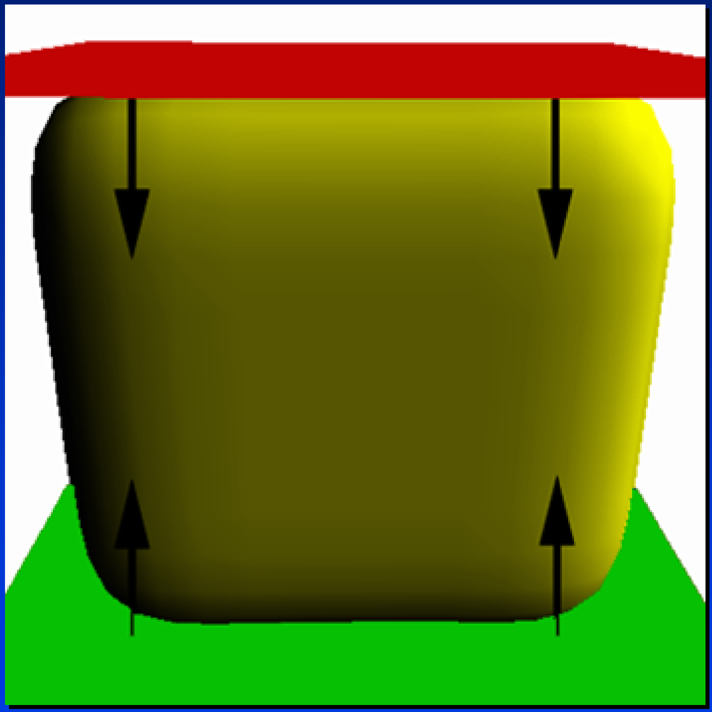
Simulation Loop



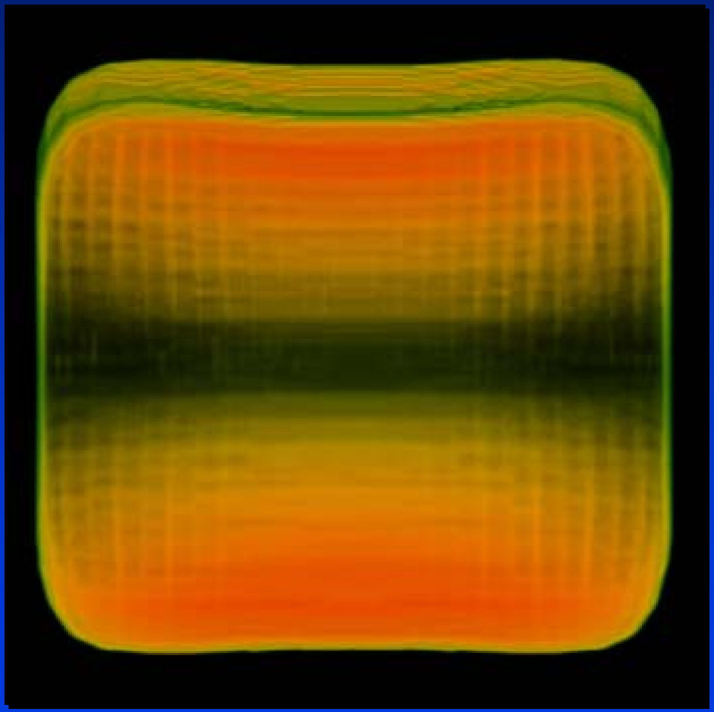
Haptics Loop



Material Simulation and Analysis



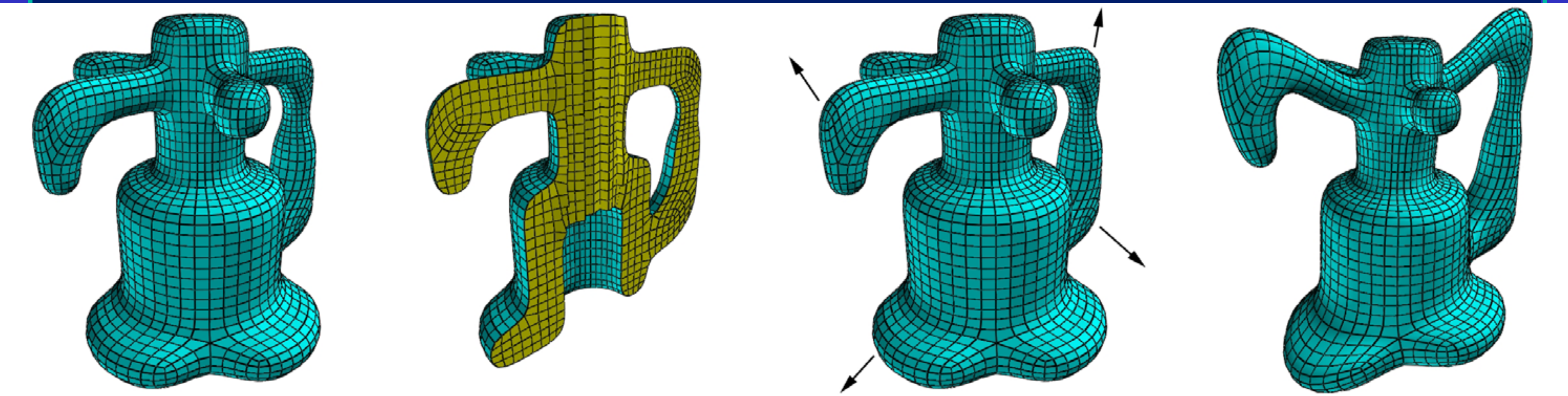
compressive forces

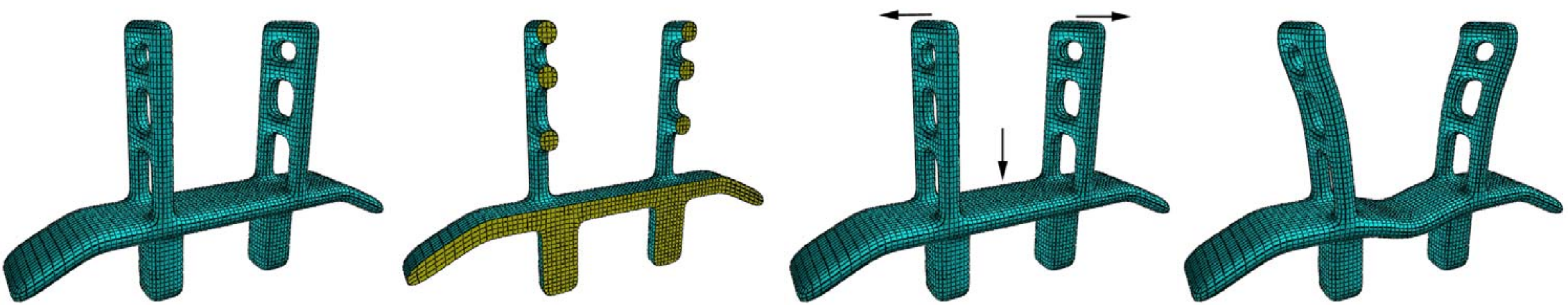


displacement mapping

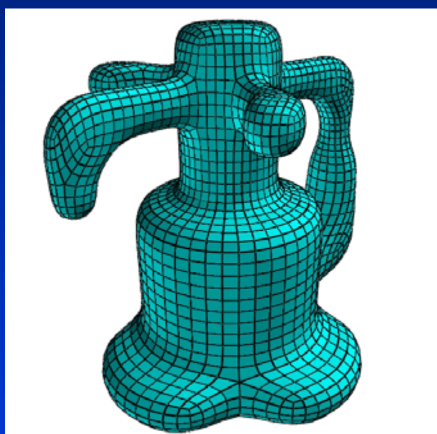
Scenes from DYNASOAR System

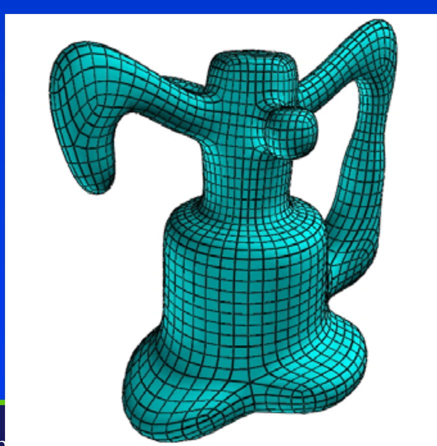
Finite Element Formulation

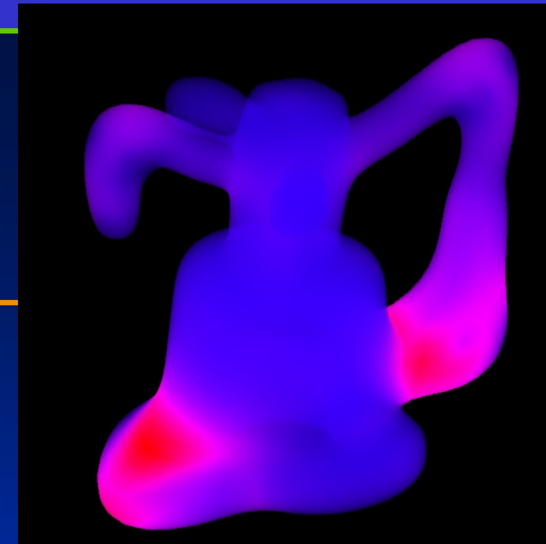




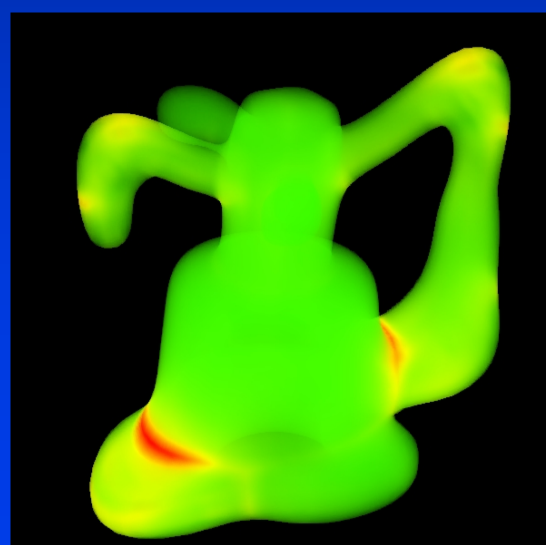
DYNASOAR (FEM) Visualization



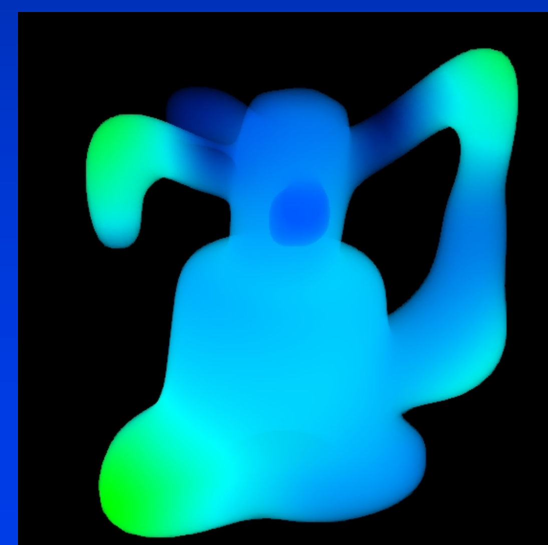




volumetric distortion



strains



displacements

FEM-Based Animation

Conclusions

- DYNASOAR: the next-generation, physics-based, volumetric CAD system with haptic interaction for virtual engineering
- Integration of subdivision solids with dynamic behaviors and material properties for various solid modeling applications
- Intuitive sculpting tools permit real-time manipulation of virtual clay-like material
- Geometry-based, force-based, and haptics-based virtual toolkits offer natural impression and intuitive interface

Research Foci & Activities

- Research group on Physics-based modeling and simulation
- MAGIC Lab (Modeling, Animation, and Geometry for Interactive Computing)
- Technical vision and strategy: Geometry + Physics
- Founded upon a novel graphical modeling methodology --- Dynamic geometry for shape design based on interactive physics
  - Integration of geometry and physics
  - Intuitive force-based CAD tools
  - Unifying modeling, design, analysis, and manufacturing
  - Virtual engineering without physical prototyping
- Applications
  - Graphics, geometric design, finite element analysis, CAD/CAM, computer animation, scientific and information visualization, haptic interaction, computer vision, virtual environments, etc.

Engineering Impacts

- Industrial significance
- Improve product quality
  - supply intuitive & effective CAD tools
- Shorten product development cycle
  - incorporate manufacturing constraints in design process
  - unify geometry, design, analysis, assembly, rapid prototyping, and manufacturing
- Reduce product cost
- Enhance the effectiveness of design engineers
- Stimulate future technologies for virtual engineering

Motivation for Future Research

- Ever-increasing, high expectations of
  - Improved product quality, reduced product prices, accelerated performance
- Challenges
  - New design theory and methodology
  - Advanced simulation methods
  - Efficient analysis tools
  - More powerful human-computer interaction
- New strategy in CAGD, FEM, CIMS, CAE
  - Subdivision-based representation, modeling, design, analysis, and manufacturing techniques for the next generation CAD/CAM system
- Geometric design and computing as a theoretical and algorithmic foundation for multi-disciplinary research and development activities in the future

Broader Impacts in IT

- Promote computer-centered, graphics-driven modeling, design, simulation, analysis technologies
- Broaden user access through multi-modal interface for both computer professionals and naïve users
- Afford vision-impaired users and computer illiterates a natural and intuitive interaction via human hands
- Advance the state-of-the-knowledge in information technology and computer science
- Revolutionize scientific and engineering education in mathematics and physics through hands-on experiences
- Alleviate the intimidation of abstract mathematics and physics
- Attract a larger population in young high-school students to study science and engineering disciplines in colleges and universities

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Future Research Focus

- Efficient and robust algorithm for design and analysis
- Physics-based sculpting toolkits
- Formulation of new powerful dynamic models
- Advanced user interaction techniques
- Various applications
- Industrial collaboration and support
- Technology transfer to commercial CAD/CAM systems

Future Research Directions

- Fundamental theory
- Interactive modeling environments with physics-based programming toolkits
- Advanced user interaction techniques
- Multidisciplinary advances from applied & computational mathematics, physics, and engineering sciences
- Visual computing & engineering applications
- Integration with engineering design systems
- Commercial software & system products

Physics-Based Modeling Theory

- Efficient and robust algorithm design and analysis
- Physics-based programming toolkits
- Advanced user interaction techniques
- Integration of multi-disciplinary advances
  - Computational sciences
  - Applied and computational mathematics
  - Physics (e.g., fluid dynamics)
  - Engineering sciences

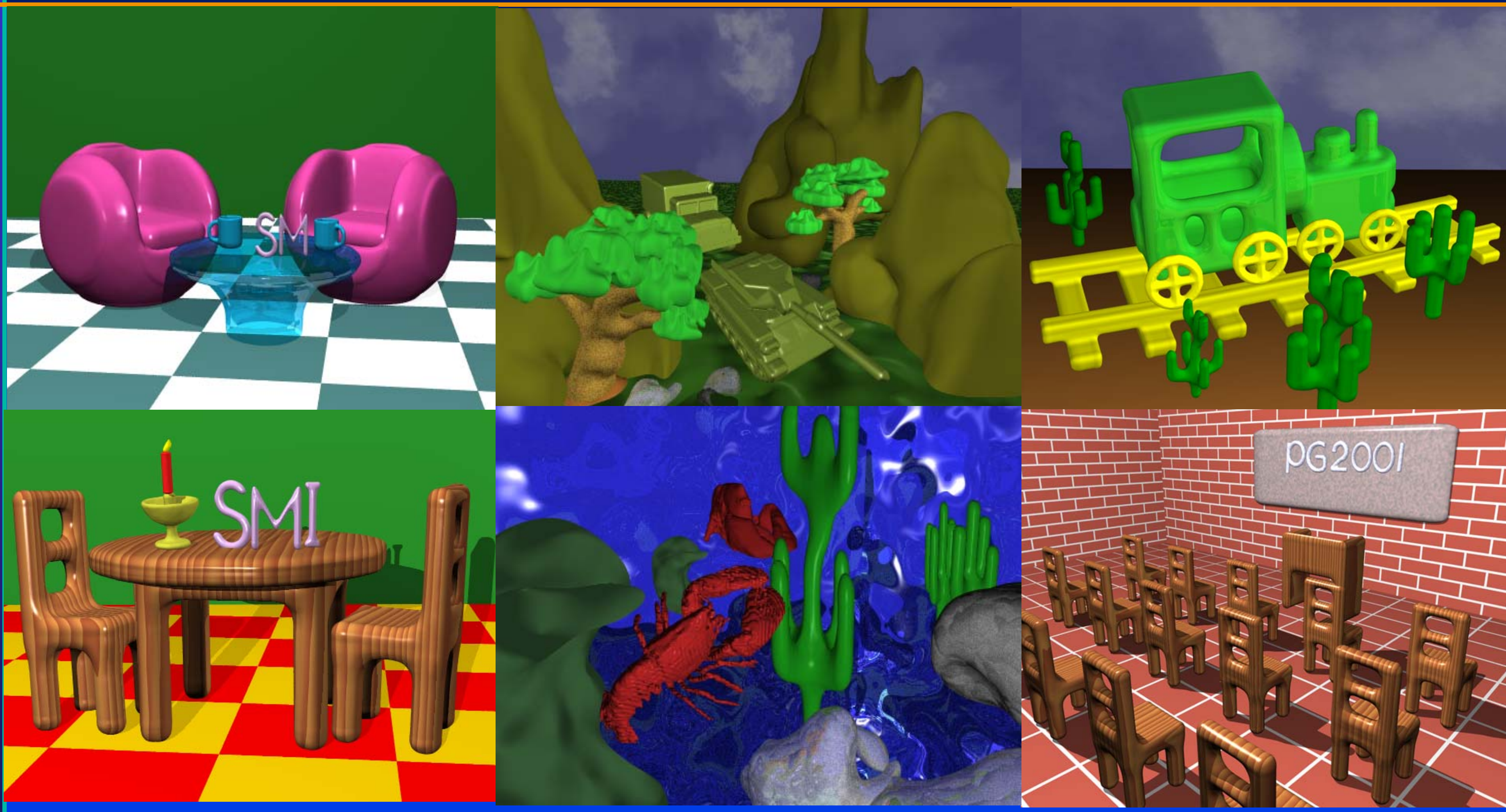
Interactive Modeling Environment

- Physics-based design tools
- Various engineering applications
  - Solid rounding, scattered data fitting, shape reconstruction, interactive sculpting, reverse engineering, data visualization, hierarchical control
- Unified approach for CAD/CAM
  - Variational design
  - User interaction
  - Shape control
  - Weight selection

Simulation-Based Virtual Environments

- Complex real-world models and phenomena
- Parallel algorithms + collaboration tools for concurrent engineering
- Distributed physics-based simulation
- Virtual engineering without physical prototyping

Thank you



Driving Applications

- Computer graphics and animation
- Geometric modeling and shape design
- CAD/CAM/CAE
- Scientific and information visualization
- Physical and haptic interaction
- Multi-modal HCI
- Computer vision
- Finite element method and numerical techniques
- Virtual engineering and virtual environments
- Applied mathematics and computational physics

Applications and Beyond

- Computer animation
- Virtual reality
- Computer vision and robotics
- Medicine and medical imaging
- Artificial life
- Scientific visualization
- Industrial collaboration and support
- Technology transfer to commercial systems

Hot Research Projects

- Dynamic NURBS theory and applications
- DYNASOAR: DYNAmic Solid Objects of ARbitrary topology
- Intelligent Balloon (subdivision surfaces for unknown topology)
- PDE surfaces and solids
- Haptics-based interface and VR
- Multiresolution analysis, wavelets
- Implicit functions

On-going Research Projects

- Dynamic NURBS theory & applications
- Subdivision surfaces and their non-uniform, rational generalizations
- Subdivision-based solid modeling
- Geometric modeling and design based on PDEs
- Intuitive force-based CAD tools
- Novel numerical solvers based on signal processing theory
- Energy-based optimization techniques
- Wavelet and implicit functions for shape design

Available Projects

- Virtual cosmetics, surgery simulation
- 3D painting environment for artists, decorating solids
- Haptics-based sculpting and its integration with VEs
- Inferring material, physical, dynamical properties from images, videos
- Digital clay, shape recovery from scattered data
- PDE-based models
- Implicit functions
- Subdivision schemes for polyhedral splines
- Point-based modeling
- Multi-resolution techniques
- Applications: morphing, facial animation, flow, …….