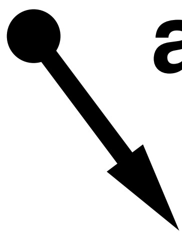
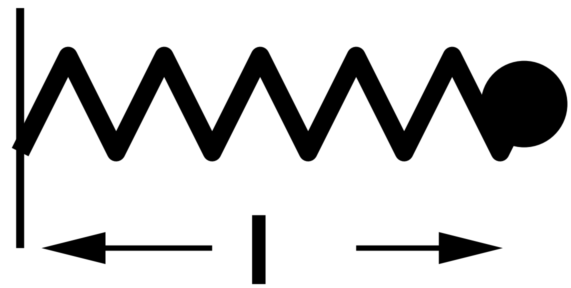
Physics Basics



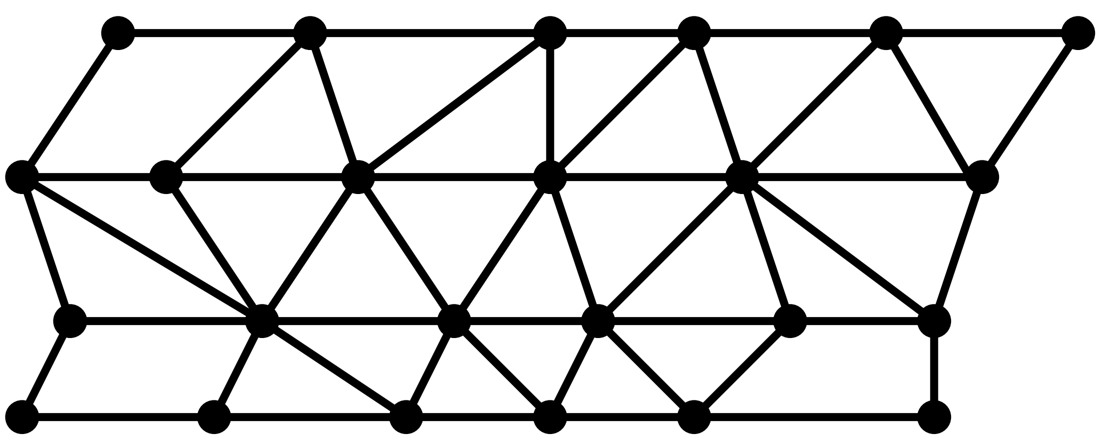
$a = f/m$



energy: $1/2\ k\ (l - l_0)^2$

force: $k(l - l_0)$

mass–spring system:



Physics Basics

$$a = f/m; \quad a = dv/dt; \quad v = dp/dt$$

$$dE/dp = f$$

$$a \;= (\ldots + f^{i} + \ldots)/m$$

$$a \;= ((\ldots + f^{i}_{e} + \ldots) - (\ldots + f^{j}_{i} + \ldots)/m$$

$$ma + cv + (\ldots + f^{j}_{i} + \ldots) = \ldots f^{i}_{e} \ldots$$

$$Ma + Cv + Kp = F$$

Physics Basics

- **One mass-point**

$$m\mathbf{a} = \mathbf{f}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

$$\mathbf{v} = \frac{d\mathbf{p}}{dt}$$

$$\frac{d^2\mathbf{p}}{dt^2} = \mathbf{f}$$

$$m\frac{d^2\mathbf{p}}{dt^2} + c\frac{d\mathbf{p}}{dt} + \sum_i \mathbf{f}_{int}^i = \sum_j \mathbf{f}_{ext}^j$$

- **Mass-spring system**

$$\mathbf{M}\frac{d^2\mathbf{p}}{dt^2} + \mathbf{C}\frac{d\mathbf{p}}{dt} + \mathbf{K}\mathbf{p} = \mathbf{f}$$

- **Numerical Simulation**

$$\mathbf{a}^t = \frac{\mathbf{v}^t - \mathbf{v}^{t-\delta t}}{\delta t}$$

$$\mathbf{v}^t = \frac{\mathbf{p}^t - \mathbf{p}^{t-\delta t}}{\delta t}$$

Physics-Based Framework

- **Motion equation**

$$\mathbf{M}_q\ddot{\mathbf{q}} + \mathbf{D}_q\dot{\mathbf{q}} + \mathbf{K}_q\mathbf{q} = \mathbf{f}_q$$

- **Equation with constraints**

$$(\mathbf{A}^\top\mathbf{M}_q\mathbf{A})\ddot{\mathbf{p}} + (\mathbf{A}^\top\mathbf{D}_q\mathbf{A})\dot{\mathbf{p}} + (\mathbf{A}^\top\mathbf{K}_q\mathbf{A})\mathbf{p} = (\mathbf{A}^\top)\mathbf{f}_q$$

Haptics Projects

- **Subdivision modeling**
- **Trimming**
- **Wavelets**
- **Multiple patches**

Other Projects

- **Advanced sculpting tools**
- **Normal,**
- **Curvatures,**
- **Geometric constraints,**
- **Pressure**
- **Multi-patch surfaces**
- **Material properties**
- **Industrial examples and applications**

Surface Discretization

- Use $n \times n$ meshes to *approximate* one Bezier patch defined by 4×4 control points

$$\mathbf{s}(u,v) = \sum_{i=0}^{3} \sum_{j=0}^{3} \mathbf{p}_{i,j} B_i(u) B_j(v)$$

$$\mathbf{s}(u,v) \approx \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \mathbf{q}_{i,j} N_i(u) N_j(v)$$

  where $N_i(u)$ and $N_j(v)$ are piecewise linear functions

- $\mathbf{q}_{i,j}$'s are on this Bezier patch

- $\mathbf{q}_{i,j}$'s are NO-LONGER independent!

$$\mathbf{q}_{i,j} = \sum_{k=0}^{3} \sum_{l=0}^{3} \mathbf{p}_{i,j} B_k(a_i) B_l(b_j)$$

- Matrix expression

$$\mathbf{q} = \mathbf{A}\mathbf{p}$$

- A is a $(n \times n)$ by (4×4) matrix
- Entries of **A** are computed using Bezier basis functior and parametric values
- Assume all parametric values are constants

$$\dot{\mathrm{q}} = \mathrm{A}\dot{\mathrm{p}}$$

$$\ddot{\mathrm{q}} = \mathrm{A}\ddot{\mathrm{p}}$$

- Dynamic equation

$$(\mathbf{A}^\top \mathbf{M}_q A)\ddot{\mathrm{p}} + (\mathbf{A}^\top \mathbf{D}_q \mathbf{A})\dot{\mathrm{p}} + (\mathbf{A}^\top \mathbf{K}_q \mathbf{A})\mathbf{p} = (\mathbf{A}^\top)\mathbf{f}_q$$

- Numerical integration (explicit method)

$$(\mathbf{A}^\top \mathbf{M}_q A)\ddot{\mathrm{p}} = (\mathbf{A}^\top)\mathbf{f}_q - (\mathbf{A}^\top \mathbf{D}_q \mathbf{A})\dot{\mathrm{p}} - (\mathbf{A}^\top \mathbf{K}_q \mathbf{A})\mathbf{p}$$

- What is the purpose?
  - integration of modeling and rendering
  - approximate higher-order finite element using a set linear finite elements
  - computationally efficient

- possible to develop more advanced tools
- hierarchical models
- non-elastic models

- **Parameters to be controlled:**
  - orders of Bezier patches $(4, 4)$
  - sampling rate (n, m)

- **Generalization**
  - B-spline surface
  - parameters
  - orders of B-spline (k, l)
  - control points (n, m)
  - sampling rate

- **B-spline surface**

$$\mathrm{s}(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{p}_{i,j} B_{i,k}(u) B_{j,l}(v)$$

- **NURBS**

- **Subdivision surfaces**