Parametric Model

- Consider the active contour or "snake" as a simple example $\mathbf{c}(u) = (x(u), y(u))$
- Functional analysis (calculus of variations)

$$E(\mathbf{c}) = (1/2)\int w_1(\mathbf{c})(\frac{\partial \mathbf{c}}{\partial u})^2 + w_2(\mathbf{c})(\frac{\partial^2 \mathbf{c}}{\partial u^2})^2 du$$

- Scalar potential function defined on the image plane

$$P(x, y) = -c\left|\nabla(G_\sigma * I(x, y))\right|$$

Parametric Model

- Lagrangian mechanics (partial differential equations)

$$\mu \frac{\partial^2 \mathbf{c}}{\partial t^2} + \gamma \frac{\partial \mathbf{c}}{\partial t} - \frac{\partial}{\partial u}\left(w_1 \frac{\partial \mathbf{c}}{\partial u}\right) + \frac{\partial^2}{\partial u^2}\left(w_2 \frac{\partial^2 \mathbf{c}}{\partial u^2}\right) = -\nabla P(\mathbf{c}(u,t))$$

Level-set Method

- Scalar field function: $\phi(\mathbf{x}, t)$
- Interface front:

$$\mathbf{s}(t) = \{(x, y, z) \mid \phi(x, y, z, t) = c\}$$

$$\phi(\mathbf{s}(t), t) = c$$

- Velocity field (PDEs):

$$\frac{\partial \phi}{\partial t} + \frac{\partial \mathbf{s}}{\partial t} \bullet \nabla \phi = 0$$

$$\frac{\partial \phi}{\partial t} + S_n \|\nabla \phi\| = 0$$

Motivation of Our Research

- Objectives: (1) Integrate subdivision geometry and level-set methods with deformable model paradigm; (2) Bridge the gap between parametric models and level-set models
- A novel PDE-driven, flow-based subdivision model
  - Overcome the topology limitation
  - Maintain the explicit representation
  - Extract (unknown) geometry and topology simultaneously
  - Multi-resolution representation and hierarchical structure
  - The level-set based PDEs give rise to model deformation and topological flexibility

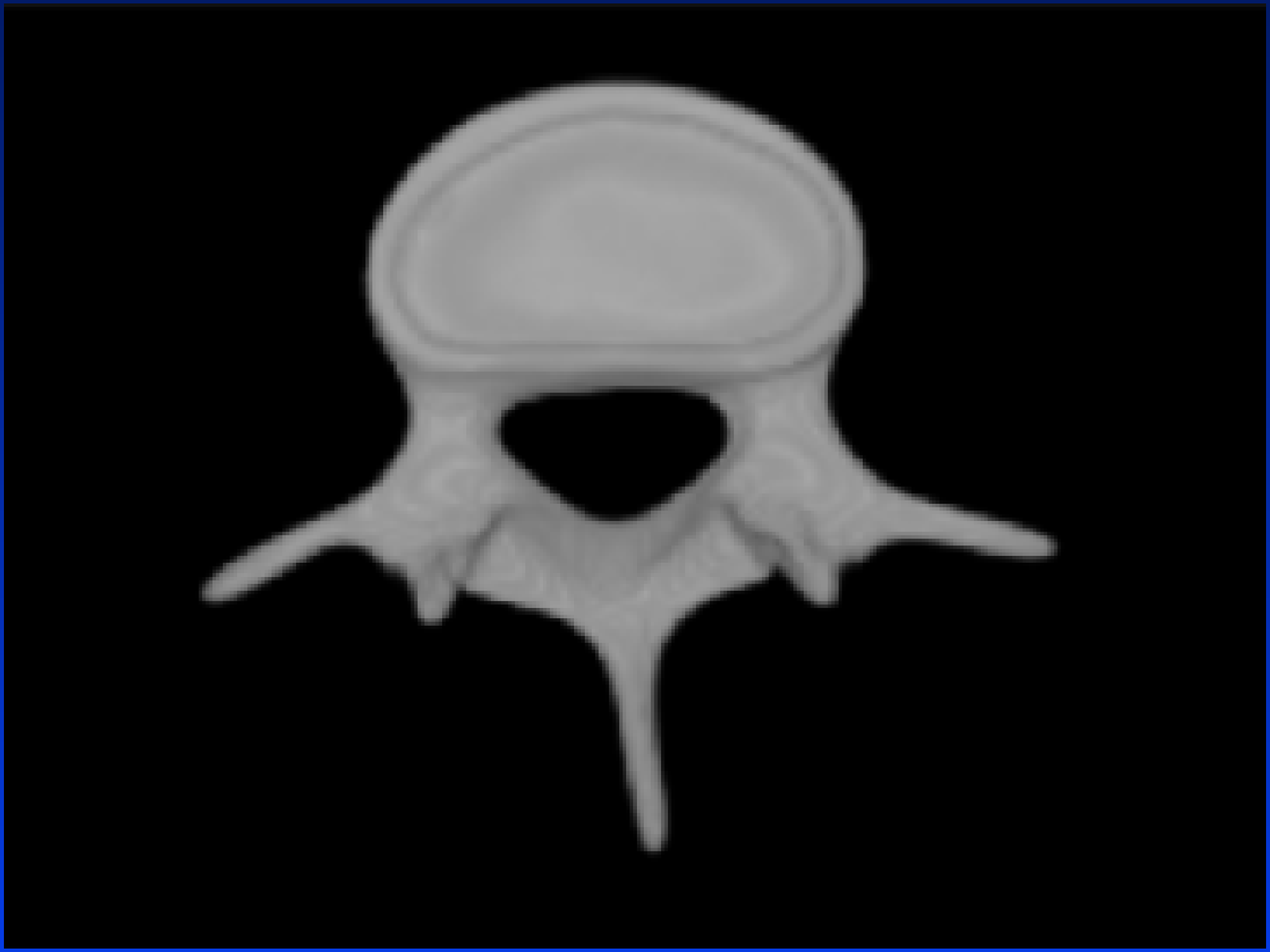
Talk Outline

- Overview of deformable models
- Motivation of our research
- Our research & main contributions
  - Intelligent Balloon
  - Intelligent Flow
  - Tangential Flow
- Conclusion & future research directions

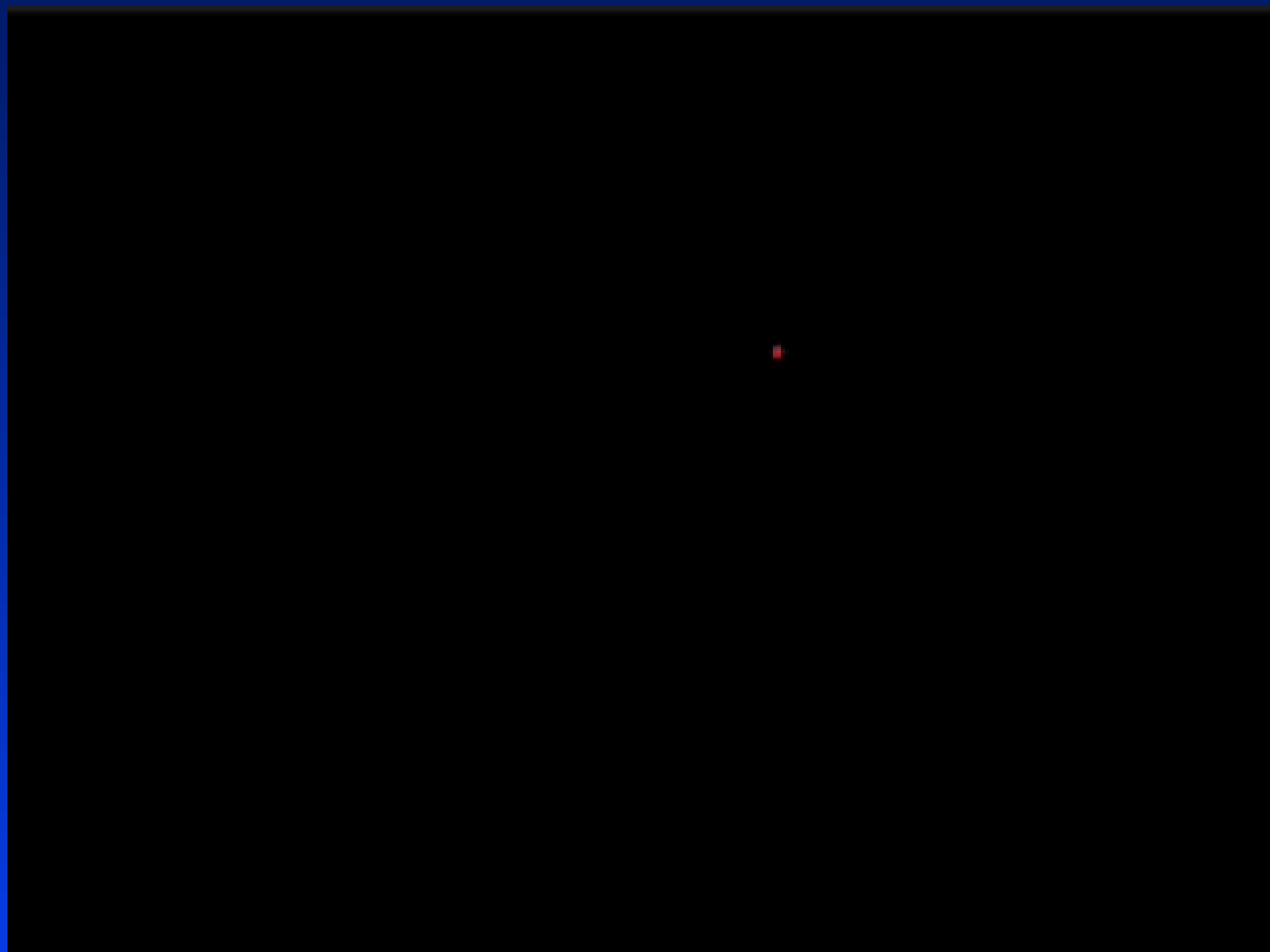
Intelligent Balloon

- A subdivision-based deformable surface whose dynamic behavior is governed by the principle of energy minimization

Vertebral Dataset



Vertebral Model



Model Refinement

Algorithmic Structure

1. Model initialization.
2. Stressed edge resolution.
3. Model growing.
4. Local subdivision.
5. Mesh optimization.
6. Collision detection and topology changes.
7. Global subdivision.

Algorithmic Structure

1. Model initialization.
2. Stressed edge resolution.
3. Model growing.
4. Local subdivision.
5. Mesh optimization.
6. Collision detection and topology changes.
7. Global subdivision.

Cost Function

$$C_i(x,y,z) = a_0 D(x,y,z) + a_1 B(x,y,z) + a_2 V(x,y,z) + a_3 A(x,y,z)$$

$C_i(x,y,z)$ is the cost function.

$D(x,y,z)$ is the deformation potential.

$B(x,y,z)$ is the boundary constraint.

$V(x,y,z)$ is the curvature constraint.

$A(x,y,z)$ is the angular constraint.

$a_{0,} a_{1,} a_{2,} a_3$ are the weighting coefficients.

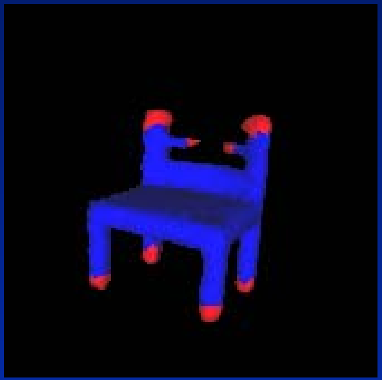
Intelligent Balloon

- The deformation behavior of the model is governed by the principle of energy minimization
- A locally defined cost function is associated with each vertex of the model, and it is a weighted linear combination of four constraints
- Through the minimization process of the cost function, the model will inflate like a balloon until it reaches the boundary of the dataset

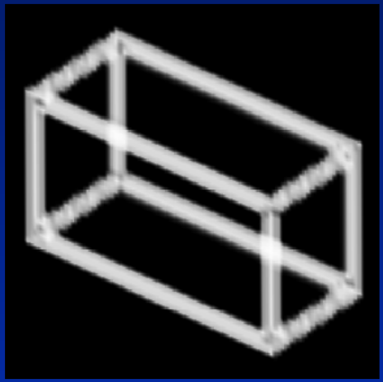
Gradient-Descent Energy Minimization

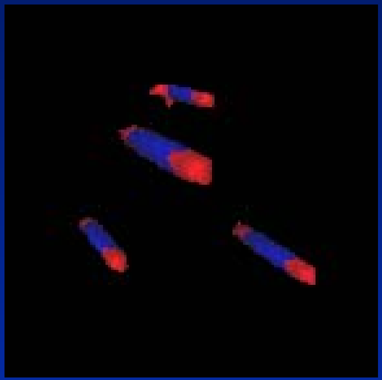
- A vertex of the model will move along the direction opposite to the gradient of the cost function $C_i(x, y, z)$
- The gradient $(\frac{\partial C_i}{\partial x}, \frac{\partial C_i}{\partial y}, \frac{\partial C_i}{\partial z})$ is numerically approximated using central differences

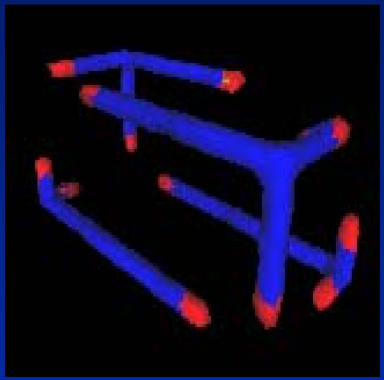
Chair

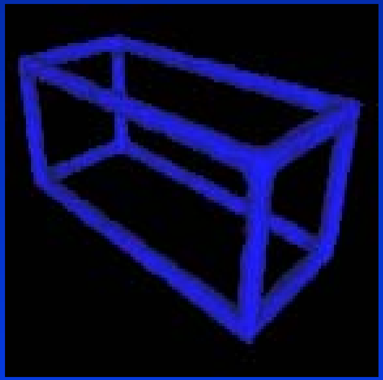


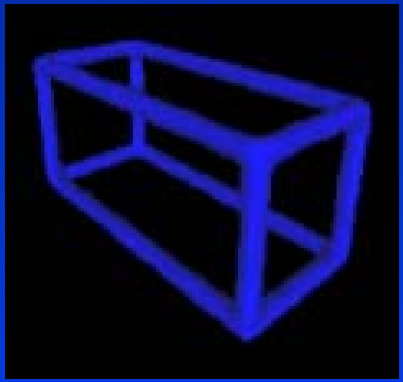
Multiple Seeds



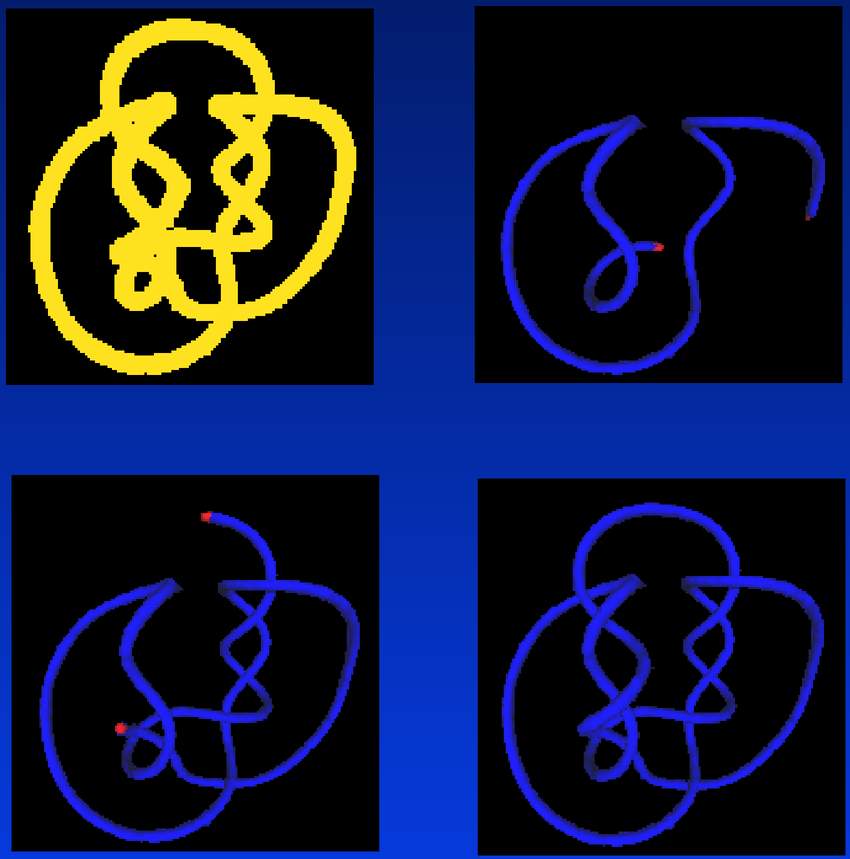








Range Data



Intelligent Balloon

- Recovers shape of arbitrary topology
- Handles both volumetric and range data
- Supports different levels of detail
- Enables parallel computation

Intelligent Balloon

Summary

- Overcomes the topology limitation
- Handles both volumetric data and range data
- Supports different levels of detail
- Enables parallel computation

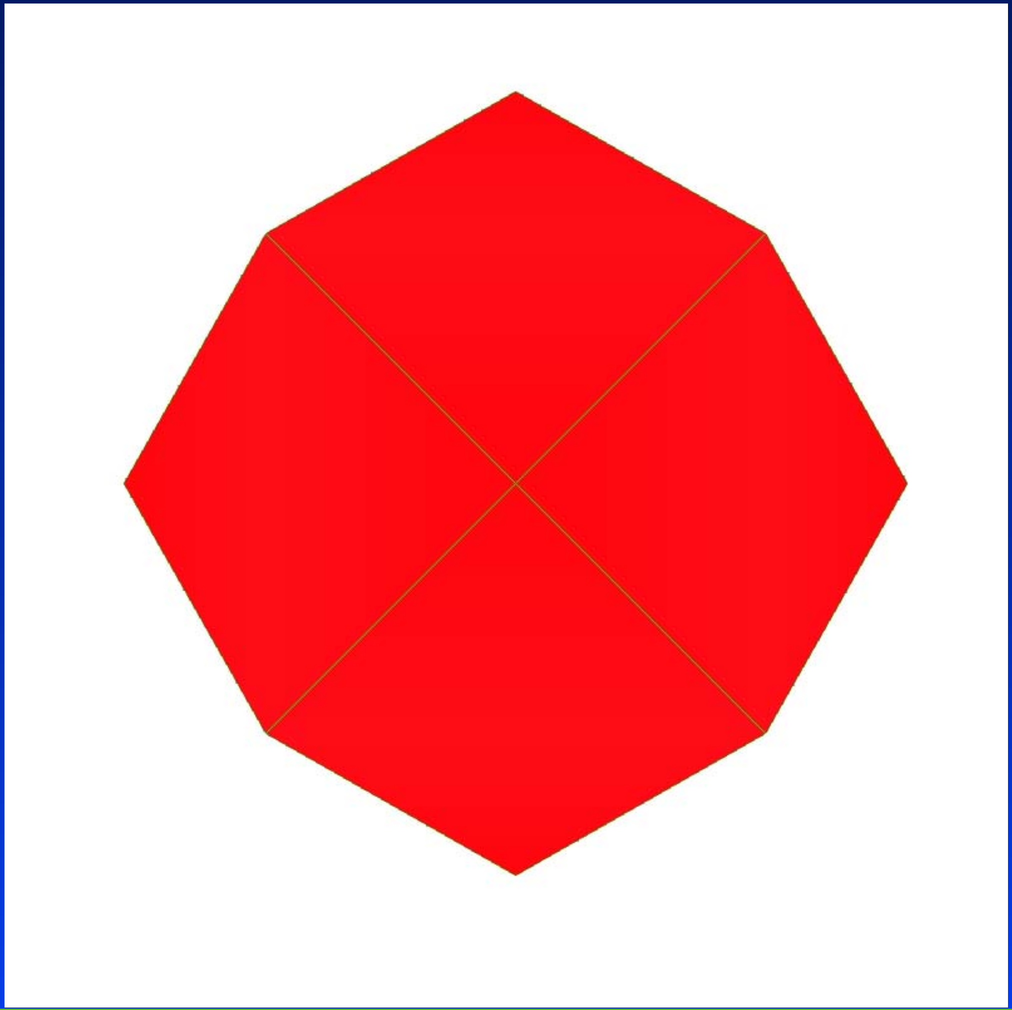
Further enhancements

- Automatic initialization
- Sharp features and fine details

Intelligent Flow

- A PDE-based deformable surface whose dynamic behavior is governed by the principle of variational analysis and/or partial differential equations

Iso-surface Extraction



Intelligent Flow

- More general deformation behaviors
- Growing from inside or shrinking from outside
- Automatic initialization
- Model can merge or split on the fly
- Can recover sharp features
- Can recover fine details

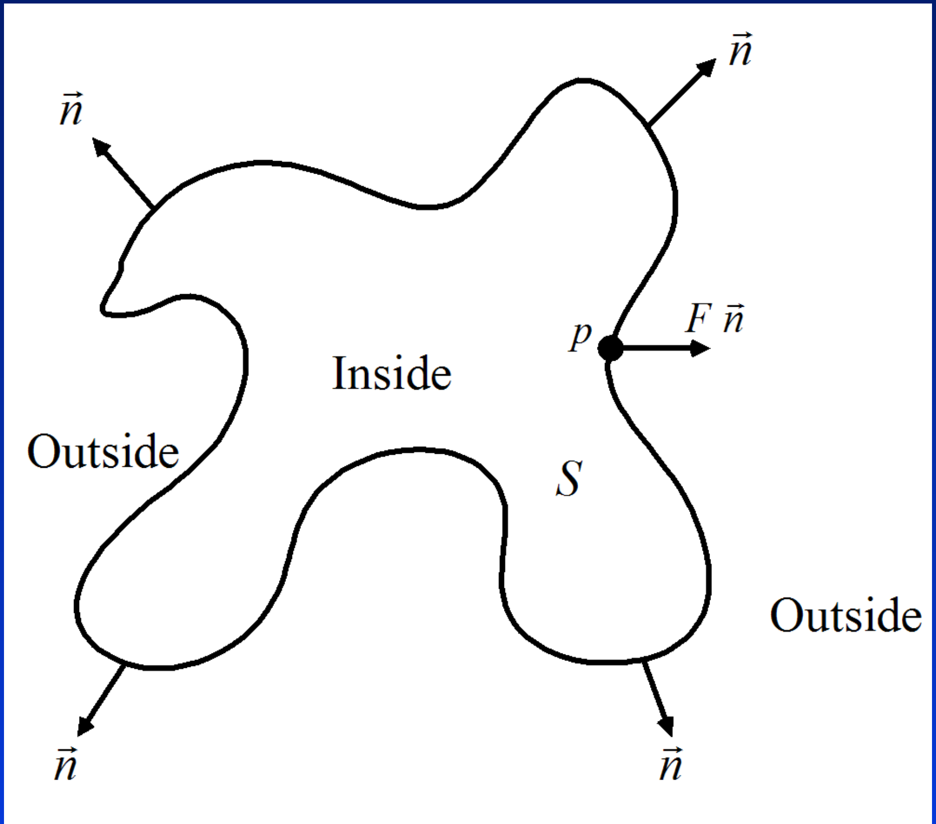
Intelligent Flow

- The deformation behavior is governed by an evolutionary system of nonlinear initial-value PDEs:

$$\frac{\partial \mathbf{s}(\mathbf{p})}{\partial t} = F(\mathbf{p}, t, k, k', \ldots) * \mathbf{n}(\mathbf{p}, t),$$
$$\mathbf{s}(\mathbf{p}, 0) = \mathbf{s}_0(\mathbf{p}).$$

p:	*Surface position*
k:	*Surface curvature*
n:	*Surface normal vector*
F:	*Application-dependent speed function*

Intelligent Flow



Weighted Minimal Surface Flow

- Many visual computing problems can be formulated as finding surfaces of minimal energy $E(S)$:

$$E(\mathbf{s}) := \iint g(\mathbf{s})\,\mathrm{d}a,$$

- The gradient descent flow is:

$$\frac{\partial \mathbf{s}}{\partial t} = g(H)\mathbf{n} - (\nabla g \cdot \mathbf{n})\mathbf{n}$$
$$\mathbf{s}(0) = \mathbf{s}_0$$

H : *Mean curvature*
n : *Surface normal*
g(S): *Data-dependent weighting function*

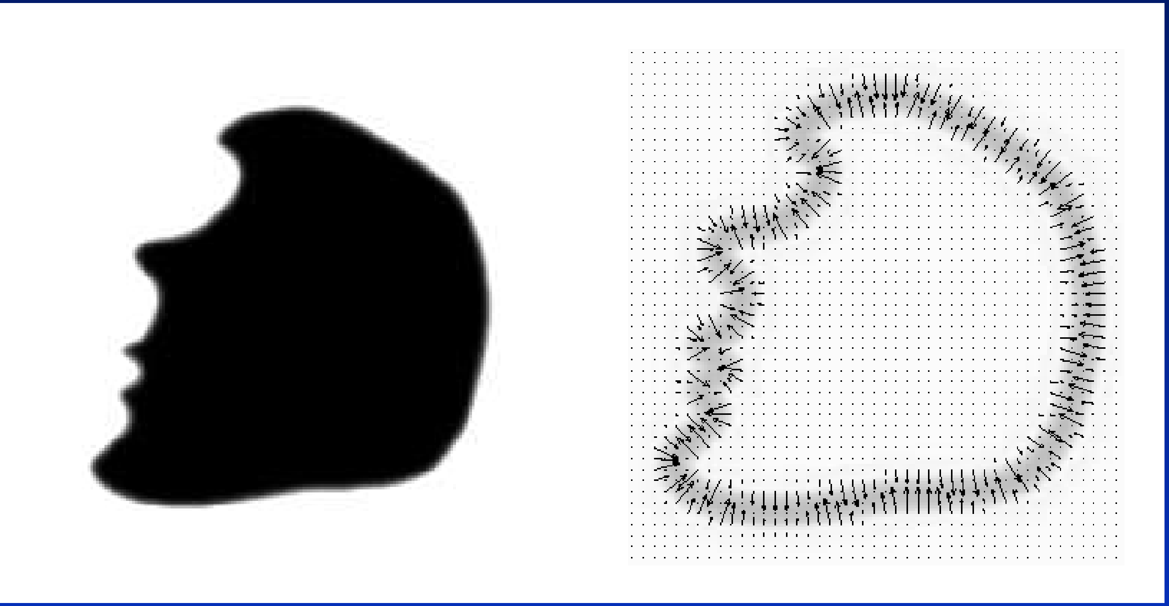
Weighted Minimal Surface Flow

- $g(S)$ is an non-negative, non-increasing, monotone function
- For surface reconstruction from 3D volumetric image datasets, $g(S)$ is defined as:

$$g(S) = \frac{1}{1 + |\nabla(G_\sigma * I(S))|^2}$$

- For surface reconstruction from 2D images, $g(S)$ is defined as the photo consistency metric between images

Weighted Minimal Surface Flow



(a): $I(x)$

(b): $g(x) = \dfrac{1}{1 + |\nabla(G_\sigma * I(x))|^2}$

2D example

Weighted Minimal Surface Flow

- In practice, in order to accelerate the deformation, a constant velocity term *v* is added to the previous equation

$$\frac{\partial \mathbf{s}}{\partial t} = g(v + H)\mathbf{n} - (\nabla g \cdot \mathbf{n})\mathbf{n}$$

Intelligent Flow

Applications:

- Iso-surface extraction for interactive visualization
- Multiple-view images surface reconstruction
- Interactive shape design
- Medical image segmentation
- Shape tracking

…

Intelligent Flow

Applications:

- Iso-surface extraction for interactive visualization
- Multiple-view images surface reconstruction
- Interactive shape design
- Medical image segmentation
- Shape tracking

…

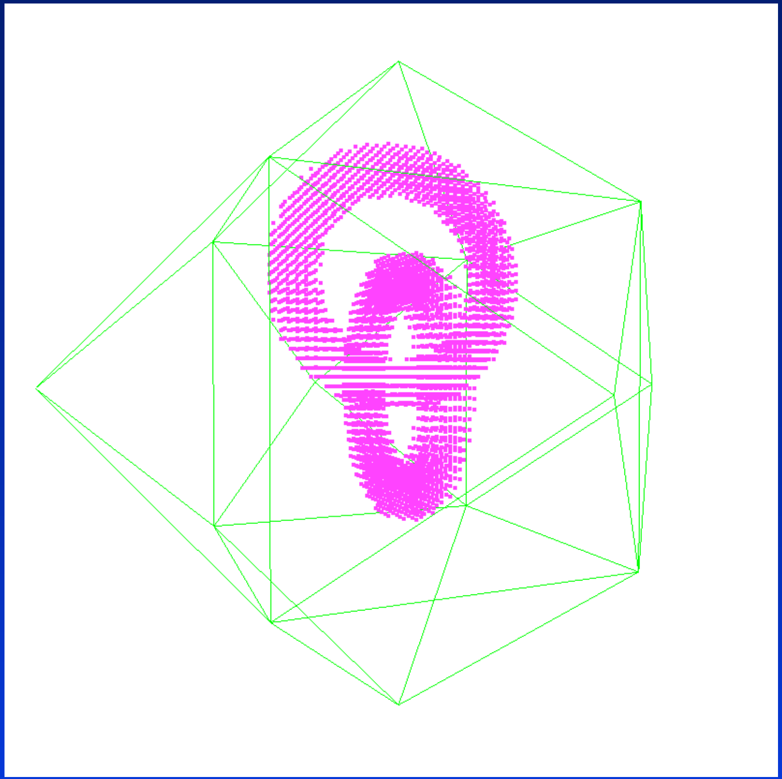
Intelligent Flow

Applications:

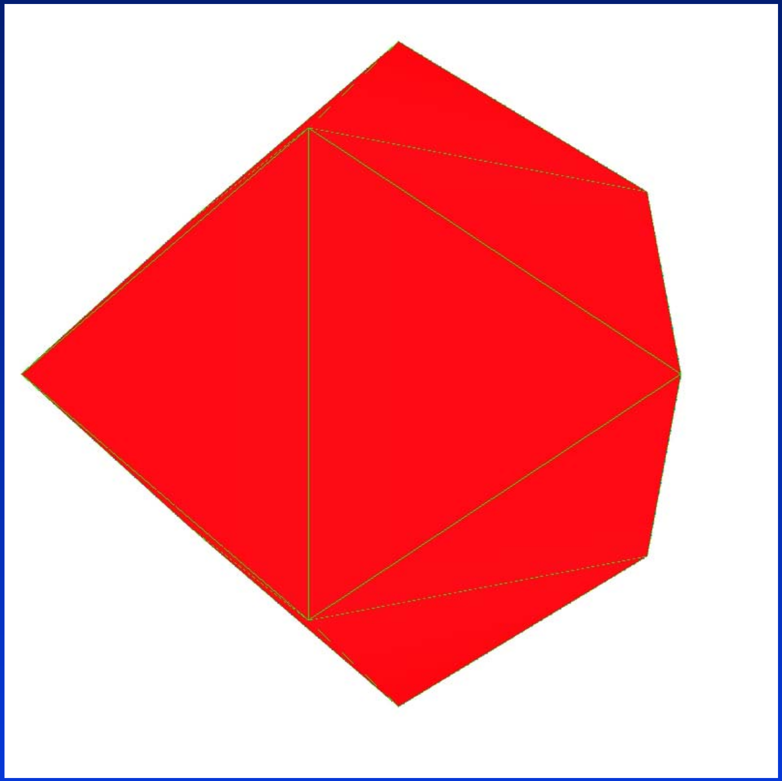
- Iso-surface extraction for interactive visualization
- Multiple-view images surface reconstruction
- Interactive shape design
- Medical image segmentation
- Shape tracking

…

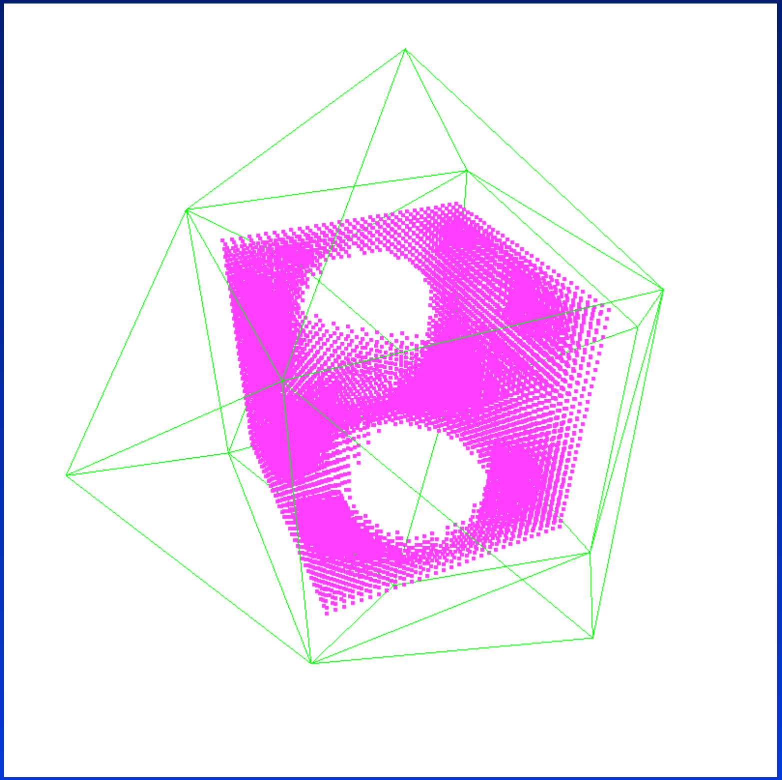
Iso-surface Extraction



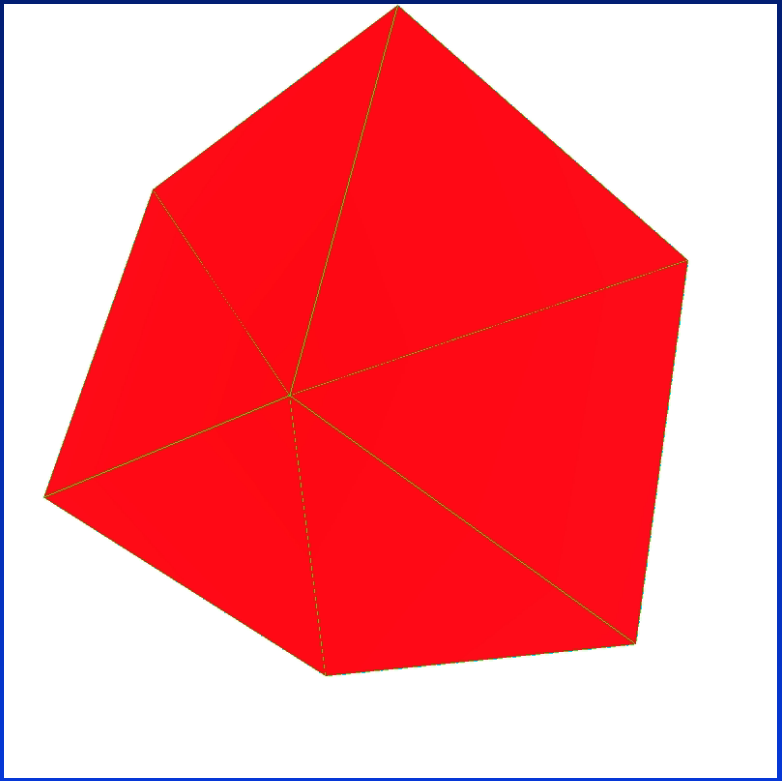
Iso-surface Extraction



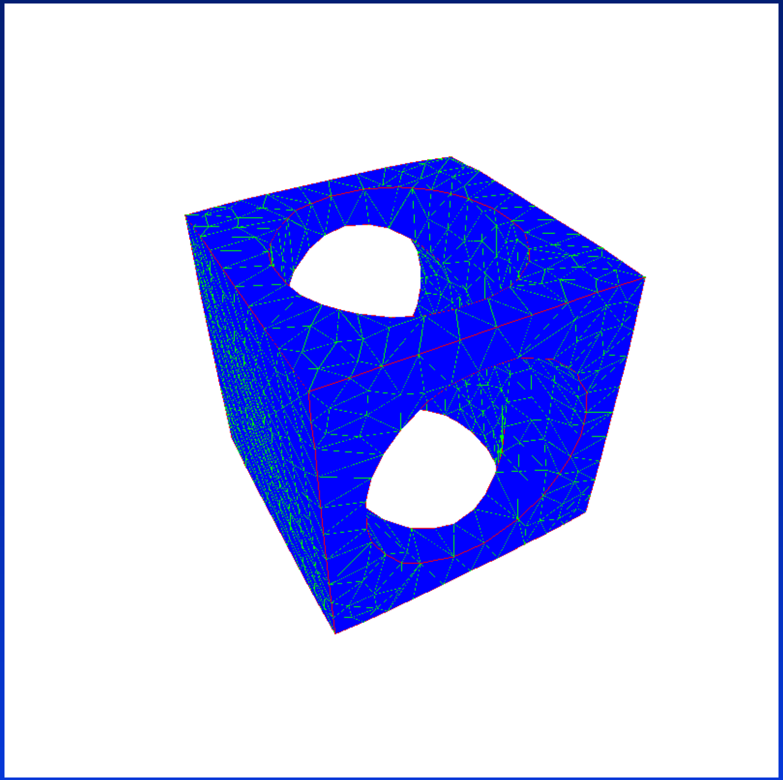
Sharp Features Reconstruction



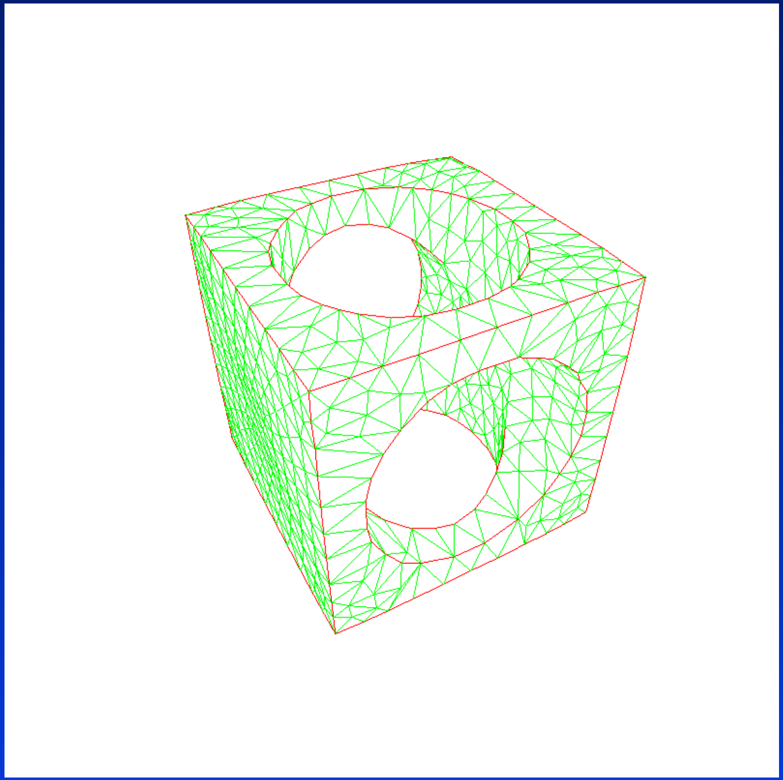
Sharp Features Reconstruction



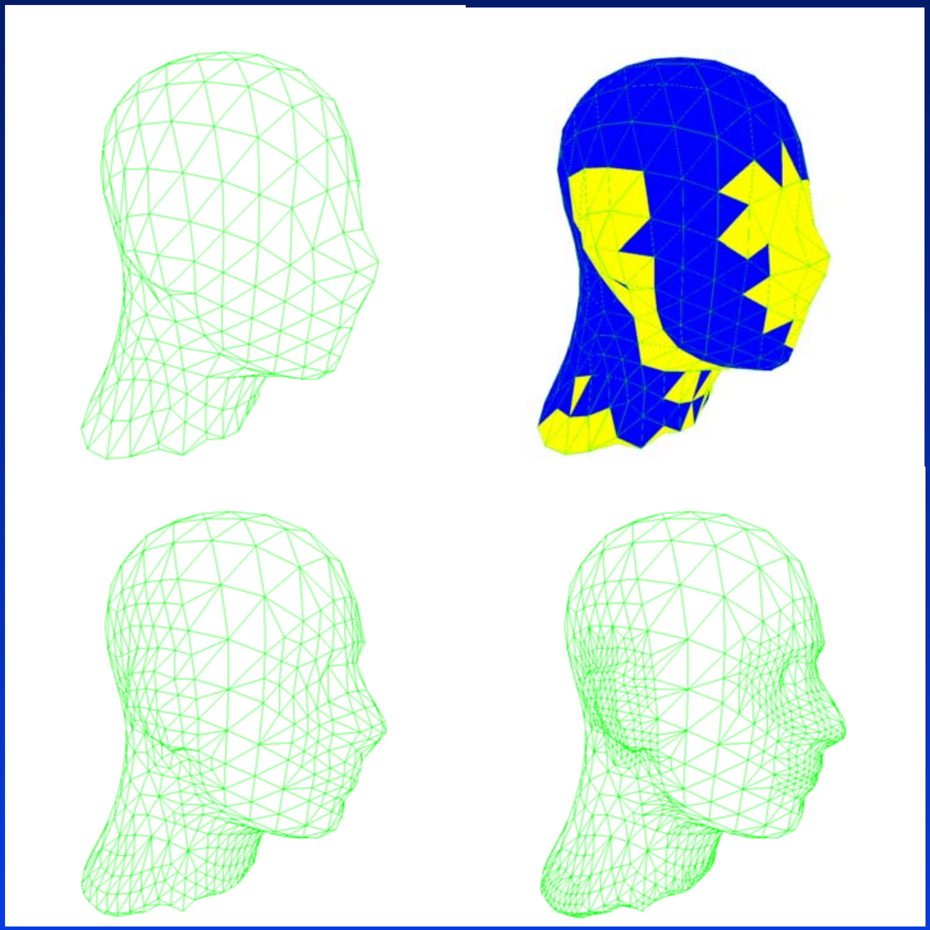
Sharp Features Reconstruction



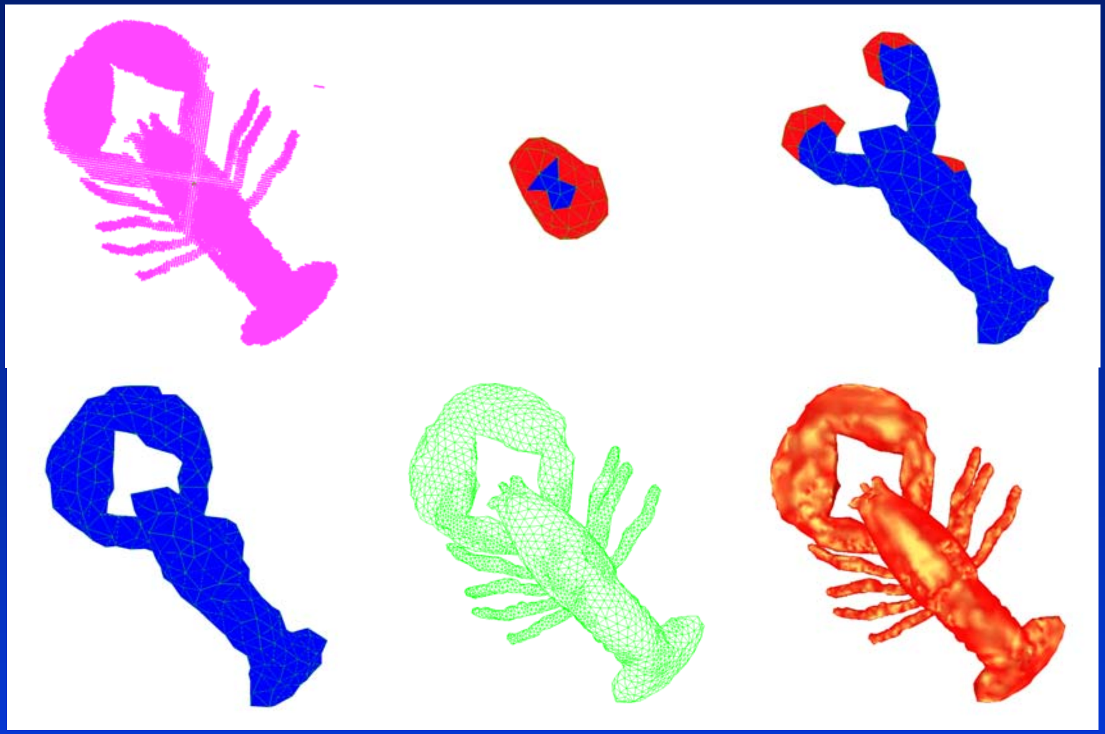
Sharp Features Reconstruction



Adaptive Refinement



Adaptive Refinement



Intelligent Flow

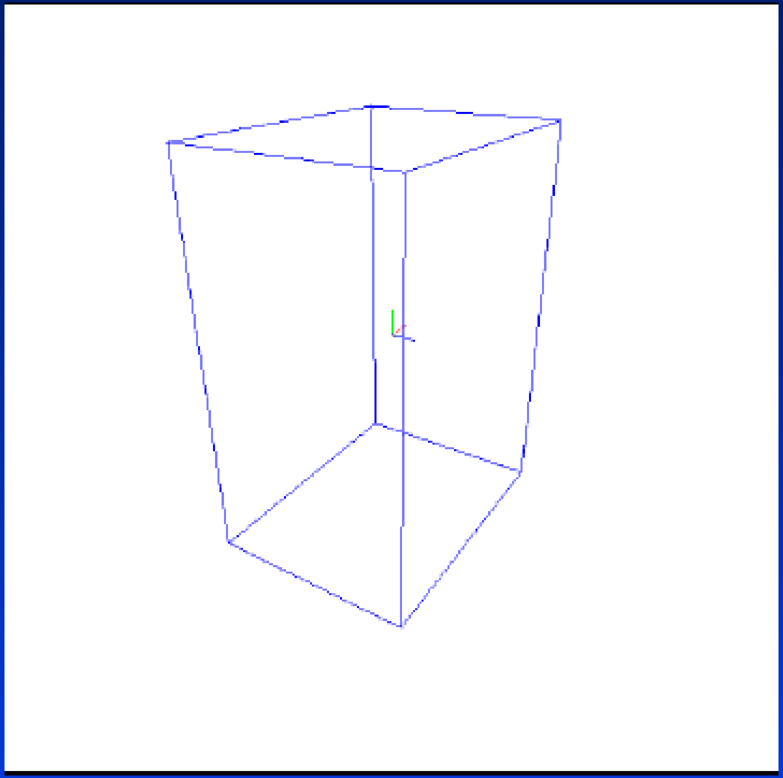
Applications:

- Iso-surface extraction for interactive visualization
- Multiple-view images surface reconstruction
- Interactive shape design
- Medical image segmentation
- Shape tracking

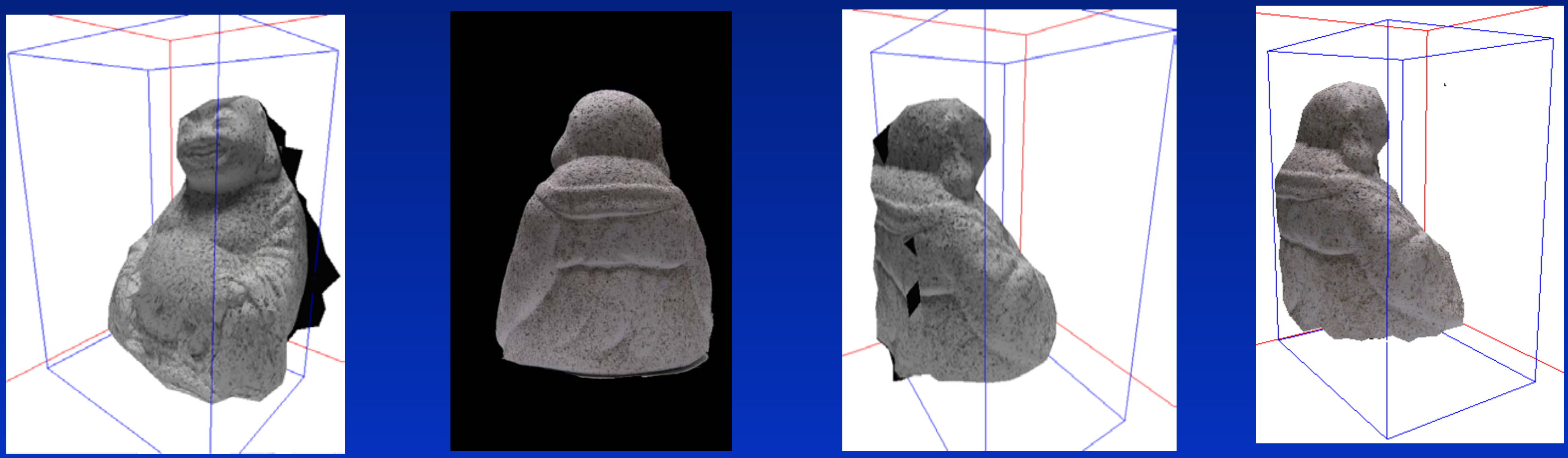
…

Multiple-View Images Surface Reconstruction

Multiple-View Images Surface Reconstruction



Multiple-View Images Surface Reconstruction



Progressive reconstruction

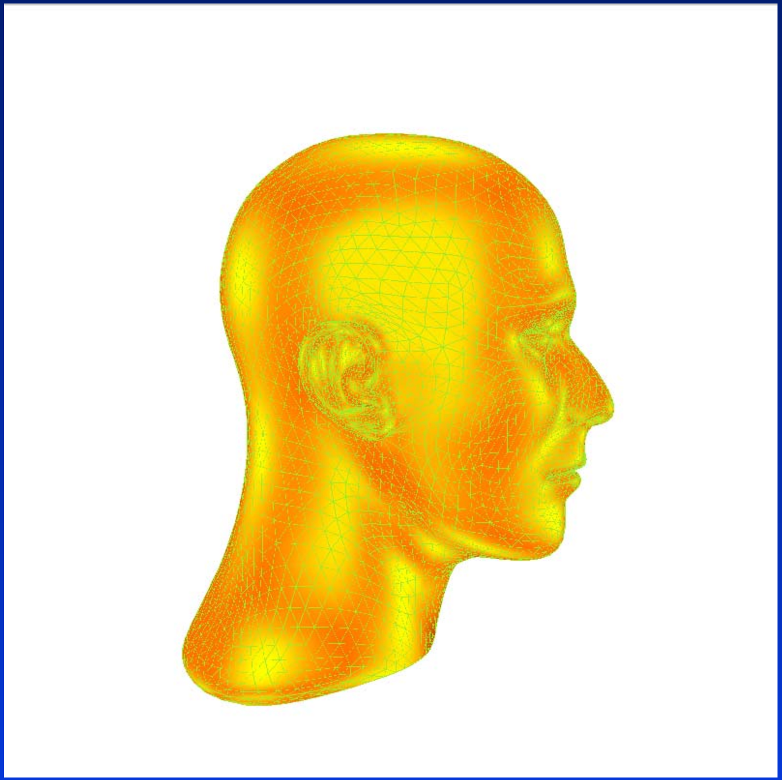
Intelligent Flow

Applications:

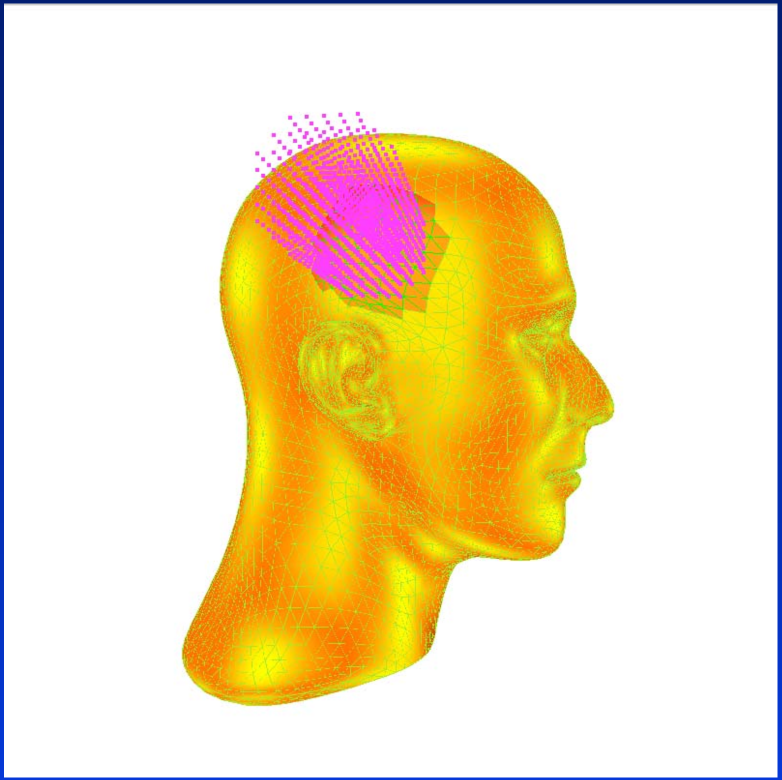
- Iso-surface extraction for interactive visualization
- Multiple-view images surface reconstruction
- Interactive shape design
- Medical image segmentation
- Shape tracking

…

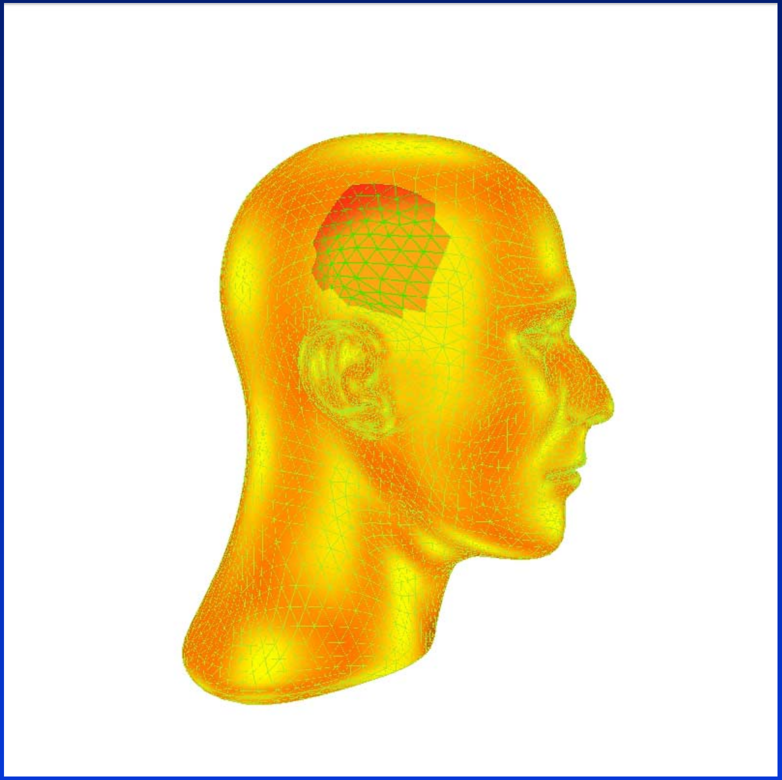
Interactive Shape Design



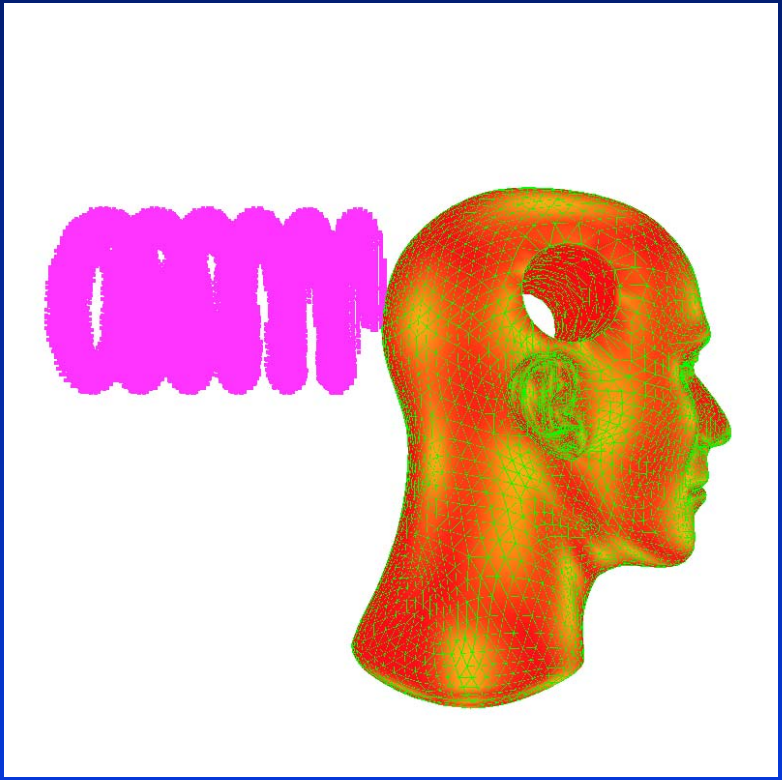
Interactive Shape Design



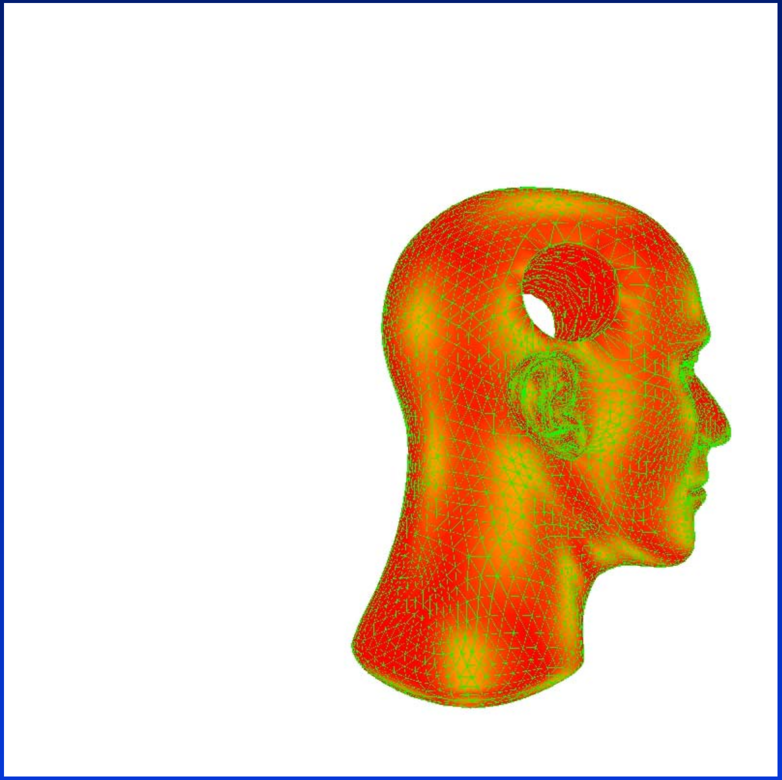
Interactive Shape Design



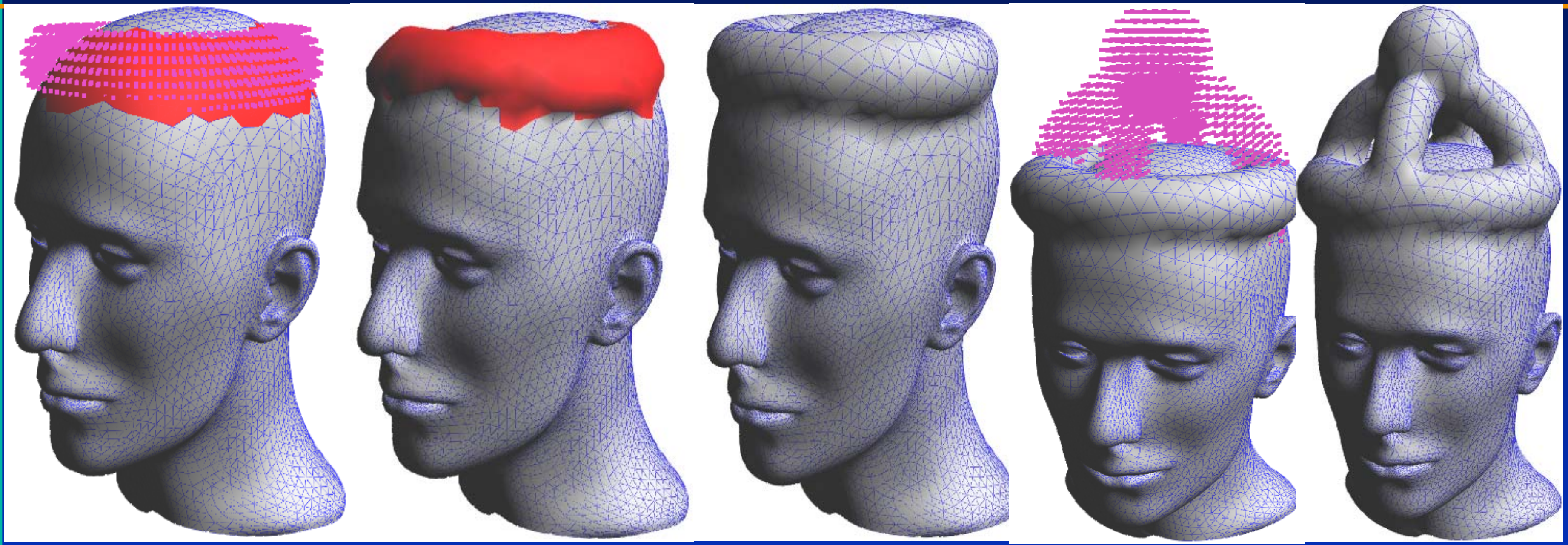
Interactive Shape Design



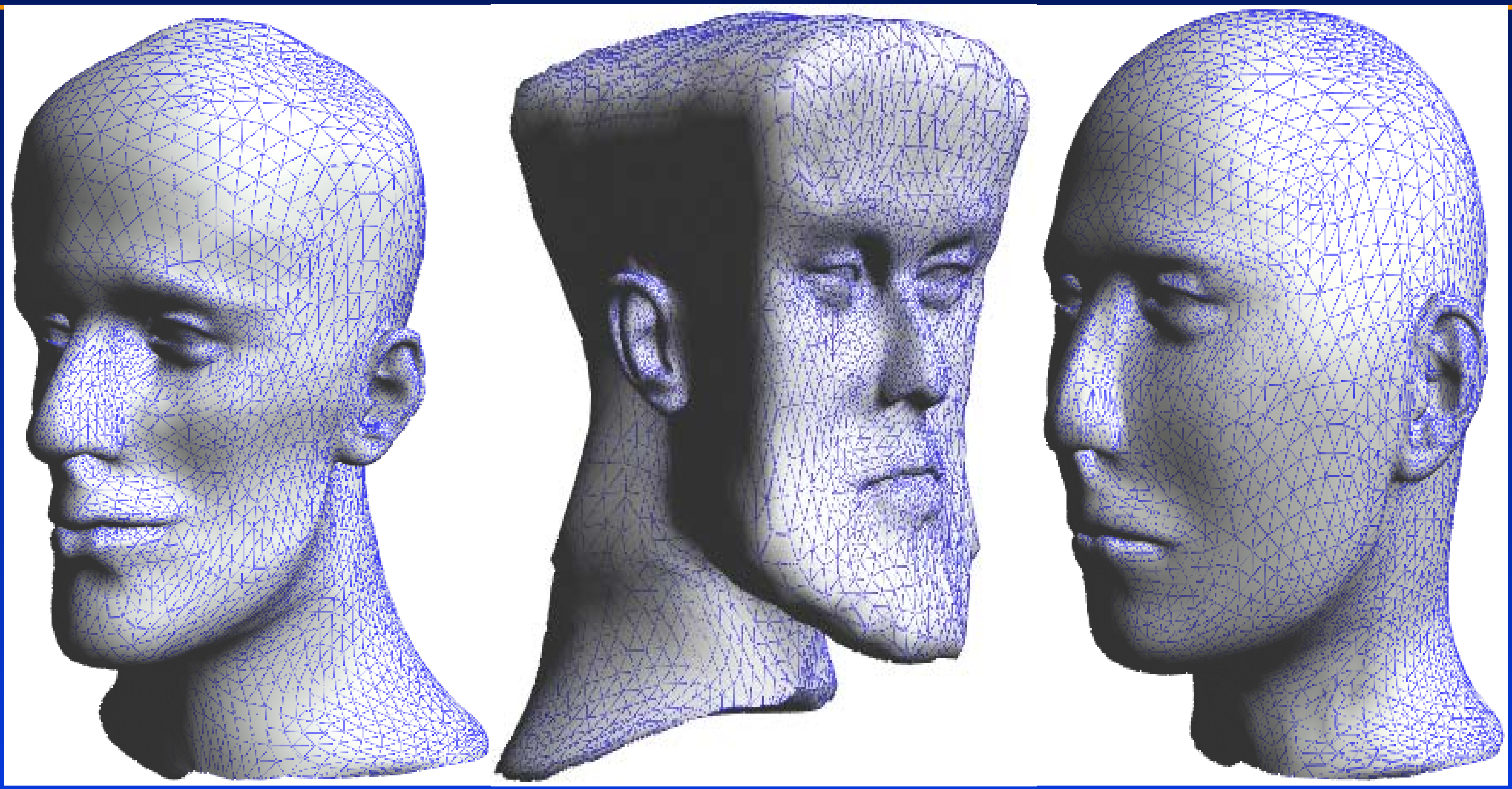
Interactive Shape Design



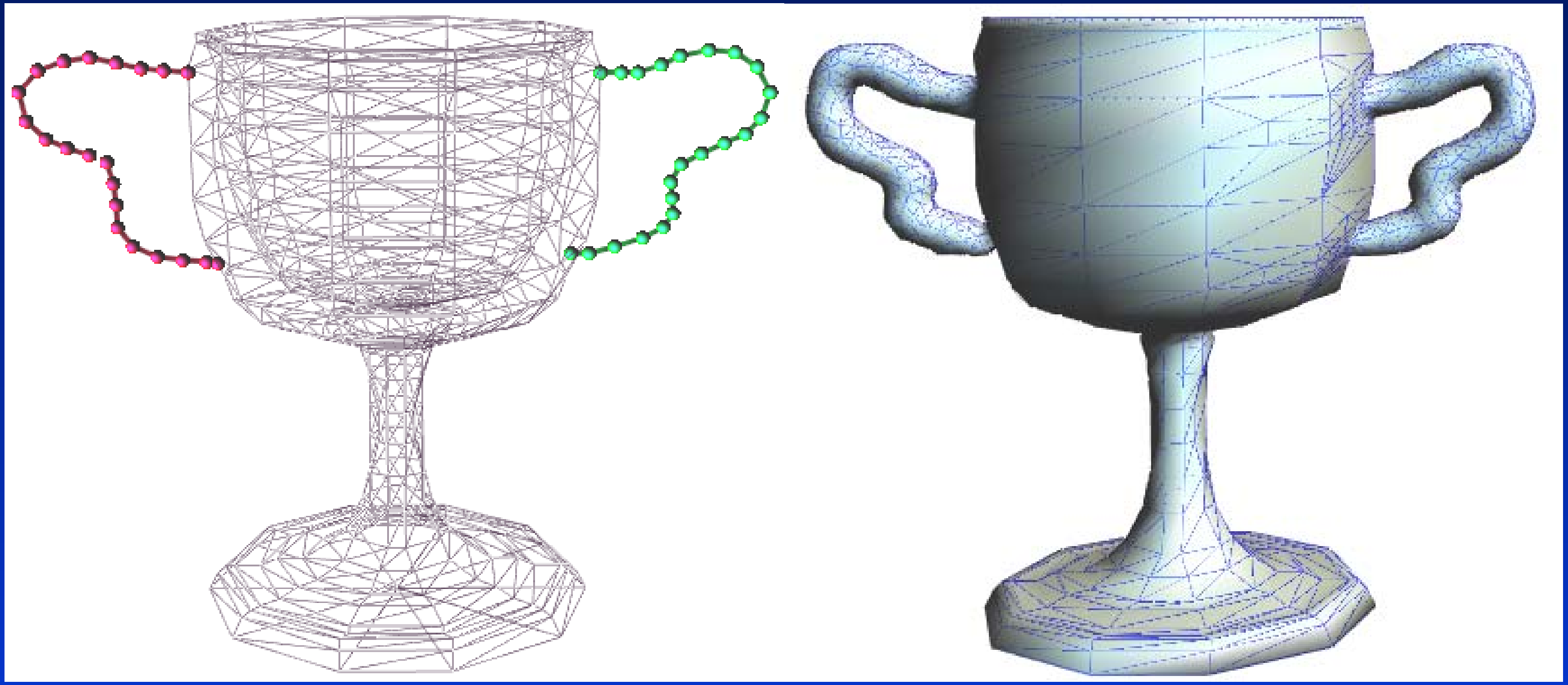
Interactive Design



Scalar-Field Free-form Deformation



Shape from Rough Sketch



Intelligent Flow

Summary

- Automatic initialization
- Multiple objects per seed
- Recover sharp features and fine details
- More general deformation behaviors
- Broader range of applications

How about open surfaces, non-manifolds?

Tangential Flow --- Basic Idea

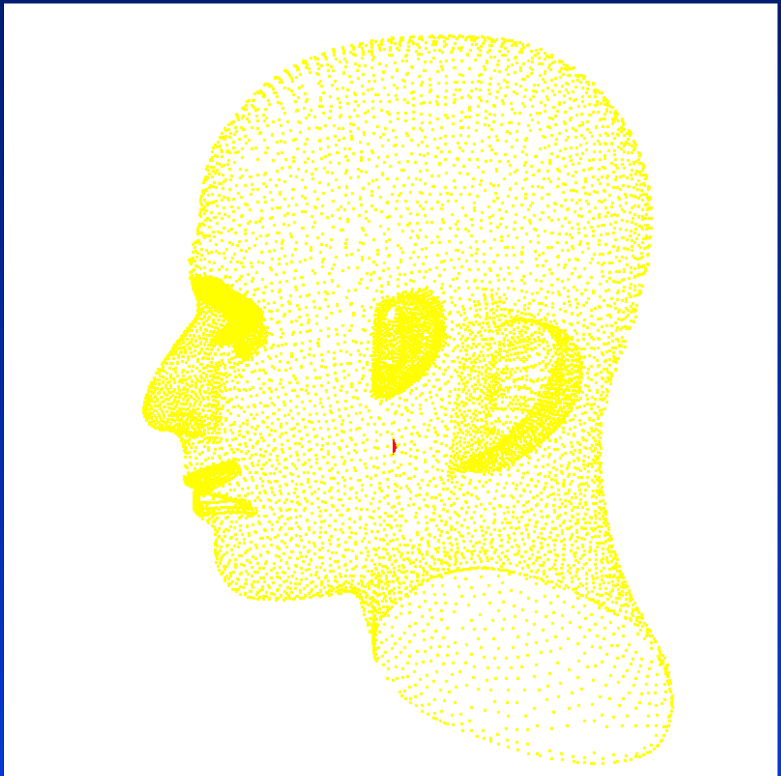
- Flow directly on the object boundary through the expansion of its active boundary contour
- Can represent shapes of disparate types (open, closed, and non-manifolds)

Tangential Flow - Model Growing

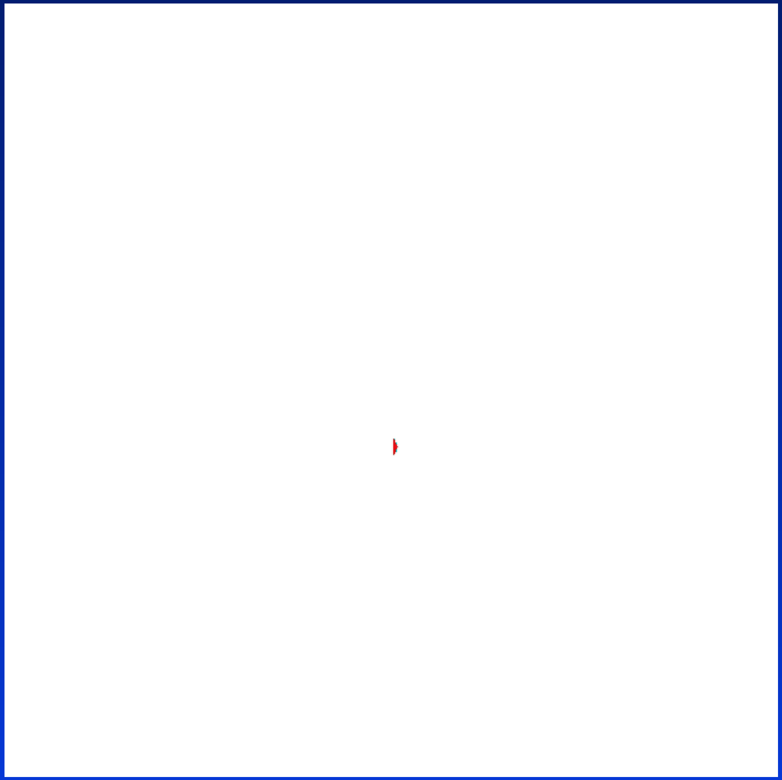
- Only the boundary contour of the model is active
- The deformation of the boundary contour is governed by:

$$\frac{\partial \mathbf{c}(\mathbf{p},t)}{\partial t} = F(\mathbf{p},t)\mathbf{n}(\mathbf{p},t),$$
$$F(\mathbf{p},t) = (v+k)g(\mathbf{p}),$$
$$\mathbf{c}(\mathbf{p},0) = \mathbf{c}_0(\mathbf{p}).$$

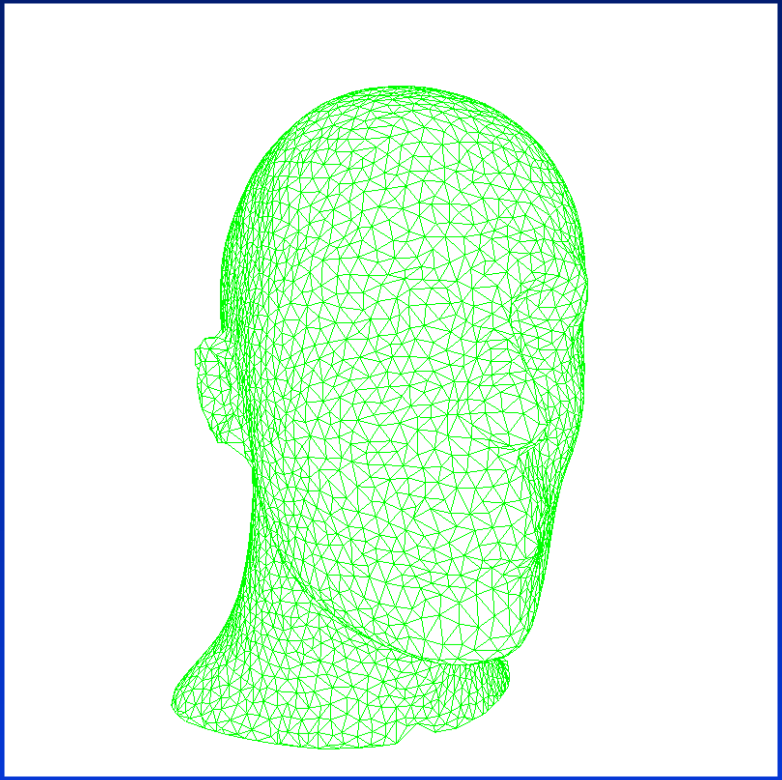
Mannequin



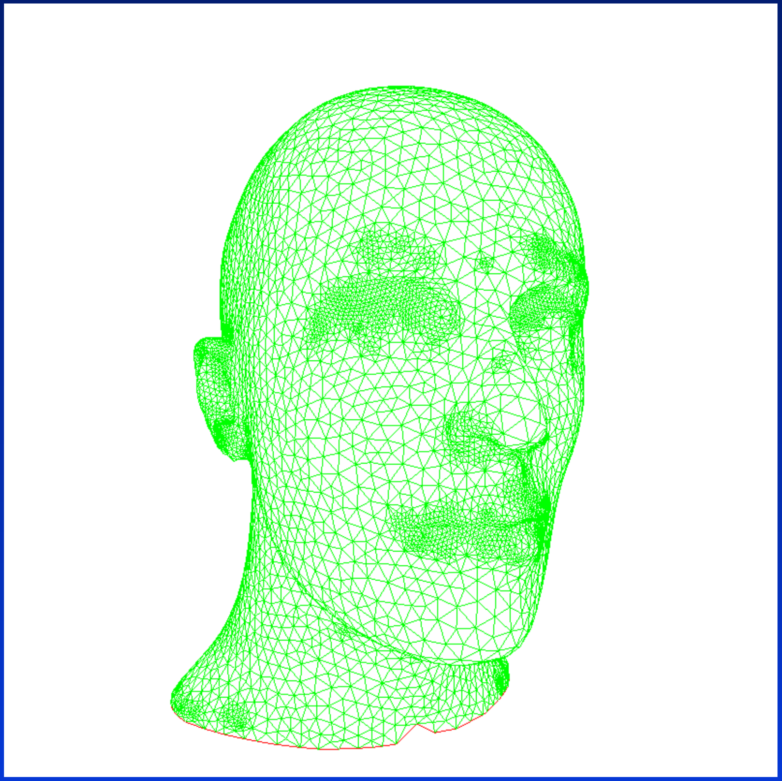
Mannequin



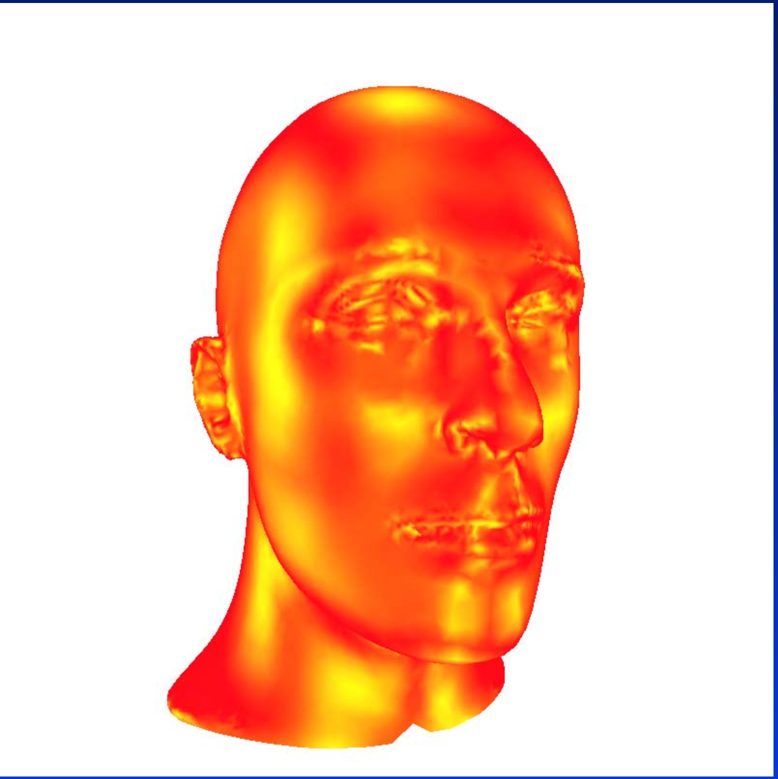
Mannequin— Initial Shape



Mannequin— Refined Shape



Mannequin— Refined Shape



Algorithm

Six main steps:

1. Model initialization.
2. Model growing.
3. Model relaxation.
4. Collision detection.
5. Mesh stitching.
6. Model refinement.

Algorithm

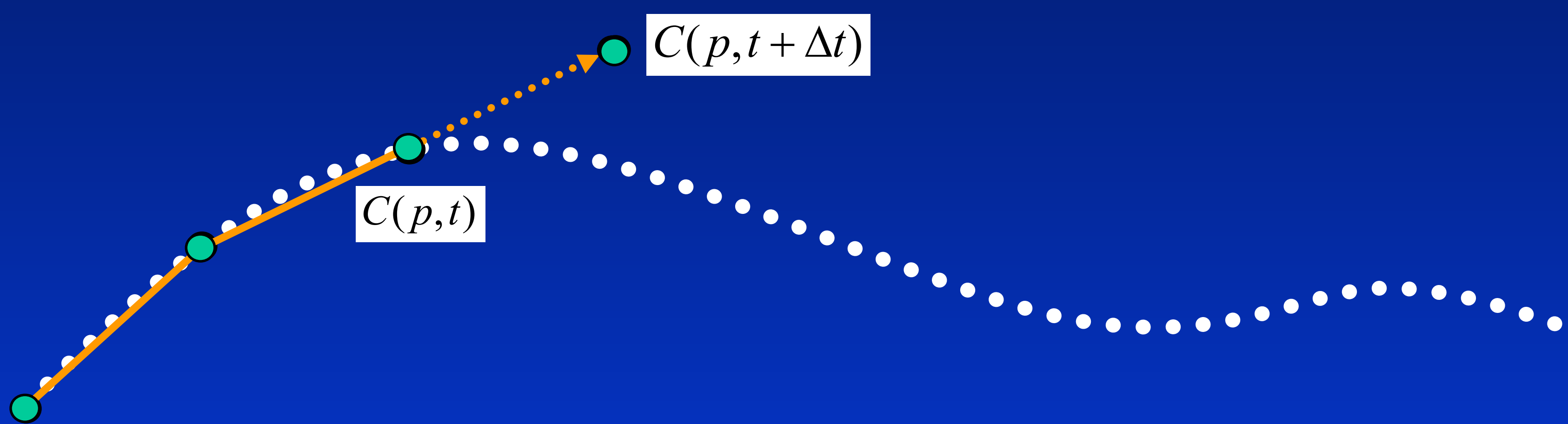
Six main steps:

1. Model initialization.
2. Model growing.
3. Model relaxation.
4. Collision detection.
5. Mesh stitching.
6. Model refinement.

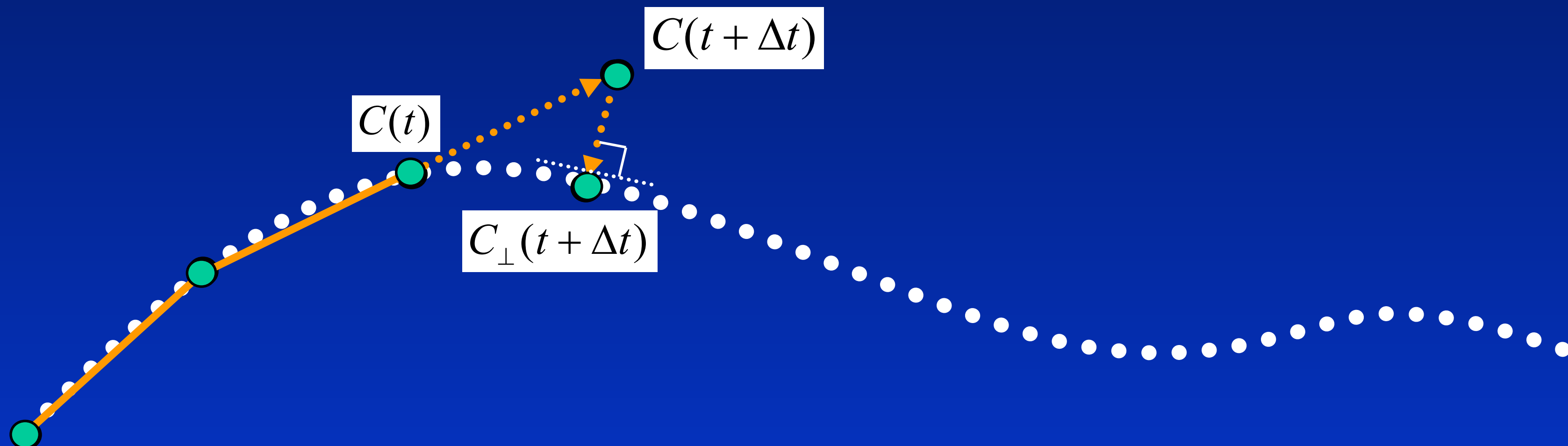
Model Initialization

- A seed model (e.g., a triangle) is automatically placed on the boundary of the object
- Multiple seeds can be initialized either simultaneously (for parallel processing) or iteratively (for multiple objects)

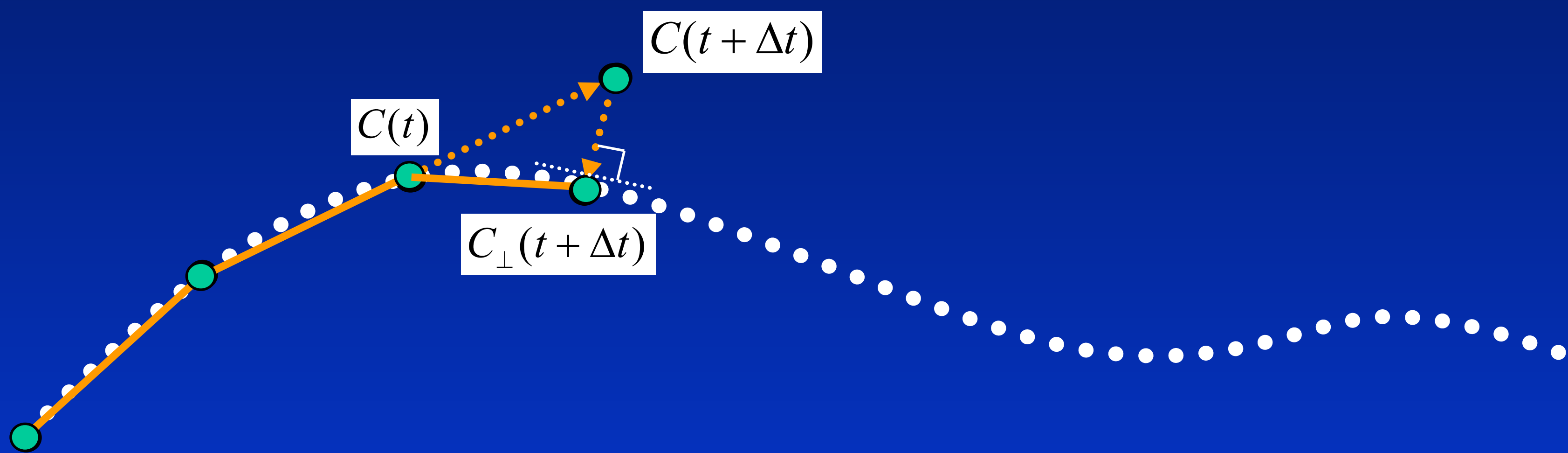
Model Growing



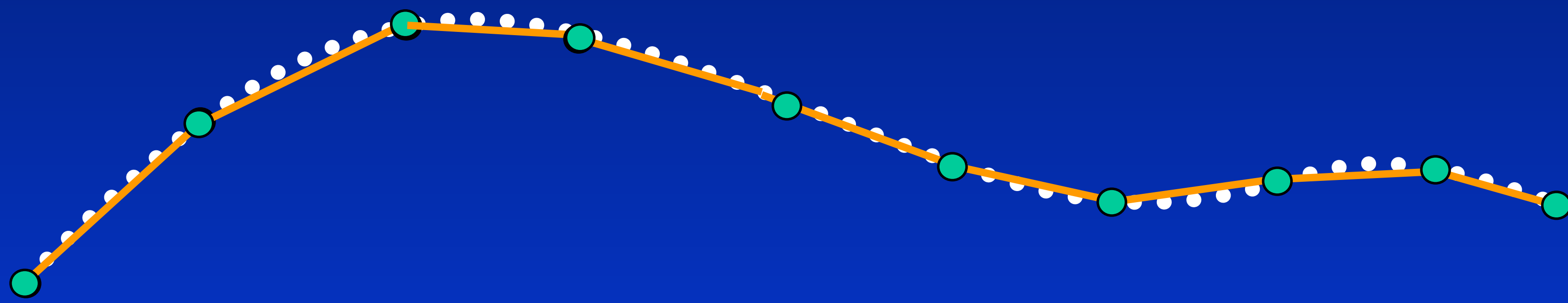
Model Growing



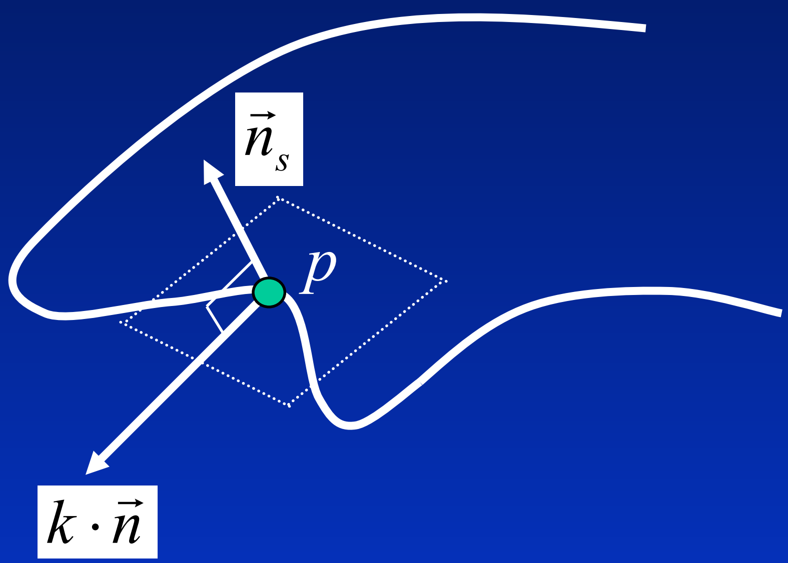
Model Growing



Model Growing



Model Growing



Local Tangent Plane Approximation

- Estimated by principle component analysis (PCA) of the *k*-nearest neighbors

- The normal vector n is the eigenvector associated with the smallest eigenvalue of the covariance matrix C:

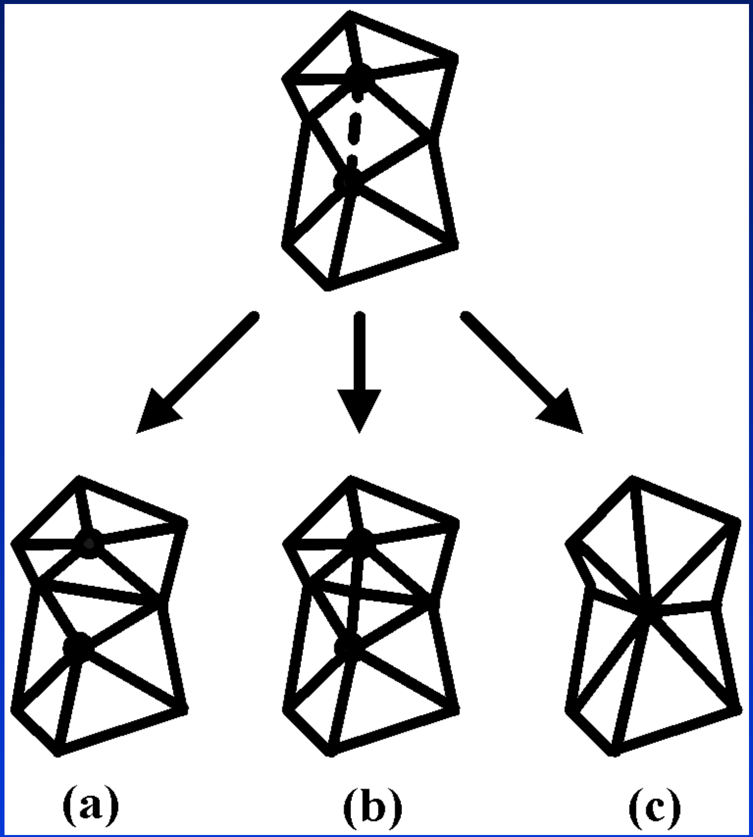
$$C = \sum_{p_i \in Nbhd(p)} (p_i - c) \otimes (p_i - c)$$

Algorithm

Six main steps:

1. Model initialization.
2. Model growing.
3. Model relaxation.
4. Collision detection.
5. Mesh stitching.
6. Model refinement.

Model Relaxation



edge swap edge split edge collapse

Collision Detection

- Distance based algorithm
- Three kinds of collisions:
  - Vertex-to-vertex collision
  - Vertex-to-edge collision
  - Vertex-to-face collision

Mesh Stitching

Three kinds of stitching:

- On-the-fly stitching
- Post stitching
- Non-manifold stitching

Model Refinement

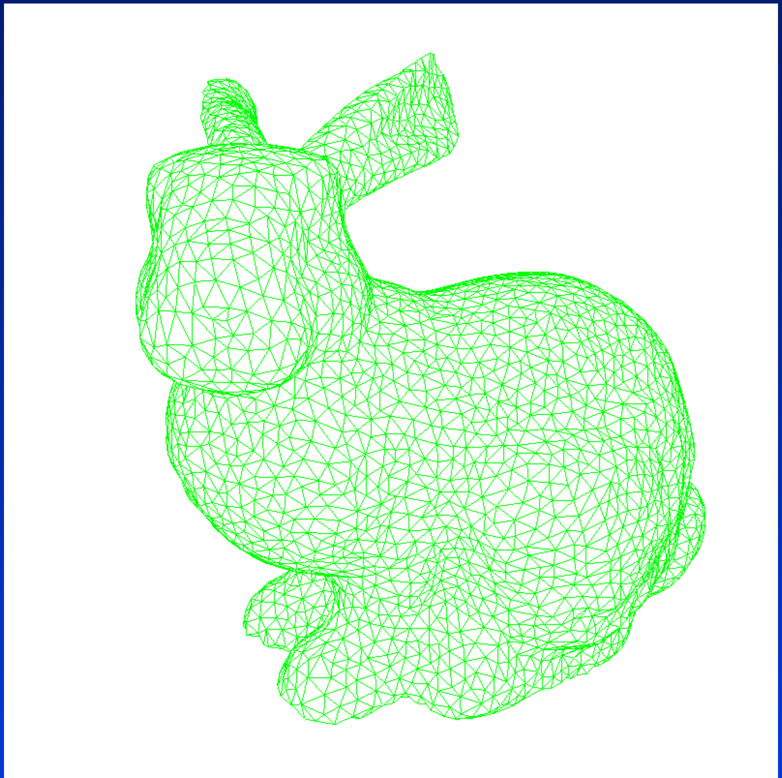
- Global refinement is conducted by subdivision
  - Piecewise smooth Loop's scheme
- Adaptive refinement is guided by error metric
  - Variance of the distance from the triangle to the dataset:

$$Var(dist) = (\sum_{i=1}^{n} d_i^{\ 2}) / n - ((\sum_{i=1}^{n} d_i) / n)^2$$

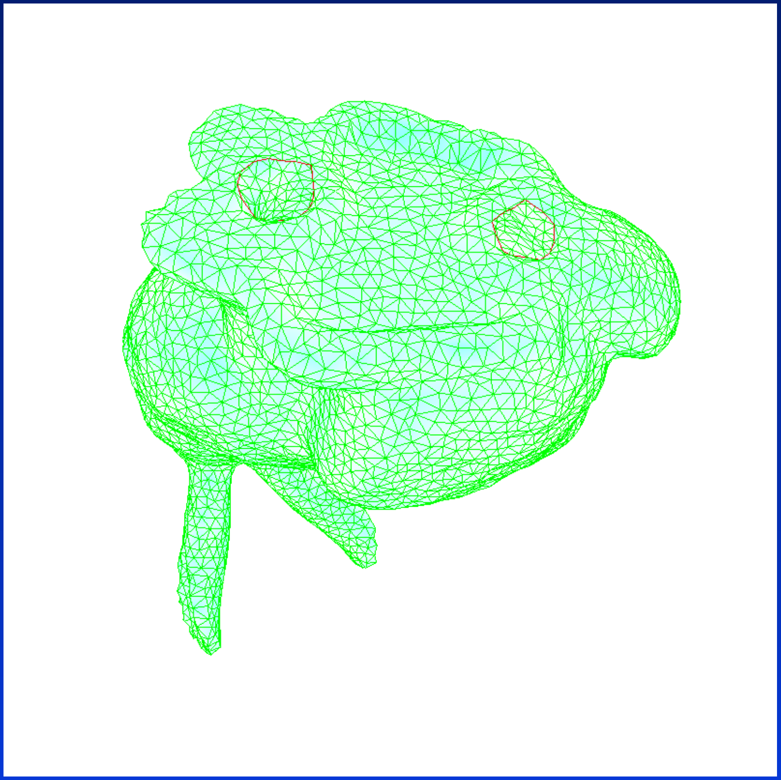
Bunny— Point Clouds Dataset

Bunny

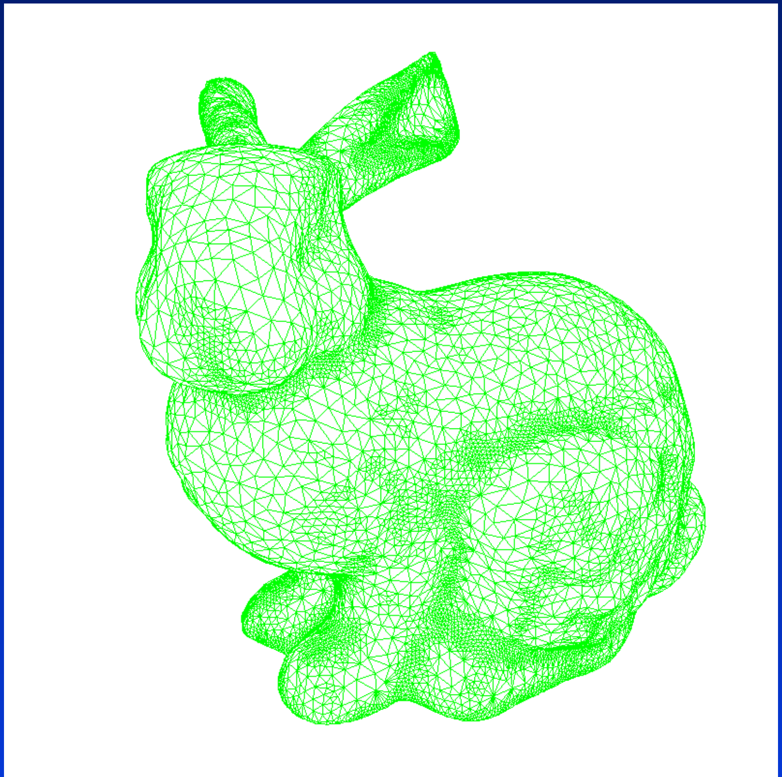
Bunny– Initial Recovered Shape



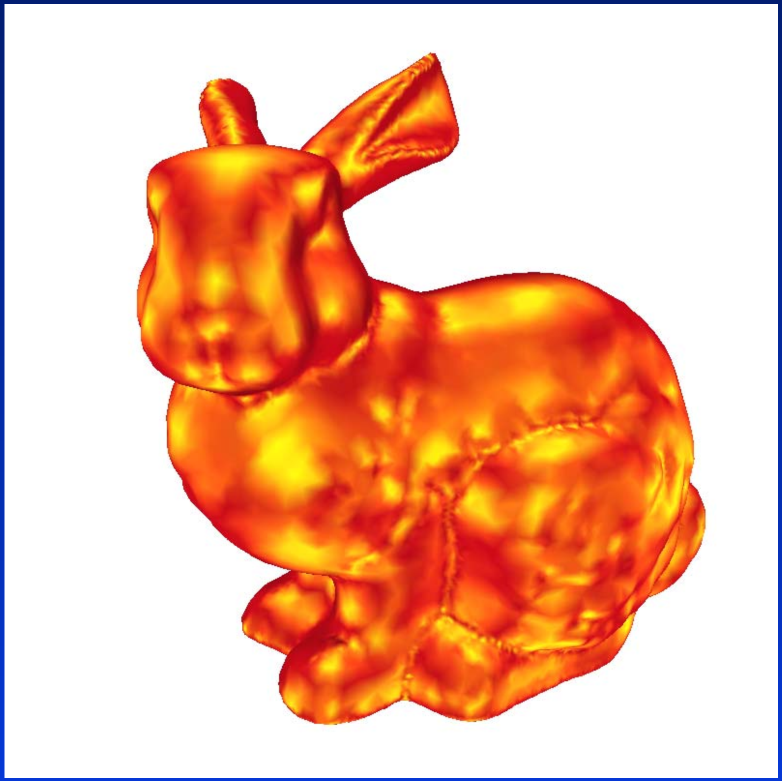
Bunny— "Bottom Up View"



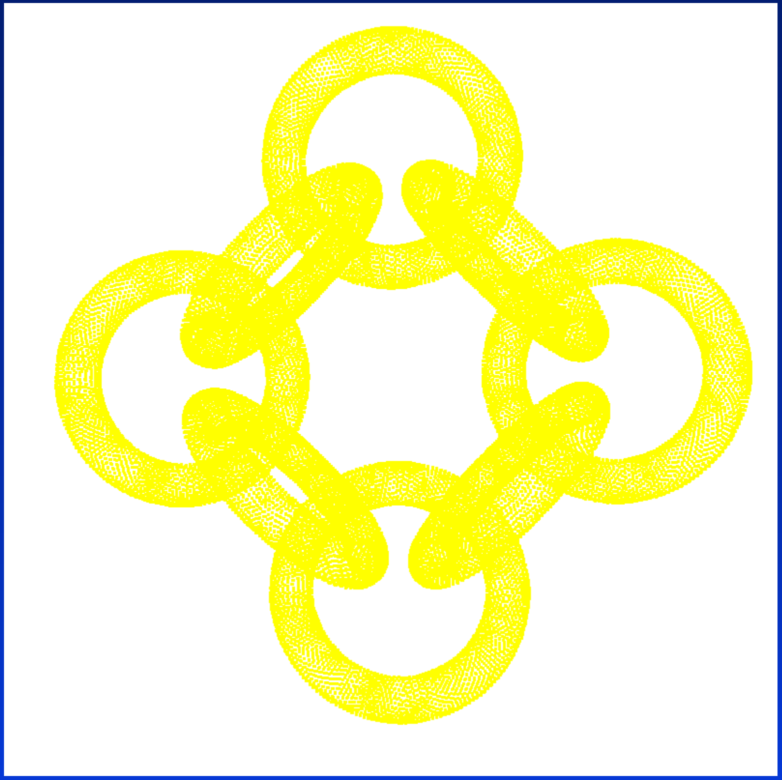
Bunny— Adaptively Refined Shape



Bunny— Adaptively Refined Shape

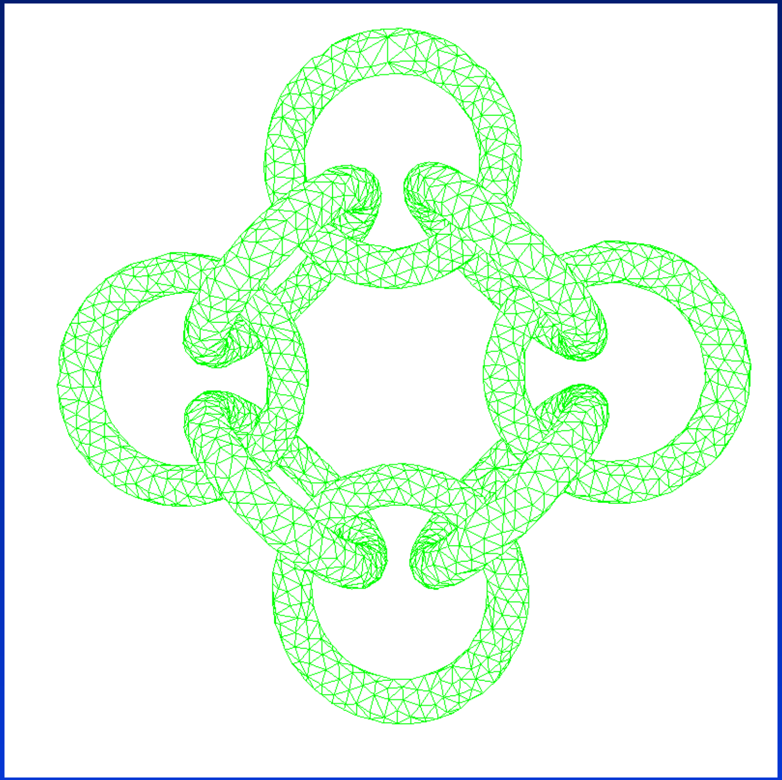


Eight-Tori Knot

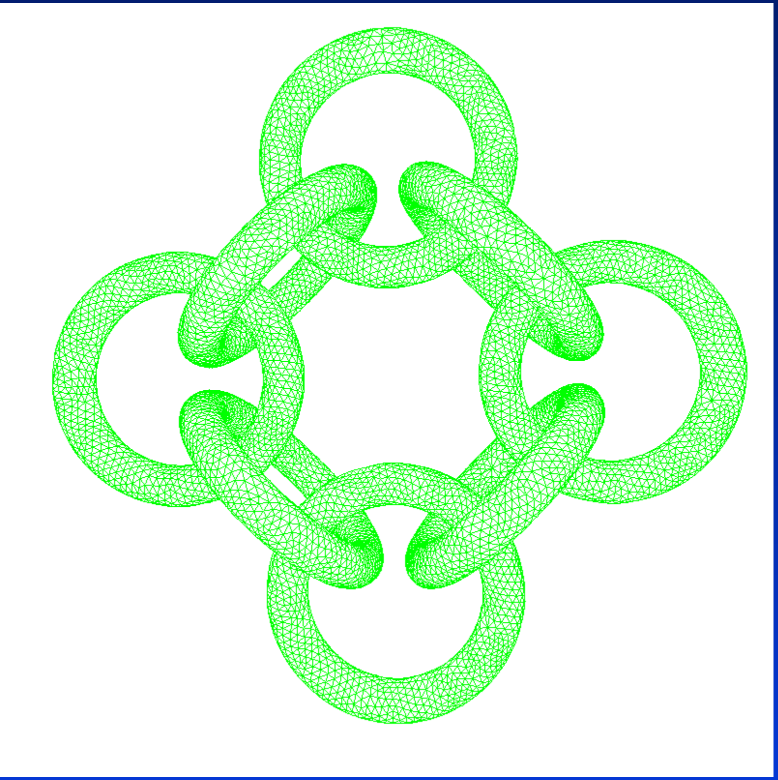


Eight-Tori Knot

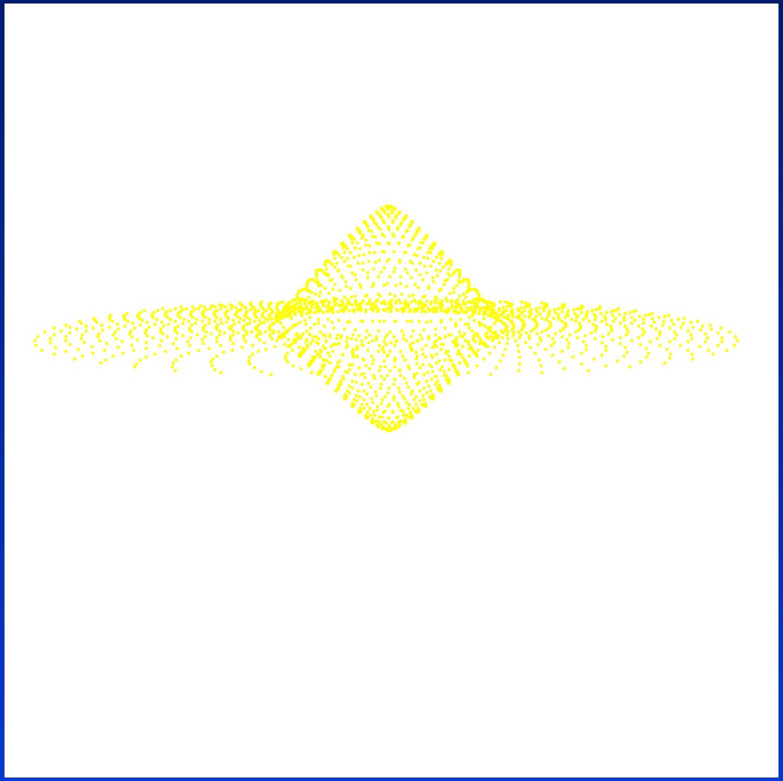
Eight-Tori Knot— Initial Shape



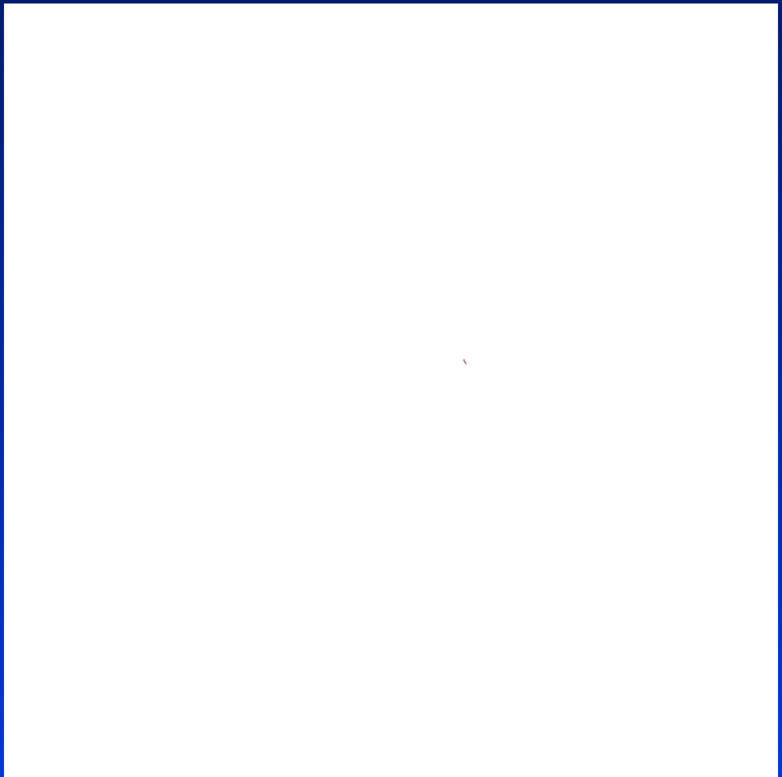
Eight-Tori Knot— Refined Shape



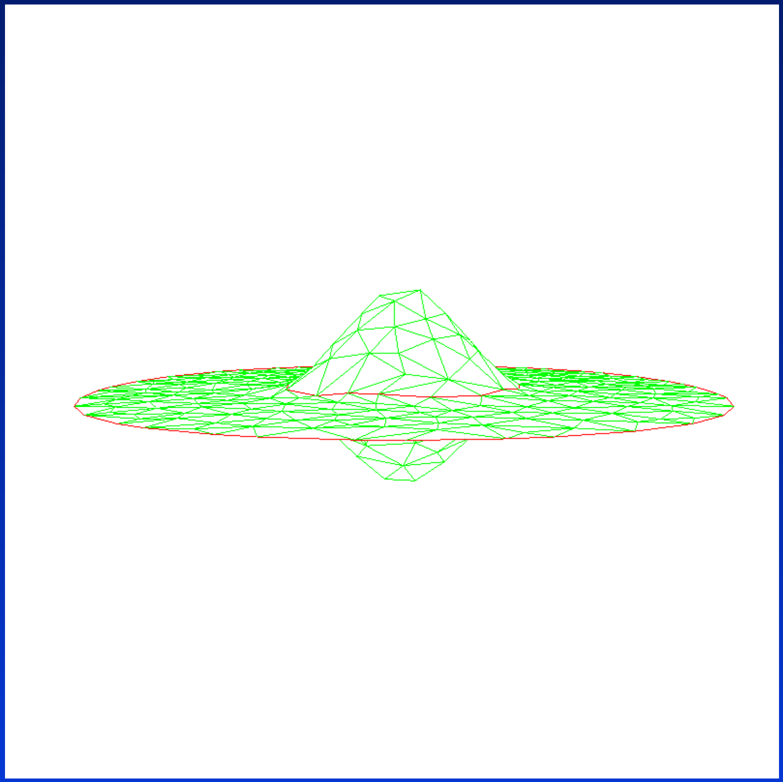
Non-manifold Shape



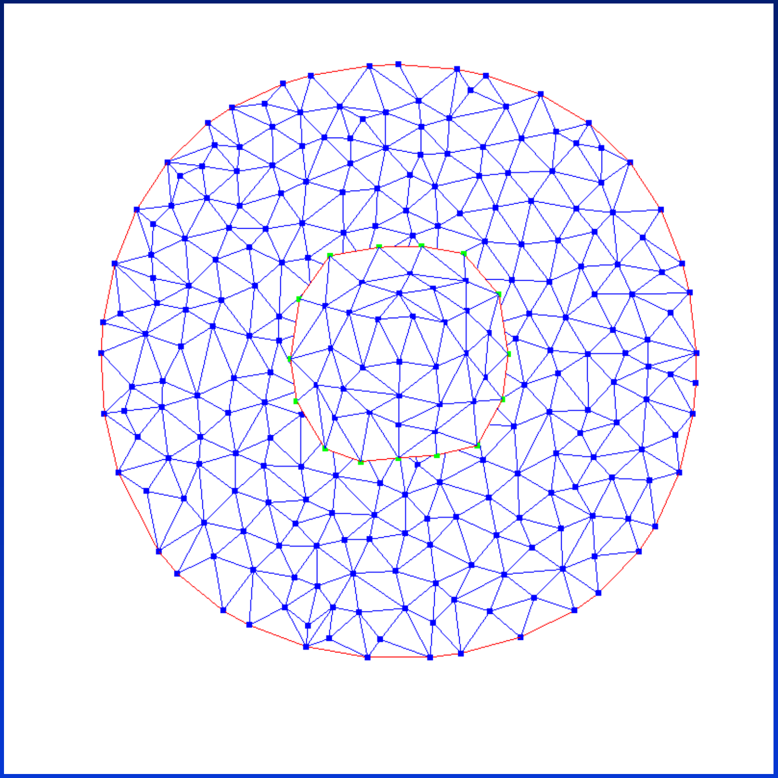
Non-manifold Shape



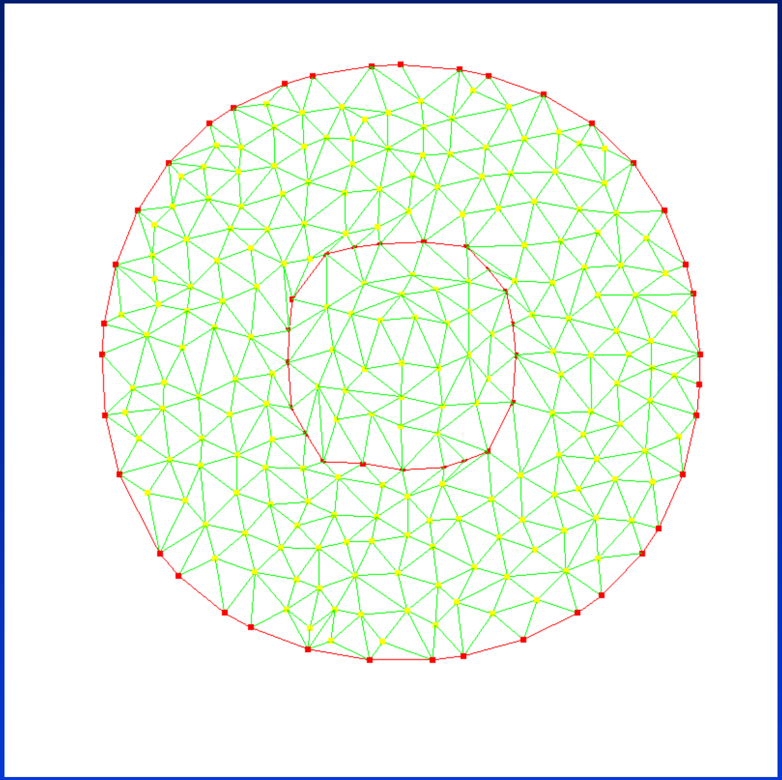
Non-manifold Shape



Non-manifold Stitching



Non-manifold Stitching



Timing and Complexity

Figure#	Data size (# Points)	# Vertices	# Faces	# Edges	Time (sec)
Mannequin	35947	4842	9653	14495	247
Bunny	12772	1844	3647	5490	156
Eight-tori	38560	3655	7310	10965	267
Non-manifold	2050	297	573	868	35

Contributions

- Three new deformable models
- Extract complicated geometry and topology
- Represent diverse types of shape
  - Open, closed, and even non-manifolds
- Works for all kinds of datasets directly
- Level-of-details representation and hierarchical structure
- Versatile applications (surface reconstruction and beyond)
- Unify modeling, rendering, visualization, and simulation

Future Applications

- Medical planning, surgical simulation, biomechanical modeling
- Haptics-based interactive shape design
- 3D morphing
- Motion tracking
- Medical image segmentation
- Computational fluid dynamics
- Feature extraction from time-varying phenomena

Future Work

- Broaden application scopes
  - Medical imaging, computer-aided engineering, visual computing, computational sciences
- PDE theory and numerical implementation
- Parallel processing and algorithmic improvement
- Robust self-collision handling for deformable models
- Multi-scale computation
- Software environment: development, evaluation, commercialization
- Academic and industrial collaboration

Conclusion

- Offer a single, unified solution to a suite of modeling, rendering, and simulation tasks
- Provide a unified framework for a variety of applications
- Broaden the application scopes of deformable models so that they become data analysis and visualization tools
- Handle application-specific tasks by way of domain-specific PDEs
- Capture geometric and non-geometric features of user interest
- Have a potential to become a powerful scientific tool for data understanding and new scientific discovery

Conclusion

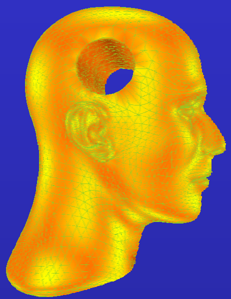
- Tackle an important challenge in visual computing
- Bridge the gap of parametric model and level-set model
- Overcome the limitation of the currently available models
- Improve both the performance and robustness of deformable models
- Extend capabilities of deformable modeling techniques
- Provide a unified framework for a variety of applications
- Prove to be a powerful visual computing tool
- Broaden the application scopes of deformable models so that they become powerful data analysis and visualization tools

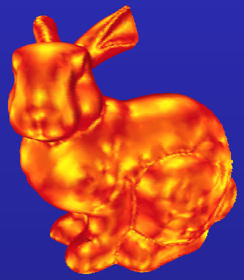
Conclusion

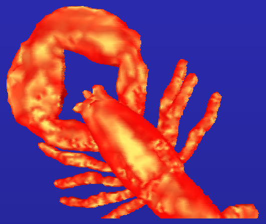
- Contribute to applied and computational mathematics
- Offer a single, unified solution to a suite of modeling, rendering, and simulation tasks
- Handle application-specific tasks by way of domain-specific PDEs
- Capture geometric and non-geometric features of user interest
- Have a potential to become a powerful scientific tool for data understanding and new scientific discovery

Acknowledgements

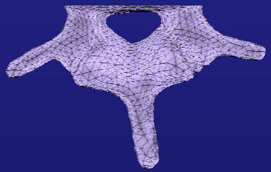
- National Science Foundation (CAREER award CCR-9896123, DMI-9896170, ITR grant IIS-0082035, IIS-0097646)
- Alfred P. Sloan Fellowship
- Ford Motor Company
- Honda America, Inc.
- My Ph.D students: Ye Duan, Jing Hua, …
- Datasets courtesy of Hughes Hoppe, University of Washington, Stanford Graphics Laboratory

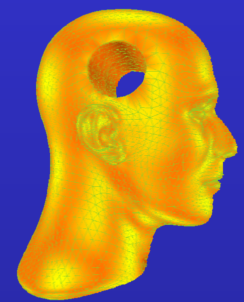


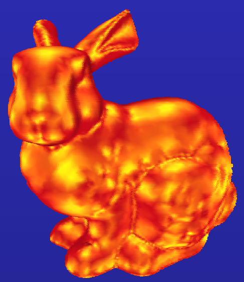


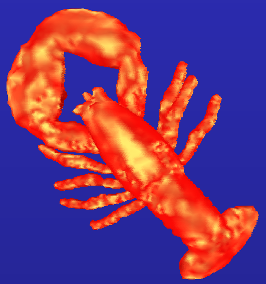


The End









Thank you!