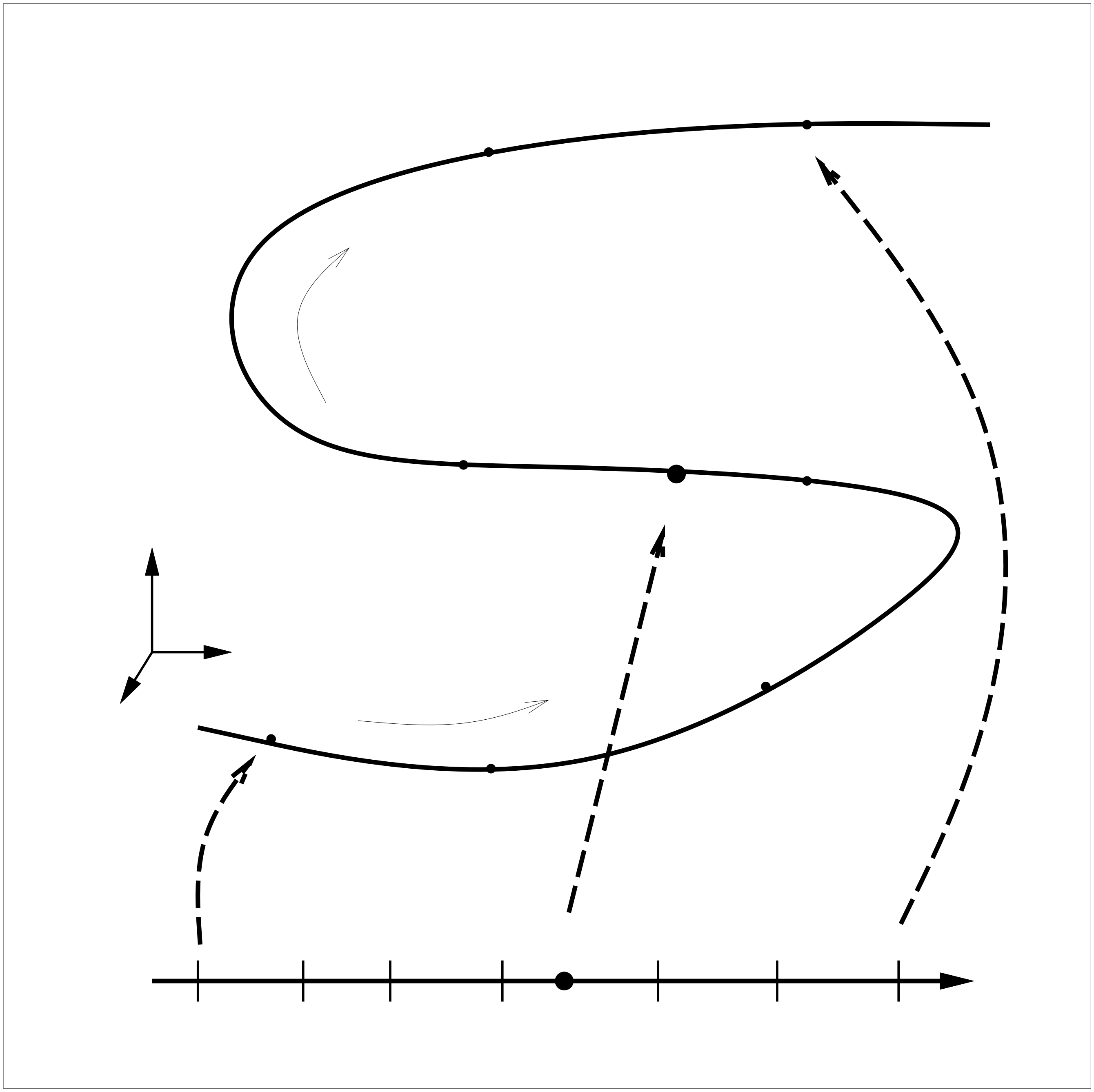
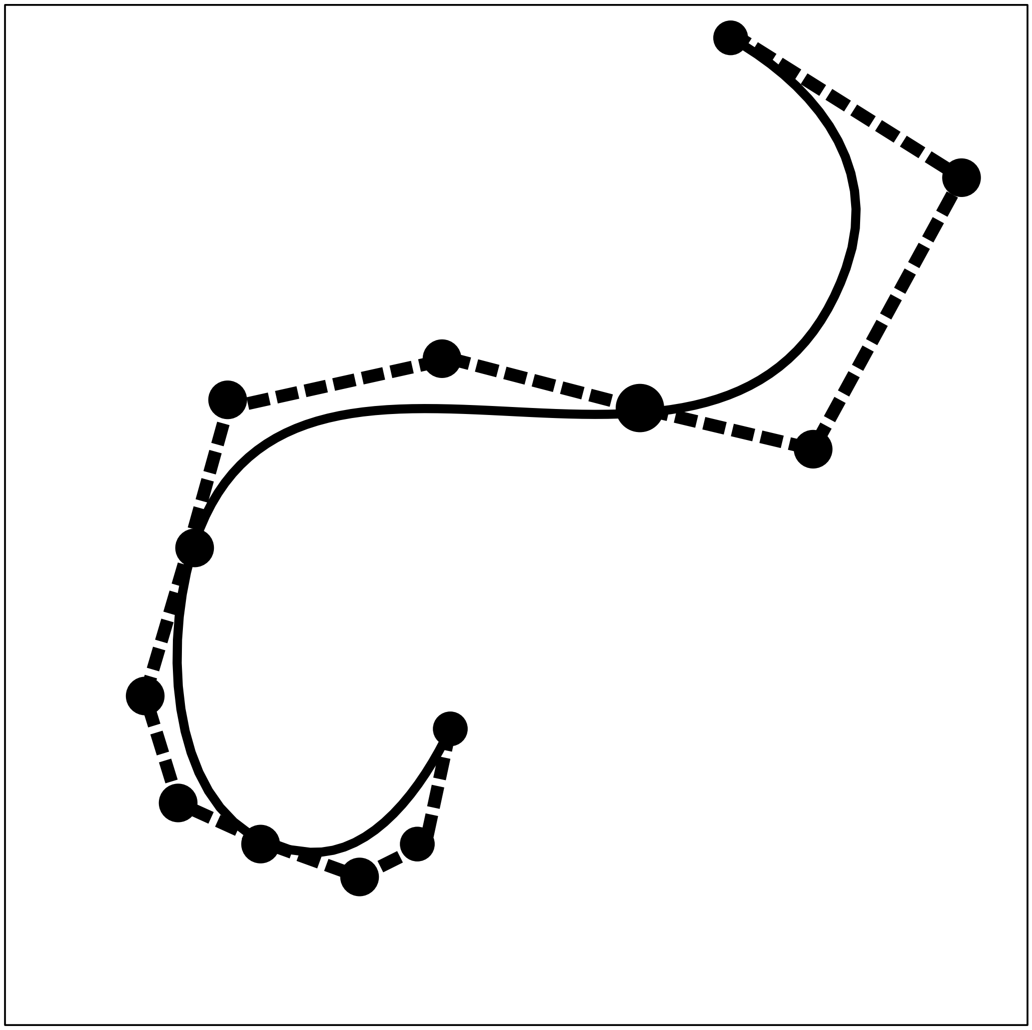
Piecewise Curves



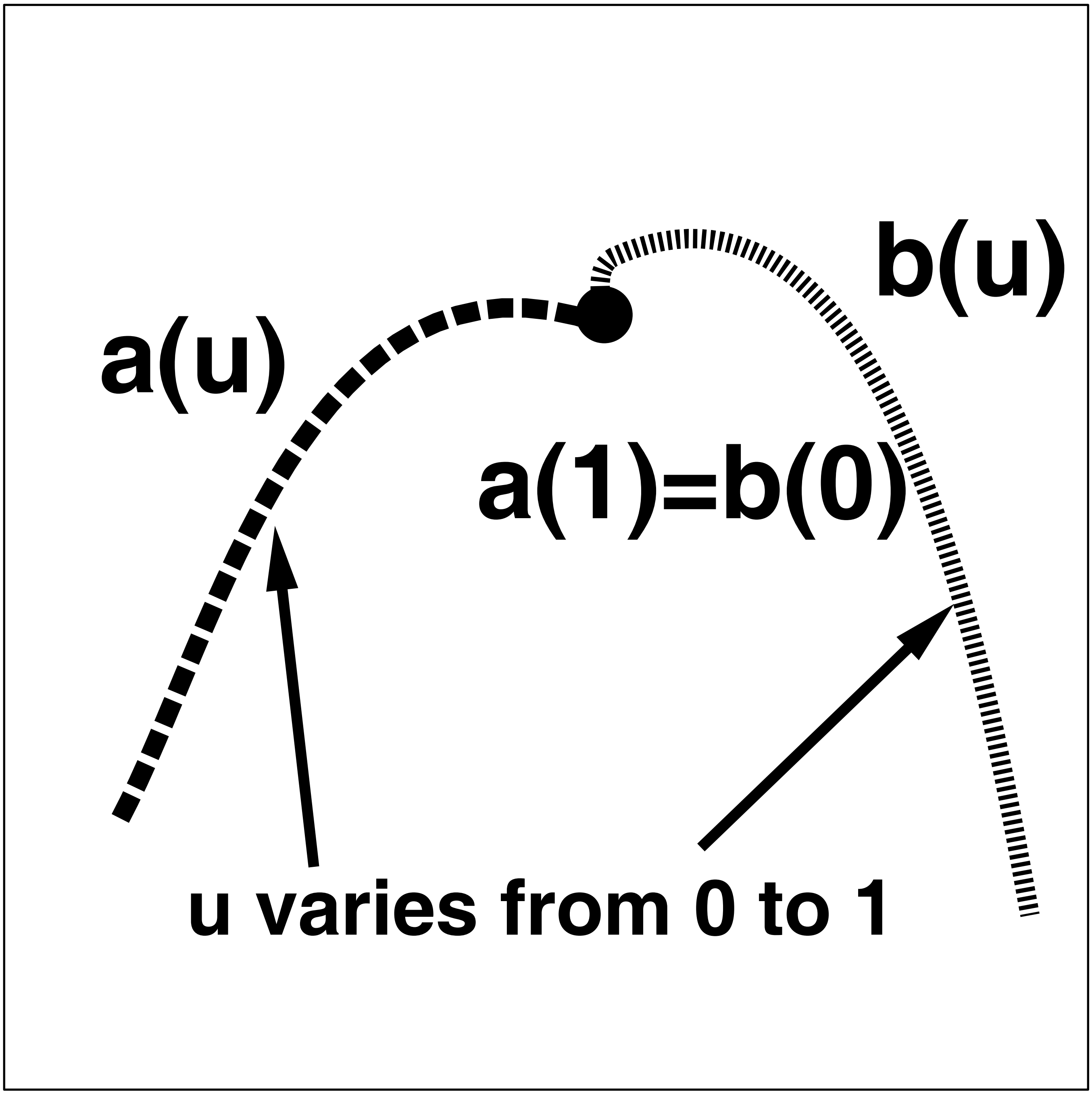
Piecewise Bezier Curve



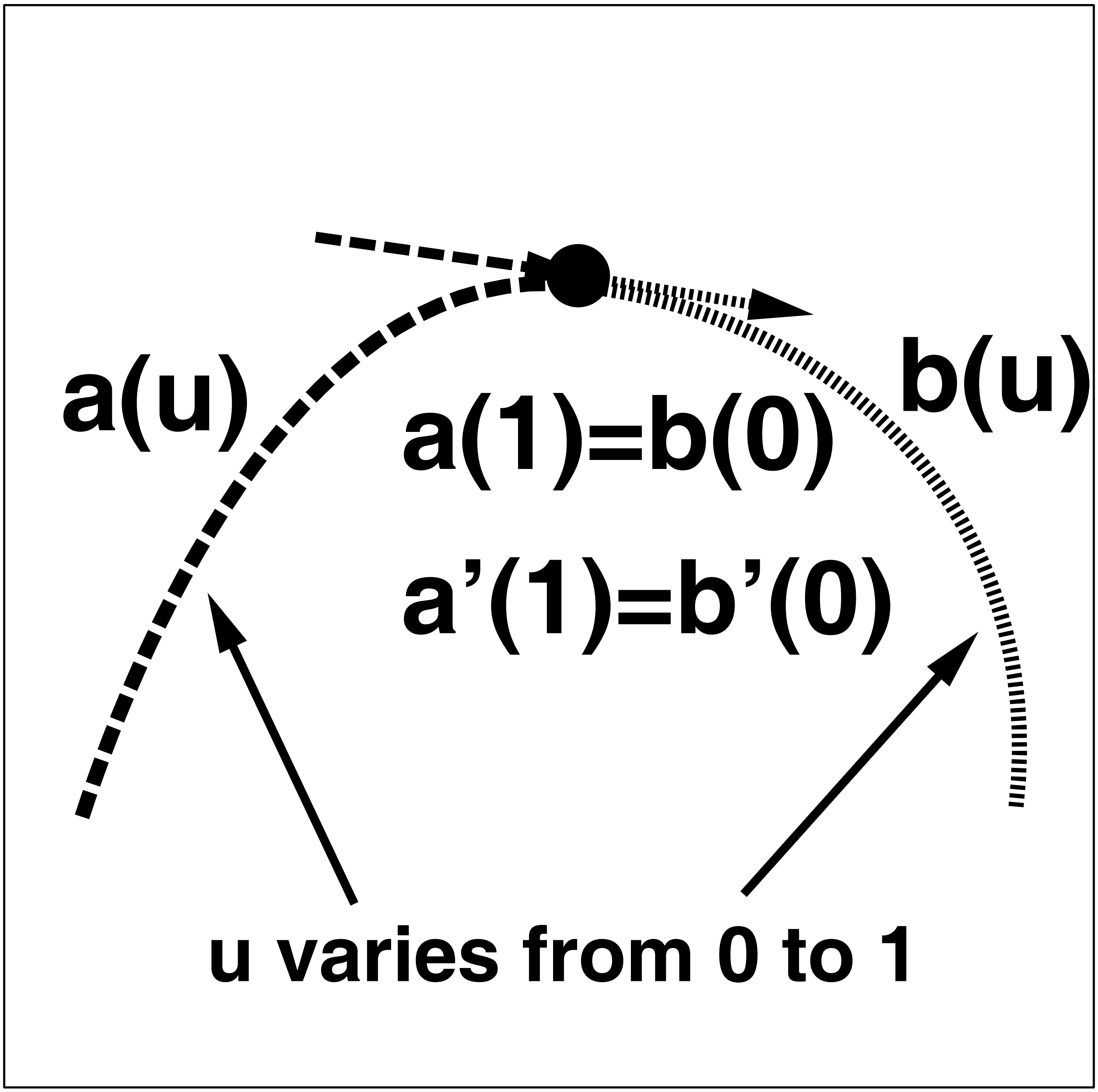
Continuity

- **One of the fundamental concepts**
- **Commonly used cases:** C^0, C^1, C^2, **etc.**

Positional Continuity



Derivative Continuity



General Continuity

- C^n continuity: derivatives (up to n-th) are the same a end points
$$\mathbf{a}^{(i)}(1) = \mathbf{b}^{(i)}(0)$$
where $i = 0, \ldots, n$

- The prior definition is for *parametric* continuity

- Parametric continuity depends on parameterization!

- Parameterization is not unique

- Different parametric representations may express the same geometry

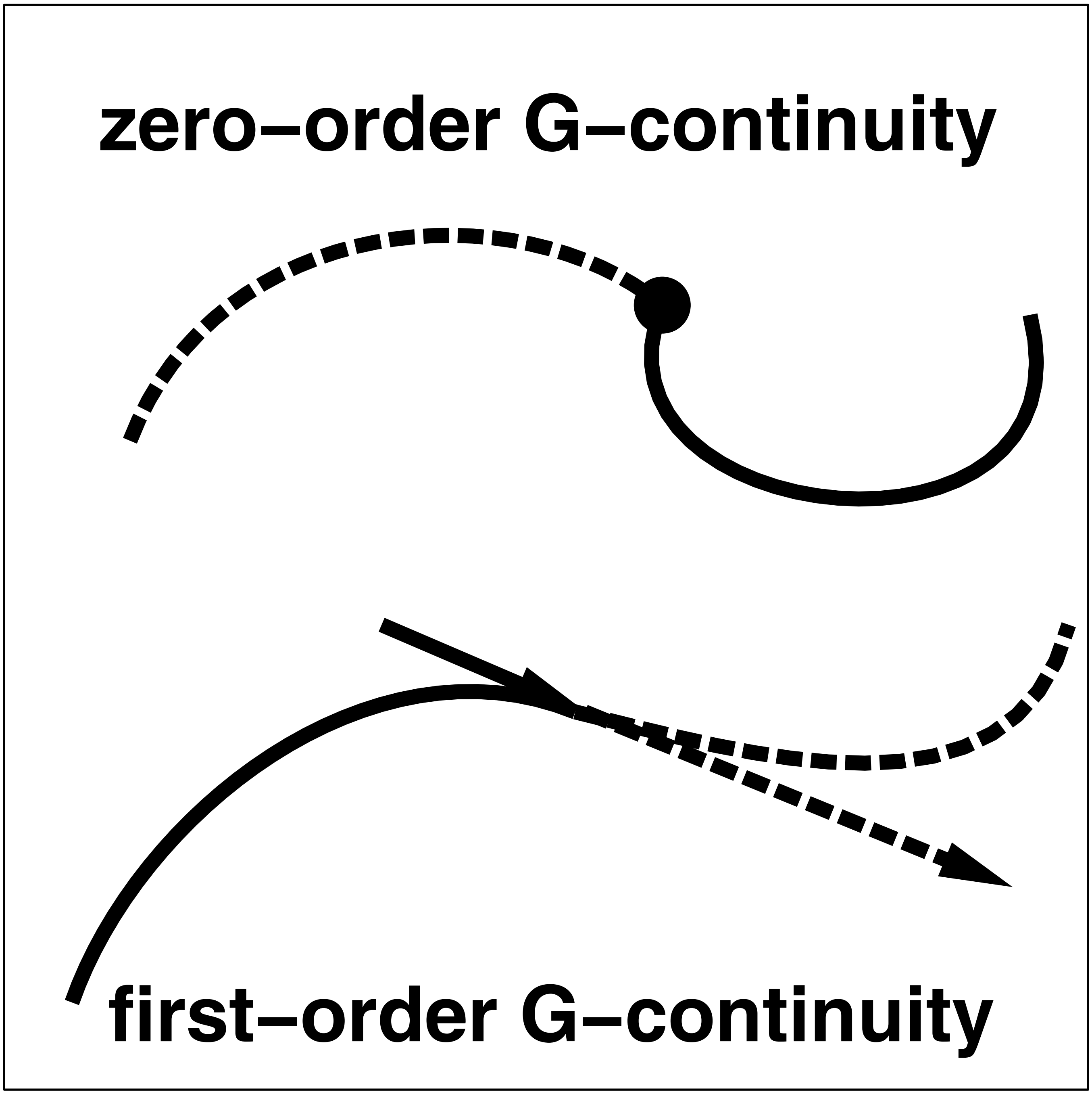
- Re-parameterization can be easily implemented

- Another type of continuity: geometric continuity, G^n

Geometric Continuity

- Depend on the curve geometry
- DO NOT depend on the underlying parameterization
- G^0: the same joint
- G^1: Two curve tangents at the joint align, but may n
have the same magnitude
- G^1: it is C^1 after the reparameterization
- Which condition is stronger???
- Examples

Geometric Continuity



Piecewise Hermite Curves

- **How to build an interactive system to satisfy various constraints**
- C^0 **continuity**

$$\mathbf{a}(1) = \mathbf{b}(0)$$

- C^1 **continuity**

$$\mathbf{a}(1) = \mathbf{b}(0)$$

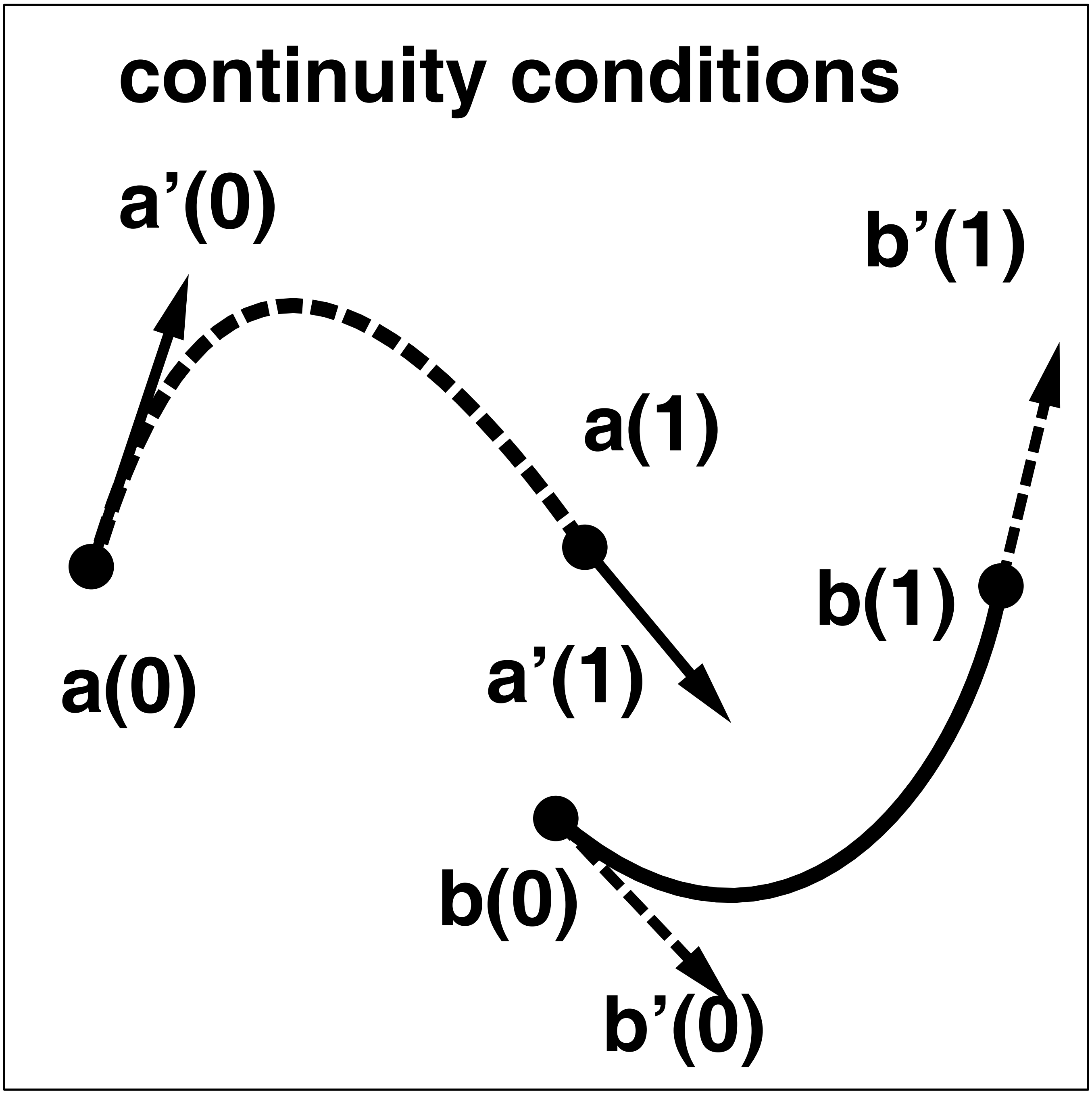
$$\mathbf{a}'(1) = \mathbf{b}'(0)$$

- G^1 **continuity**

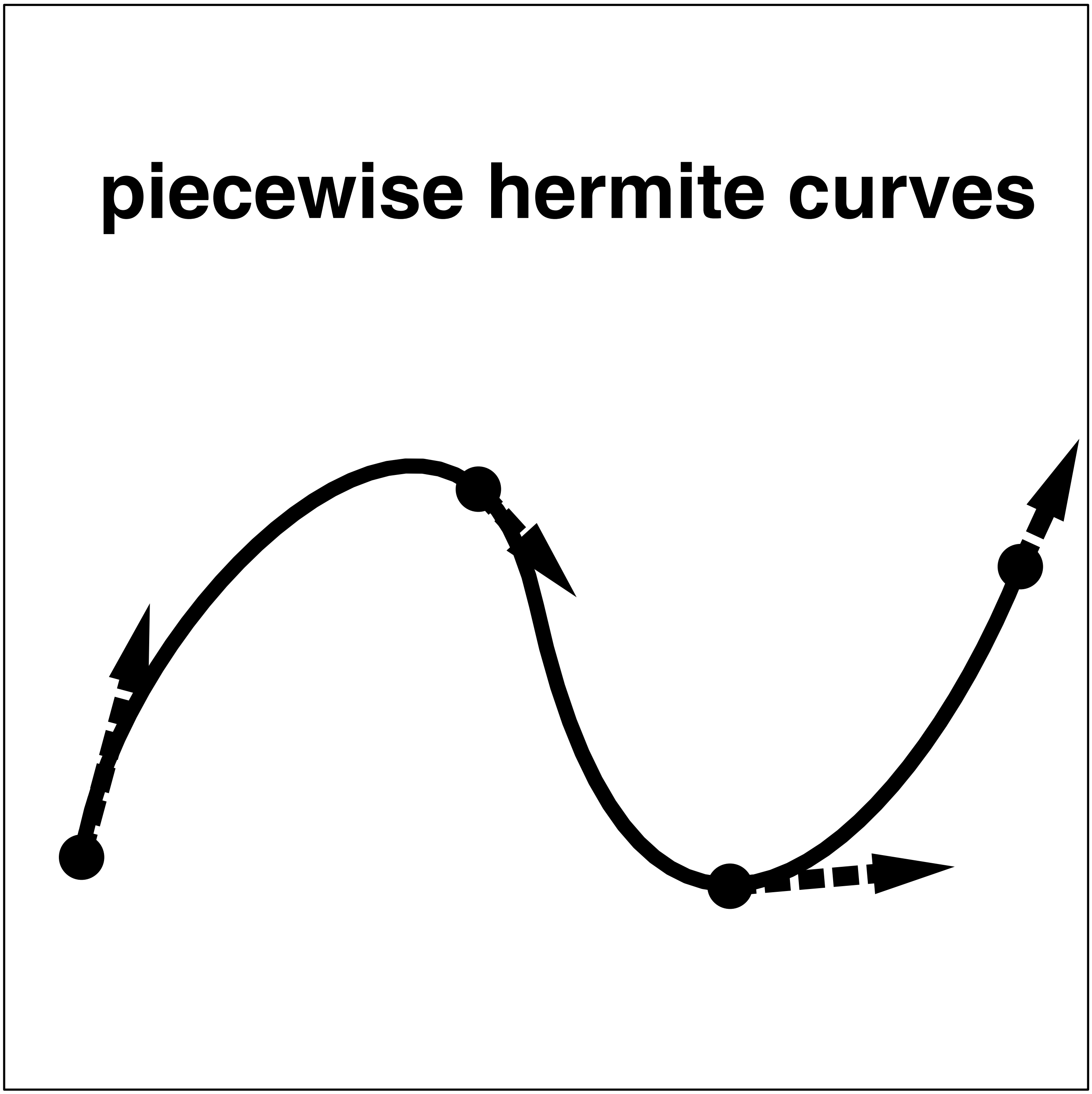
$$\mathbf{a}(1) = \mathbf{b}(0)$$

$$\mathbf{a}'(1) = \alpha \mathbf{b}'(0)$$

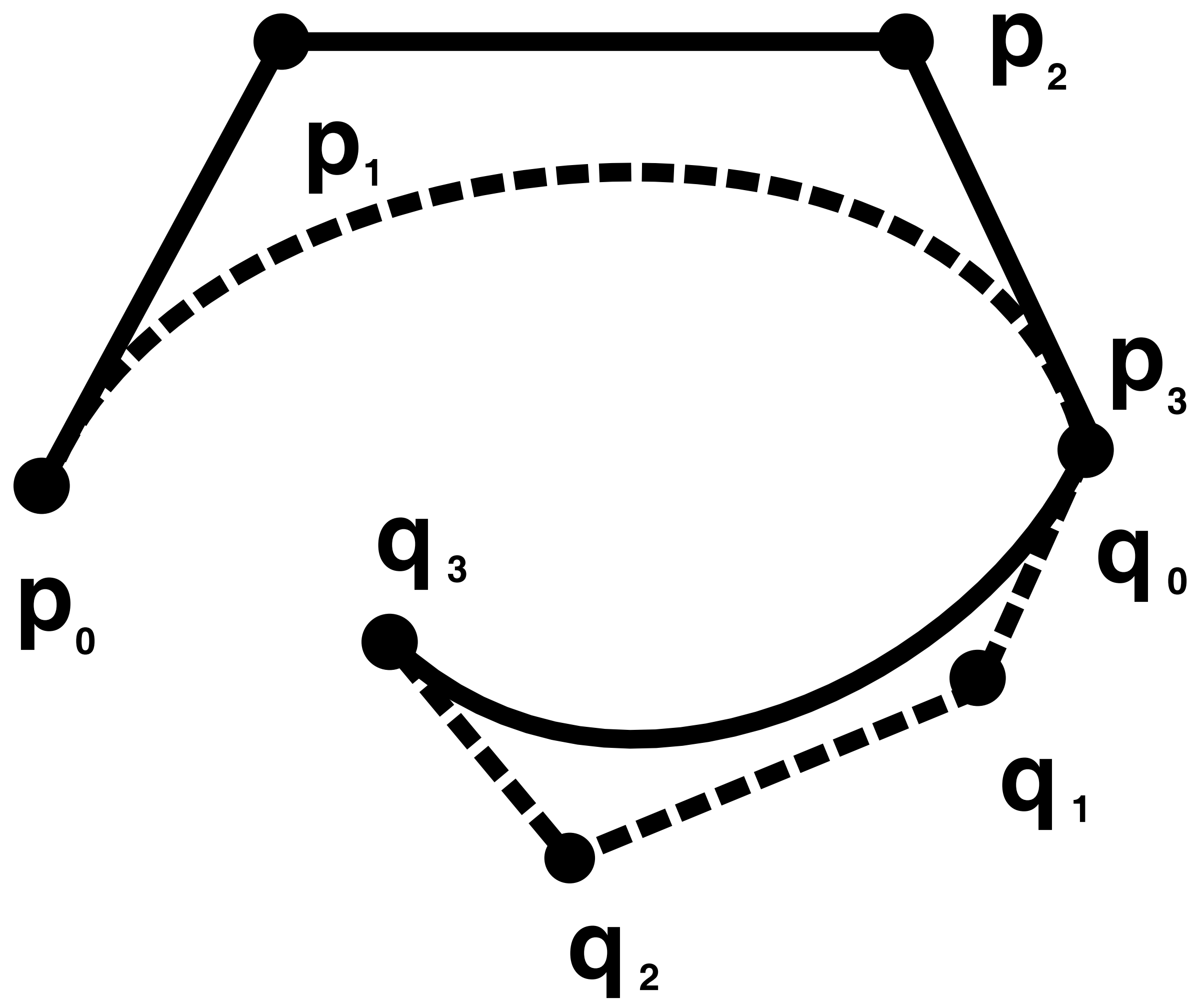
Piecewise Hermite Curves



Piecewise Hermite Curves



Piecewise Bezier Curves



piecewise Bezier curves

Piecewise Bezier Curves

- C^0 **continuity**

$$\mathbf{p}_3 = \mathbf{q}_0$$

- C^1 **continuity**

$$\mathbf{p}_3 = \mathbf{q}_0$$

$$\mathbf{p}_3 - \mathbf{p}_2 = \mathbf{q}_1 - \mathbf{q}_0$$

- G^1 **continuity**

$$\mathbf{p}_3 = \mathbf{q}_0$$

$$\mathbf{p}_3 - \mathbf{p}_2 = \alpha(\mathbf{q}_1 - \mathbf{q}_0)$$

- C^2 **continuity**

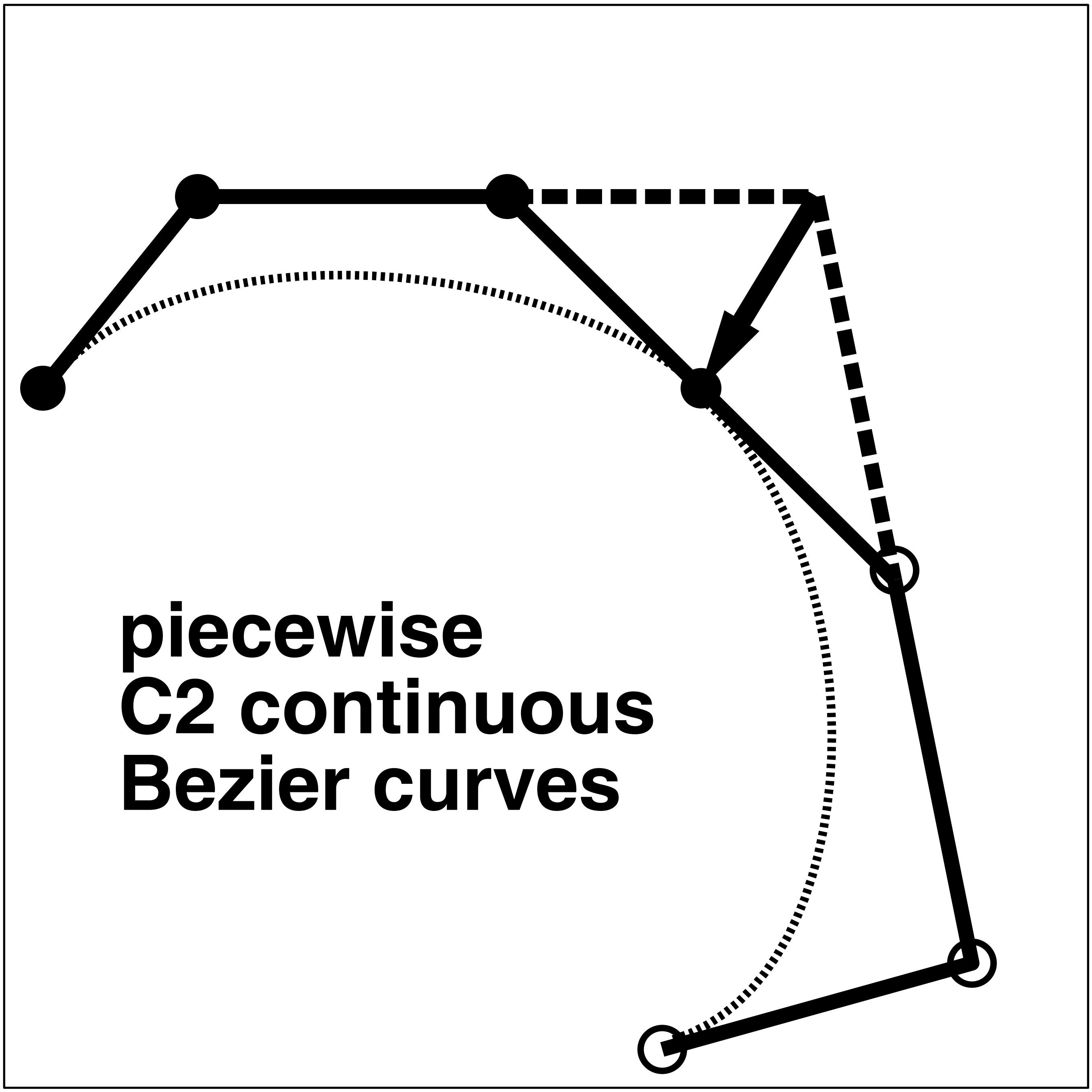
$$\mathbf{p}_3 = \mathbf{q}_0$$

$$\mathbf{p}_3 - \mathbf{p}_2 = \mathbf{q}_1 - \mathbf{q}_0$$

$$\mathbf{p}_3 - 2\mathbf{p}_2 + \mathbf{p}_1 = \mathbf{q}_2 - 2\mathbf{q}_1 + \mathbf{q}_0$$

- **Geometric interpretation**
- G^2 **continuity**

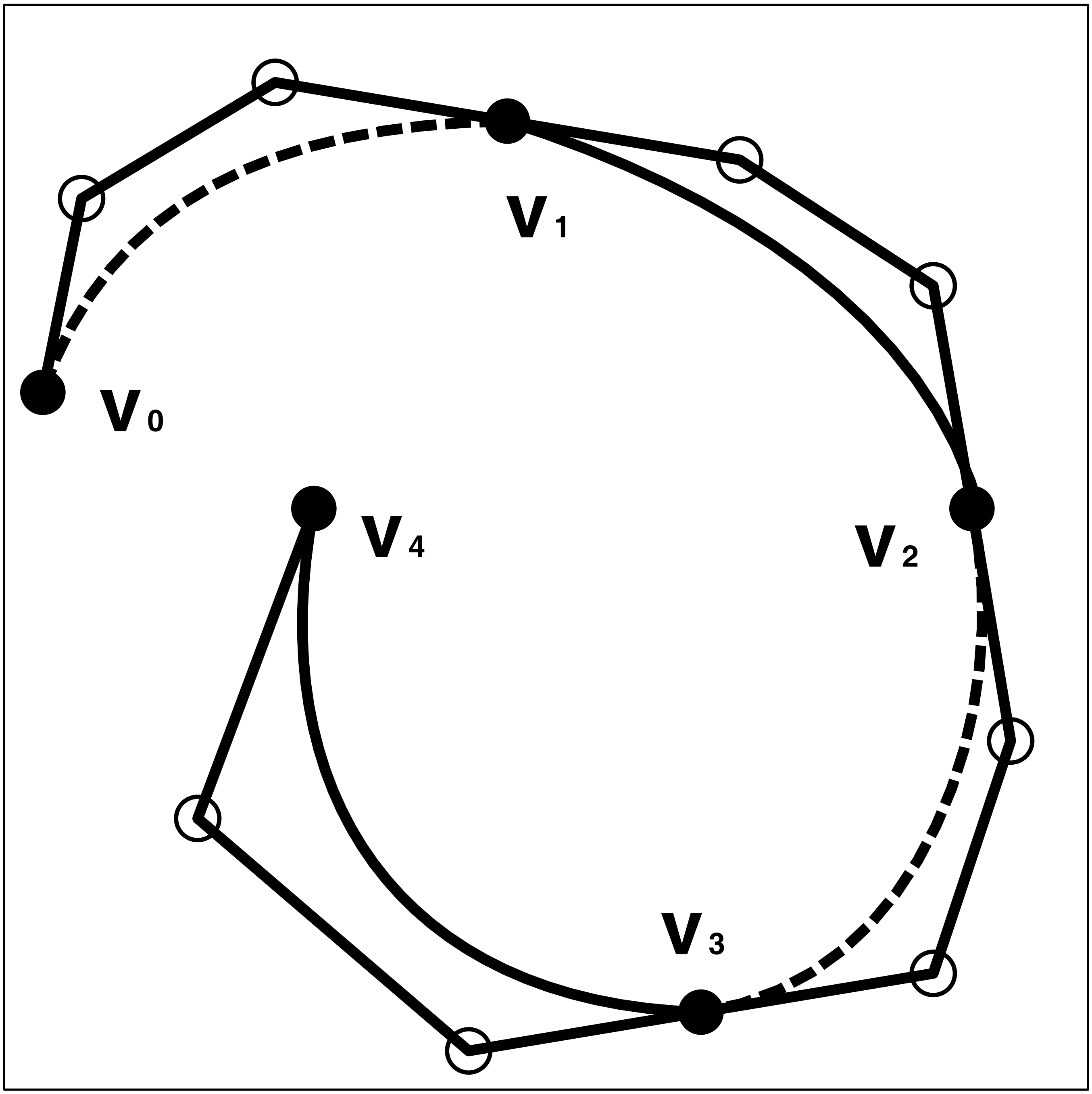
Piecewise C^2 Bezier Curves



Continuity Summary

- C^0: straightforward, but not enough
- C^3: too constrained
- Piecewise curves with hermite and Bezier representations satisfying various continuity condition
- Interactive system for C^2 interpolating splines using piecewise Bezier curves
- Advantages and disadvantages

C^2 Interpolating Splines



Natural Splines

- Interpolate all control points
- Equivalent to a thin strip of metal in a physical sense
- Forced to pass through a set of desired points
- No local control (global control)
- $n + 1$ control points
- n pieces
- $2n$ extra points
- $2(n - 1)$ conditions
- We need two additional conditions
  - specify derivatives at two end points
  - specify the two internal control points that define first curve span

- **interactive system**
- **natural end conditions: second-order derivatives at end points are defined to be zero**

- **Advantages: interpolation, C^2**

- **Disadvantages: no local control (if one point is changed, the entire curve will move)**

- **How to overcome this drawback: B-Splines**

B-Splines Motivation

- **The goal is local control!!!**
- **B-splines provide local control**
- **Do not interpolate control points**
- C^2 **continuity**
- **Alternatively**
- **Catmull-Rom Splines**
- **Keep interpolations**
- **Give up** C^2 **continuity (only** C^1 **is achieved)**
- **Will be discussed later!!!**