# Solid Modeling

- Solid: Boundary + Interior
- Volume occupied by geometry
- Solid representation schemes
- Constructive Solid Geometry (CSG)
- Boundary representations (B-reps)
- Space-partition representations
- Operations

# Constructive Solid Geometry

- Constructive (procedural) models of complex solids
- Complex solids as compositions of simpler solids
- A set of 3D objects (simple primitives)
  - blocks
  - pyramids
  - cylinders
  - cones
  - spheres
  - tori
  - algebraic halfspaces

$$\{(x, y, z) | f(x, y, z) <= 0\}$$

- Other objects as primitives
  - closed spline surfaces
  - spline solids
  - swept solids

- others

- Primitives: point-sets

- Boolean set operations
  - union, intersection, difference

- Complicated object: a binary tree
  - leaf nodes: primitive shape
  - non-leaf nodes: set operators

- Surfaces are actually defined IMPLICITLY

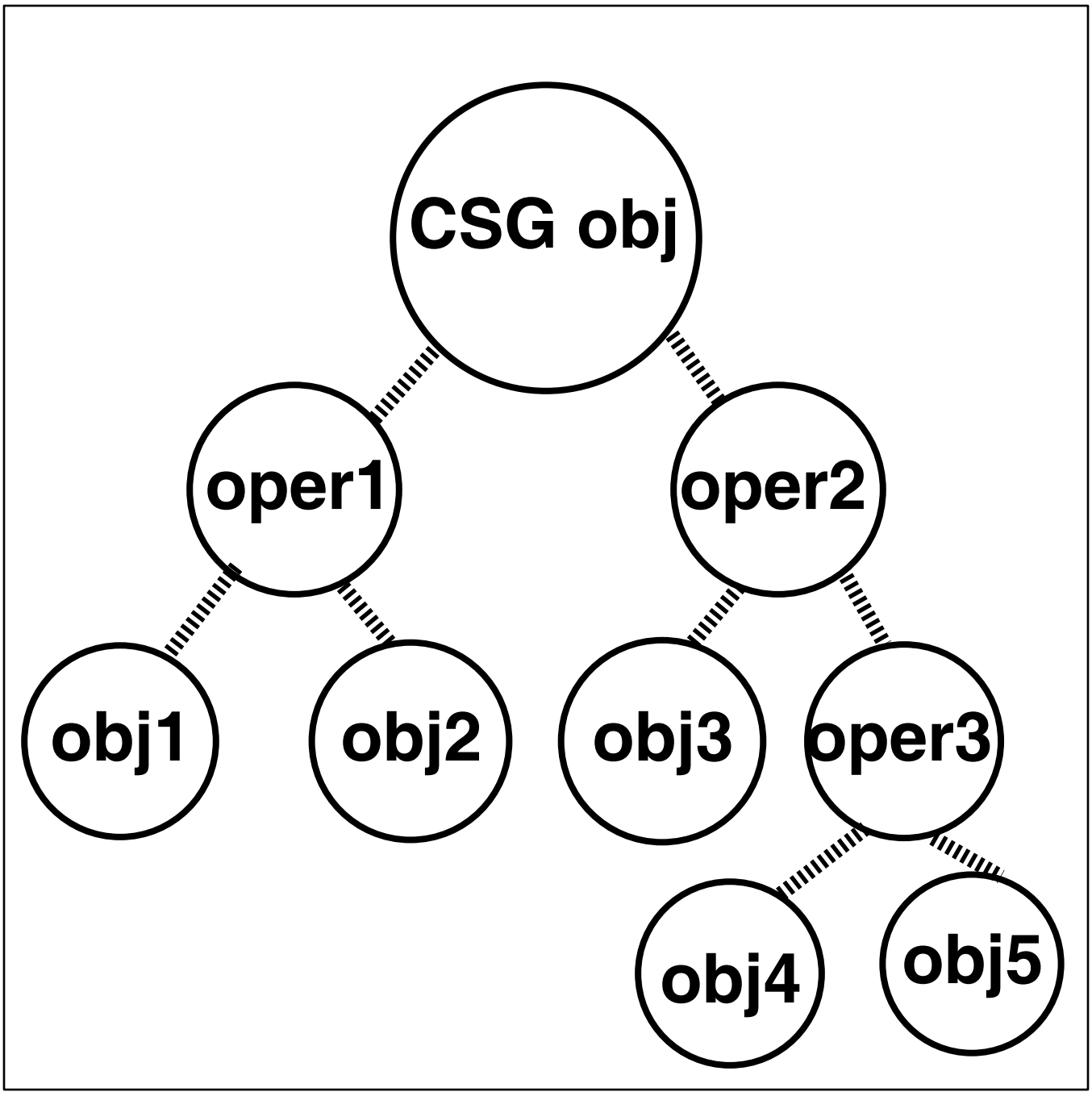
- Solids ALSO have B-reps
  - a set of surfaces
  - separate object interior from the environment
  - unambiguous interior and exterior!

- Examples
  - polygon facets, spline patches

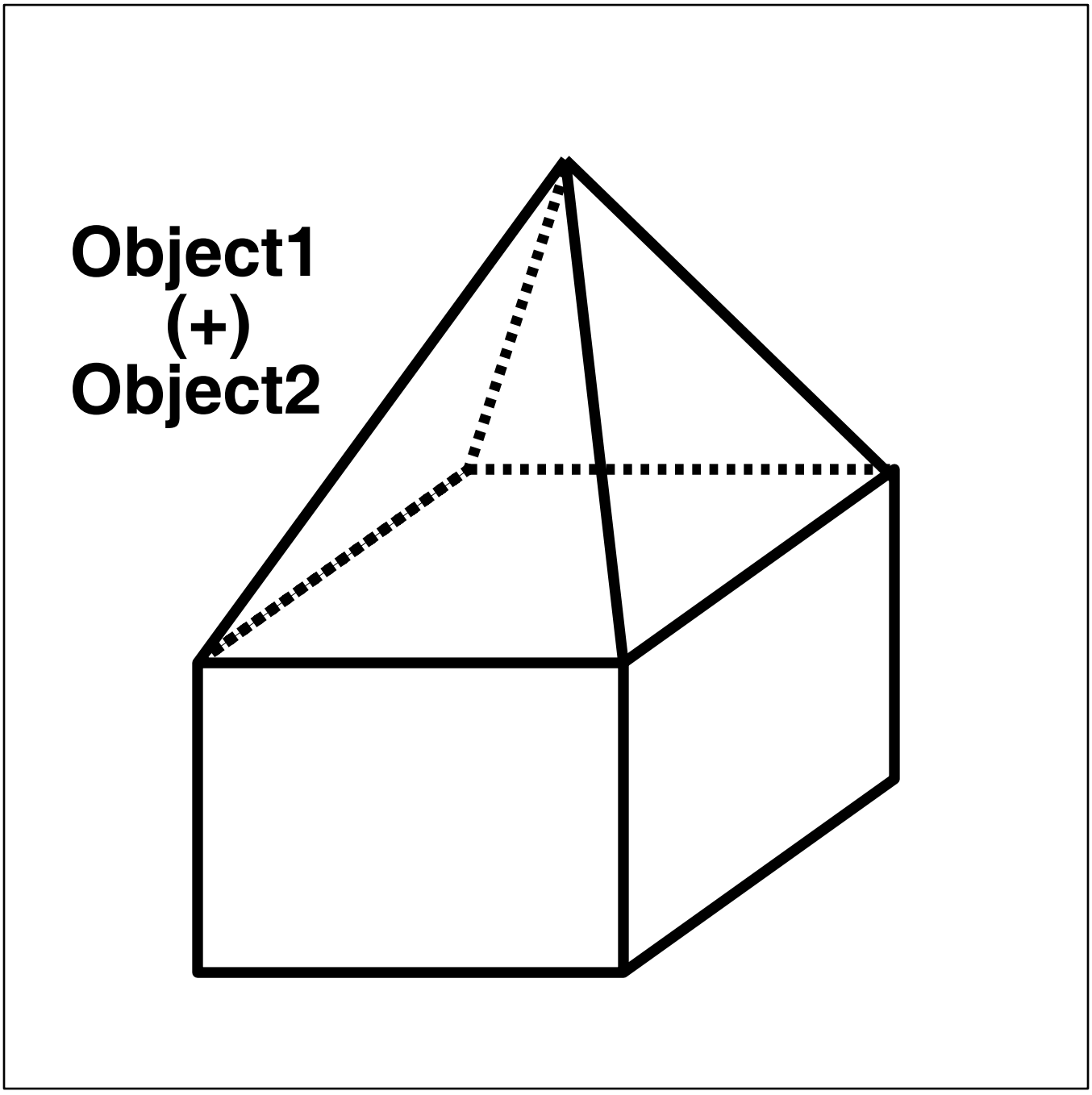
- Solid modeling field

- mathematical foundations
- representations
- algorithms
- applications
- user interfaces
- systems

# CSG Schemes



# CSG Example



# Basic Operations

- Point, curve, and surface classification/neighborhood w.r.t solids
- Local/global modification
- Representation conversion
- Topological validity
- Approximation
- Representation redundancy
- Interaction
- Manifold/nonmanifold representation
- Free-form solids
- Geometric/topological query
- Large-scale model management

# B-REPS

- Faces, edges, and vertices of solid boundary
  - topological representation (connectivity)
  - geometric (numerical data) representation
- Polyhedral (tessellated) models
- Solids bounded by curved surfaces and edges
- Separate points inside from outside the solid
- Interface between the solid and the environment
- B-reps' conditions
  - closed, orientable
  - non-self-intersecting
  - bounded
  - connected
- Internal features of objects

  - Ray-casting is a fundamental method

- Determine the new B-reps for composite solids
  - Surface intersection calculation

- Determine physical properties
  - volume
  - mass, center of mass,
  - moment of inertia
  - density

- Decomposition

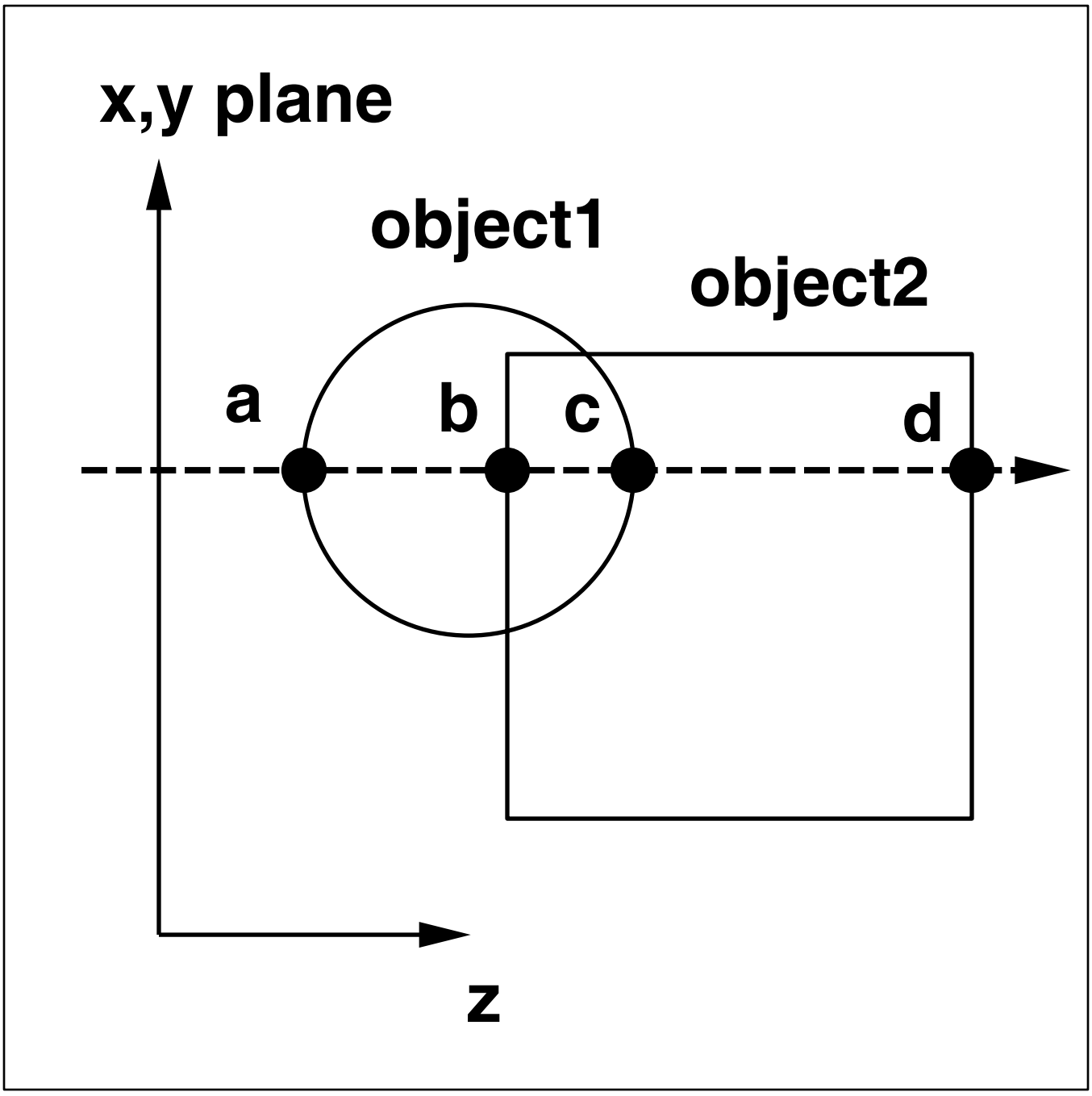
- Voxelization

- Adaptive subdivision

- Recursive subdivision

- Conversion among different schemes

# Ray Casting Method



# Spatial Partition

- Idea: cell decomposition
- Each piece is easier to describe than the original model
- Partition the space into a set of nonoverlapping regions (cubes)
- Uniform subdivision
- Predetermined resolution
- Cells are cubical in a fixed spatial grid
- Intersecting cubes
- Spatial-occupancy enumeration — spatial arrays
- (+) Easy to access points (cells) in arbitrary location

- (+) Spatial uniqueness
- (-) No explicit relationship for object parts
- (-) Large amounts of data storage
- (-) Limited accuracy
- (-) Specific coordinate system
- (-) Hard to manipulate (e.g., rotation)

# Octree representation

- Recursive subdivision
- Hierarchical tree structures
- Generalization of quadtree schemes
- Successively divide a 2D region into quadrants
- Homogeneous quadrants
  - either full or empty w.r.t. the object
- Heterogeneous quadrants
  - partially full (intersection)
- Terminating conditions
  - all quadrants are homogeneous
  - predefined resolution level
- Not restricted to solid geometry

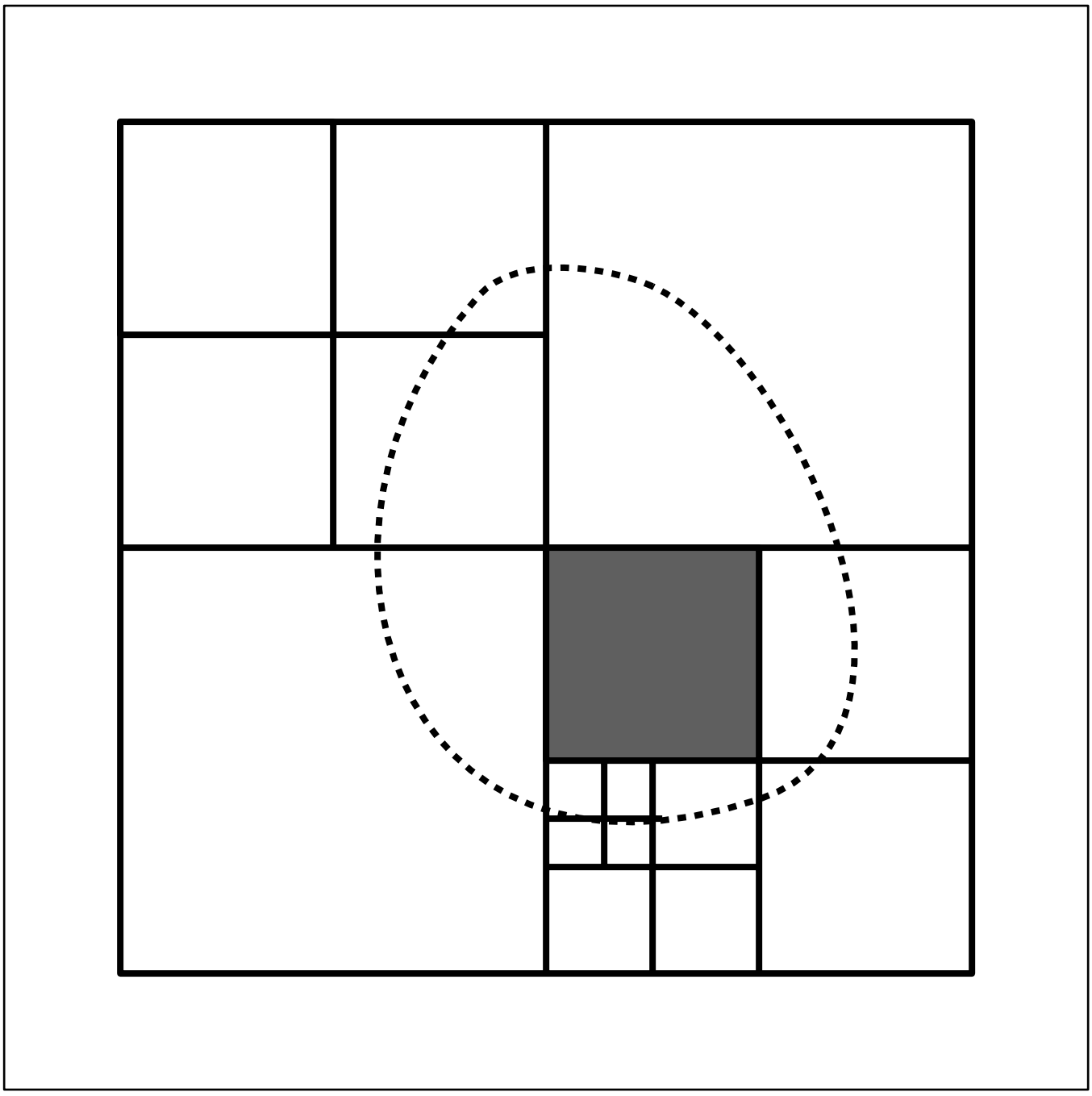
- Applicable to color, intensity, density, and others
- Divide 3D space into octants
- CSG-based set operations
- Binary space-partition
- Useful for a lot of graphics and visualization algorithms
- All computations are based on integer arithmetic
- Fast and parallel algorithms
  - translation
  - rotation
  - scaling
  - boolean operations
  - geometric and material properties
  - interference analysis
- Difficult for
  - curved objects

- free-form (sculptured) solids
- deformable modeling

- Irregular shape elements

  - triangles
  - tetrahedra

# Quadtrees



# Octrees