

Midterm Exam II (April 21, 2021, 08:30 am - 09:55 am)**CSE 215: Foundations of Computer Science**

State University of New York at Stony Brook, Spring 2021

Instructor: Prof. Pramod Ganapathi

Total points = 45. Total questions = 5. Total pages = 2.

Instructions:

- Please write your full name and SBU student ID on the answer sheet.

Problem 1. [20 points]

Use mathematical induction to prove the following identities.

- (a) [5 points] For all integers
- $n \geq 1$
- ,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

- (b) [5 points] For all integers
- $n \geq 1$
- ,
- $n(n^2 + 5)$
- is a multiple of 6.

- (c) [5 points] For all integers
- $n \geq 0$
- ,

$$1 + \frac{2}{3} + \frac{4}{9} + \cdots + \left(\frac{2}{3}\right)^n = 3 \left[1 - \left(\frac{2}{3}\right)^{n+1} \right]$$

- (d) [5 points] Suppose that
- c_1, c_2, c_3, \dots
- is a sequence defined as follows:

$$c_1 = 3, c_2 = -9$$

$$c_k = 7c_{k-1} - 10c_{k-2} \quad \text{for all integers } k \geq 3$$

Prove that $c_n = 4 \cdot 2^n - 5^n$ for all integers $n \geq 1$.**Problem 2. [5 points]**

Mention whether the following statements are true or false without giving any reasons.

Assume all sets are subsets of a universal set U .

- (a) [1 point] $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$
- (b) [1 point] $A = A \cup (A \cap B)$
- (c) [1 point] $A \subseteq A \cup B$
- (d) [1 point] $A \cap (A \cup B) = A \cap B$
- (e) [1 point] $A \subseteq B$ if and only if $A \cup B = B$

Problem 3. [5 points]Suppose the grading function $f : [0, 100] \rightarrow \{A, A-, B+, B, B-, C+, C, C-, D+, D, F\}$ is defined as follows:

Percentage		Grade
[93, 100]	—————→	A
[90, 93)	—————→	A–
[87, 90)	—————→	B+
[83, 87)	—————→	B
[80, 83)	—————→	B–
[77, 80)	—————→	C+
[73, 77)	—————→	C
[70, 73)	—————→	C–
[67, 70)	—————→	D+
[63, 67)	—————→	D
[0, 63)	—————→	F

Is this grading function a one-to-one correspondence? Prove or disprove.

Problem 4. [5 points]

The rotate-by-90-degree function $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ is defined as follows.

$$f(x, y) = (y, -x)$$

(This function is used in image processing for rotating a face or an image by 90 degree in clockwise direction.) Is this function a one-to-one correspondence? Prove or disprove.

Problem 5. [10 points]

Mention whether the following statements are true or false without giving any reasons. Assume that the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ are arbitrary functions.

- [1 point] $f \circ f = f$.
- [1 point] $f \circ g = g \circ f$.
- [1 point] f and g are both one-to-one correspondences implies that $f \circ g$ and $g \circ f$ are both one-to-one correspondences.
- [1 point] f and g are both onto does not imply that $f \circ g$ and $g \circ f$ are both onto.
- [1 point] f and g are both one-to-one implies that $f \circ g$ and $g \circ f$ are both one-to-one.
- [1 point] If $f \circ g$ is the identity function, then f and g are one-to-one correspondences.
- [1 point] Suppose f^{-1} exists. Then f^{-1} need not be an onto function.
- [1 point] The size of the set of all multiples of 6 is less than the size of the set of all multiples of 3.
- [1 point] The size of the set of rational numbers is the same as the size of the set of real numbers in the range $[0, 0.0000001]$.
- [1 point] The size of the set of real numbers in the range $[1, 2]$ is the same or larger than the size of the set of real numbers in the range $[1, 4]$.