**Final Exam (May 18, 2021, 08:00 - 10:45 am)**
**CSE 215: Foundations of Computer Science**
State University of New York at Stony Brook, Spring 2021
Instructor: Prof. Pramod Ganapathi

Total points = 60. Total questions = 11. Total pages = 2.

**Problem 1. [5 points]**

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

**Problem 2. [5 points]**

Construct a truth table for the following statement form: $(p \to q) \vee ((q \oplus r) \to \sim p)$.

**Problem 3. [5 points]**

Mention whether the following statements are true or false. Reasons are not needed.

(a) [1 point] $p \vee \sim p \equiv \mathbf{c}$

(b) [1 point] $p \vee (p \wedge q) \equiv p \wedge (p \vee q)$

(c) [1 point] $\mathbf{c} \equiv p \vee \mathbf{t}$

(d) [1 point] $p \wedge p \equiv p \vee p$

(e) [1 point] $p \wedge \mathbf{c} \equiv \sim \mathbf{t}$

**Problem 4. [5 points]**

Prove that the sum of the squares of any two consecutive odd integers is even.

**Problem 5. [5 points]**

Suppose that $x$ and $y$ are real numbers. Prove that $x = y$ if and only if $xy = (x+y)^2/4$.

**Problem 6. [5 points]**

Use algebra to prove that $0.3181818\ldots \times 0.888\ldots = 28/99$. The first number has 18 repeated and the second number has 8 repeated for an infinite number of times.

**Problem 7. [10 points]**

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

(b) [5 points] Consider the Fibonacci sequence: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$, for $n \geq 2$. For all integers $n \geq 2$,

$$f_{n+2} = 1 + \sum_{i=0}^{n} f_i.$$

**Problem 8. [5 points]**

Write the pseudocodes of the recursive algorithms for solving the Towers of Hanoi problem and computing the greatest common divisor of two integers.

**Problem 9. [5 points]**

Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

| Function | Domains | One-to-one function? | Onto function? |
|---|---|---|---|
| $f(x) = 3x$ | $f : \mathbb{Z} \to \mathbb{Z}$ | | |
| $f(x) = 3x$ | $f : \mathbb{R} \to \mathbb{R}$ | | |
| $f(x) = 3x^2$ | $f : \mathbb{Z} \to \mathbb{Z}$ | | |
| $f(x) = 3x^2$ | $f : \mathbb{R} \to \mathbb{R}$ | | |

**Problem 10. [5 points]**

Consider $f : \mathbb{R}^* \to \mathbb{R}^*$, where $\mathbb{R}^* = \mathbb{R}^+ \cup \{0\}$ is the set of all nonnegative real numbers, such that $f$ is a mapping from pounds to kilograms. A kilogram of mass in the International System of Units (SI) represents 2.2046226218 pounds. Is $f$ a one-to-one correspondence? Prove your answer.

**Problem 11. [5 points]**

Let $A$ be the set of all people. Let $R$ be the relation defined on $A$ as follows: For persons $p$ and $q$ in $A$, we have $p\ R\ q \Leftrightarrow p$ has the same birthday as $q$.
Is $R$ an equivalence relation? Prove your answer. If $R$ is an equivalence relation, what are the distinct equivalence classes of the relation?