Midterm Exam 2 (November 10, 2020, 01:15 pm - 02:35 pm)
CSE 215: Foundations of Computer Science
State University of New York at Stony Brook, Fall 2020
Instructor: Prof. Pramod Ganapathi

Total points = 45. Total questions = 6. Total pages = 2.

- Please write your full name and SBU student ID on the answer sheet.
- Please include the following integrity statement on your answer sheet:
  "Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the appropriate office and that I will be subjected to the maximum possible penalty permitted under University guidelines."

Problem 1. [20 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For integers $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

(b) [5 points] For integers $n \geq 1$,

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$$

(c) [5 points] $9^n + 3$ is divisible by 4 for integers $n \geq 1$.

(d) [5 points] Suppose that $g_1, g_2, g_3, \ldots$ is a sequence defined as follows:

$$\begin{aligned} &g_1 = 3, g_2 = 5 \\ &g_k = 3g_{k-1} - 2g_{k-2} \qquad \text{for all integers } k \geq 3. \end{aligned}$$

Prove that $g_n = 2^n + 1$ for all integers $n \geq 1$.

Problem 2. [5 points]

For each of the following statements, find a counterexample to show that the statement is false. Assume all sets are subsets of a universal set U.

1. For all sets A, B, and C, $(A \cap B) \cup C = A \cap (B \cup C)$.
2. For all sets A, B, and C, if $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.

Problem 3. [5 points]

Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$ and $Z = \{1, 2\}$. Use arrow diagrams to define functions.

1. Define a function $f : X \to Y$ that is one-to-one but not onto.

2. Define a function $g : X \to Z$ that is onto but not one-to-one.
3. Define a function $h : X \to X$ that is neither one-to-one nor onto.
4. Define a function $k : X \to X$ that is one-to-one and onto but is not the identity function on X.

Problem 4. [5 points]

Define $S : \mathbb{Z}^+ \to \mathbb{Z}^+$ by the rule: For all integers n,
$S(n) =$ the sum of the positive divisors of n.

1. Is S one-to-one? Prove or give a counterexample.
2. Is S onto? Prove or give a counterexample.
3. Is S one-to-one correspondence?

Problem 5. [5 points]

Define $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ by the formulas
$f(x) = x + 3$ and $g(x) = -x$ for all $x \in \mathbb{R}$.
Find $g \circ f, (g \circ f)^{-1}, g^{-1}, f^{-1}$, and $f^{-1} \circ g^{-1}$, and state how $(g \circ f)^{-1}$ and $f^{-1} \circ g^{-1}$ are related.

Problem 6. [5 points]

Prove that there is a one-to-one correspondence between the Fahrenheit scale and the Celsius/Centigrade scale for the interval defined between freezing point and boiling point of water. In other words, prove that there is a one-to-one correspondence between the real number line segments [32, 212] (the Fahrenheit scale) and [0, 100] (the Celsius scale). Using this one-to-one correspondence, come up with a formula to convert $x \in [32, 212]$ degree Fahrenheit to Celsius.