Final Exam (December 17, 2020, 08:00 - 10:35 am)
CSE 215: Foundations of Computer Science
State University of New York at Stony Brook, Fall 2020
Instructor: Prof. Pramod Ganapathi

Total points = 60. Total questions = 11. Total pages = 2.

- Please write your full name and SBU student ID on the answer sheet.
- Please include the following integrity statement on your answer sheet:
  "Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the appropriate office and that I will be subjected to the maximum possible penalty permitted under University guidelines."

Problem 1. [5 points]

Determine if the following deduction rule is valid.
$p \to (q \vee r)$
$\sim (p \to q)$
$\therefore\ r$

Problem 2. [5 points]

Suppose p and q are propositional statements. Prove that p and q are logically equivalent if and only if $p \leftrightarrow q$ is a tautology.

Problem 3. [5 points]

Verify using truth tables if the following two logical expressions are equivalent.
$(p \to q) \wedge (\sim p \to \sim q)$ and $\sim p \leftrightarrow \sim q$

Problem 4. [5 points]

Prove that $n^2+9n+27$ is odd for all natural numbers n. You can use any proof technique.

Problem 5. [5 points]

Prove using contradiction that the cube root of an irrational number is irrational.

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all natural numbers n,

$$1^2 \times 2 + 2^2 \times 3 + 3^2 \times 4 + \cdots + n^2 \times (n+1) = \frac{n(n+1)(n+2)(3n+1)}{12}$$

(b) [5 points] For all natural numbers n,

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{n \times (n+1)} = \frac{n}{n+1}$$

Problem 8. [5 points]

Write Euclid's recursive algorithm to compute the greatest common divisor (GCD) of two whole numbers. Show the step-by-step process to compute the GCD of 46 and 14 using the algorithm.

Problem 9. [5 points]

Functions F and G are defined by formulas. Find $G \circ F$ and $F \circ G$ and determine whether $G \circ F$ equals $F \circ G$.
$F(x) = x^5$ and $G(x) = x^{1/5}$ for all real numbers x.

Problem 10. [5 points]

Prove that the following set is countable using a diagram and a formula for the one-to-one correspondence function.
$\{\pm 1^1, \pm 2^2, \pm 3^3, \pm 4^4, \pm 5^5, \ldots\}$

Problem 11. [5 points]

Show how to find the units digit of 1357^{7531}.