Discrete Mathematics

(Predicate Logic)

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Predicates and Quantified Statements

What is a propositional function or predicate?

Definition

- A propositional function or predicate is a sentence that contains one or more variables
- A predicate is neither true nor false
- A predicate becomes a proposition when the variables are substituted with specific values
- The domain of a predicate variable is the set of all values that may be substituted for the variable

Examples

Symbol	Predicate	Domain	Propositions
$p(x)$	$x > 5$	$x \in \mathbb{R}$	$p(6), p(-3.6), p(0), \ldots$
$p(x, y)$	$x + y$ is odd	$x \in \mathbb{Z}, y \in \mathbb{Z}$	$p(4, 5), p(-4, -4), \ldots$
$p(x, y)$	$x^2 + y^2 = 4$	$x \in \mathbb{R}, y \in \mathbb{R}$	$p(-1.7, 8.9), p(-\sqrt{3}, -1), \ldots$

What is a truth set?

Definition

- A truth set of a predicate is the set of all values of the predicate that makes the predicate true
- If $p(x)$ is a predicate and x has domain D, then the truth set of $p(x)$ is the set of all elements of D that makes $p(x)$ true when the values are substituted for x. That is,

$$\text{Truth set of } p(x) = \{x \in D \mid p(x)\}$$

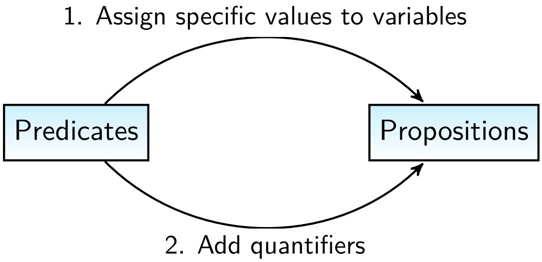
Examples

Symbol	Predicate	Domain	Truth set
$p(x)$	$x > 5$	$x \in \mathbb{R}$	$\{p(6), p(13.6), p(5.001), \ldots\}$
$p(x, y)$	$x + y$ is odd	$x \in \mathbb{Z}, y \in \mathbb{Z}$	$\{p(4, 5), p(-4, -3), \ldots\}$
$p(x, y)$	$x^2 + y^2 = 4$	$x \in \mathbb{R}, y \in \mathbb{R}$	$\{p(-2, 2), p(-\sqrt{3}, -1), \ldots\}$

Predicates to propositions

There are two methods to obtain propositions from predicates

1. Assign specific values to variables
2. Add quantifiers



What are quantifiers?

Definition

- Quantifiers are words that refer to quantities such as "all" or "some" and they tell for how many elements a given predicate is true

- Introduced into logic by logicians Charles Sanders Pierce and Gottlob Frege during late 19th century
- Two types of quantifiers:
  1. Universal quantifier ($\forall$)
  2. Existential quantifier ($\exists$)

Universal quantifier ($\forall$)

Definition

- Let $p(x)$ be a predicate and D be the domain of x
- A universal statement is a statement of the form

$$\forall x \in D, p(x)$$

- Forms:
  - "$p(x)$ is true for all values of x"
  - "For all x, $p(x)$"
  - "For each x, $p(x)$"
  - "For every x, $p(x)$"
  - "Given any x, $p(x)$"
- It is true if $p(x)$ is true for each x in D; It is false if $p(x)$ is false for at least one x in D
- A counterexample to the universal statement is the value of x for which $p(x)$ is false

Universal quantifier ($\forall$)

Examples

Universal st.s	Domain	Truth value	Method
$\forall x \in D, x^2 \geq x$	$D = \{1, 2, 3\}$	True	Method of exhaustion
$\forall x \in \mathbb{R}, x^2 \geq x$	$\mathbb{R}$	False	Counterexample $x = 0.1$

Caution

- Method of exhaustion cannot be used to prove universal statements for infinite sets

Existential quantifier ($\exists$)

Definition

- Let $p(x)$ be a predicate and D be the domain of x
- An existential statement is a statement of the form

$$\exists x \in D, p(x)$$

- Forms:
  - "There exists an x such that $p(x)$"
  - "For some x, $p(x)$"
  - "We can find an x, such that $p(x)$"
  - "There is some x such that $p(x)$"
  - "There is at least one x such that $p(x)$"
- It is true if $p(x)$ is true for at least one x in D; It is false if $p(x)$ is false for all x in D
- A counterproof to the existential statement is the proof to show that $p(x)$ is true is for no x

Existential quantifier ($\exists$)

Examples

Universal st.s	Domain	Truth value	Method
$\exists x \in D, x^2 \geq x$	$D = \{1, 2, 3\}$	True	Method of exhaust.
$\exists x \in \mathbb{R}, x^2 \geq x$	$\mathbb{R}$	True	Example
$\exists x \in \mathbb{Z}, x + 1 \leq x$	$\mathbb{Z}$	False	How?

Formal and informal languages

Example

- $\forall x \in \mathbb{R}, x^2 \geq 0$
  - Every real number has a nonnegative square
  - All real numbers have nonnegative squares
  - Any real number has a nonnegative square
  - The square of each real number is nonnegative
  - No real numbers have negative squares
  - x^2 is nonnegative for every real x
  - x^2 is not less than zero for each real number x

Universal conditional statement $(\forall, \rightarrow)$

Definition

- A universal conditional statement is of the form

$$\forall x, \text{ if } p(x) \text{ then } q(x)$$

Examples

- $\forall x \in \mathbb{R}$, if $x > 2$ then $x^2 > 4$
- $\forall$ real number x, if x is an integer then x is rational
  $\forall$ integer x, x is rational ▷ Logically equivalent
- $\forall x$, if x is a square then x is a rectangle
  $\forall$ square x, x is a rectangle ▷ Logically equivalent
- $\forall x \in U$, if $p(x)$ then $q(x)$
  $\forall x \in D$, $q(x)$ ▷ Logically equivalent
  (where, $D = \{x \in U \mid p(x) \text{ is true}\}$)

- Can be extended to existential conditional statement $(\exists, \rightarrow)$

Implicit quantification $(\Rightarrow, \Leftrightarrow)$

Examples

- If a number is an integer, then it is a rational number
  Implicit meaning: $\forall$ number x, if x is an integer, x is rational
- The number 10 can be written as a sum of two prime numbers
  Implicit meaning: $\exists$ prime numbers p and q such that $10 = p+q$
- If $x > 2$, then $x^2 > 4$
  Implicit meaning: $\forall$ real x, if $x > 2$, then $x^2 > 4$

Definition

- Let $p(x)$ and $q(x)$ be predicates and D be the common domain of x. Then implicit quant. symbols $\Rightarrow, \Leftrightarrow$ are defined as:

$$p(x) \Rightarrow q(x) \equiv \forall x, p(x) \rightarrow q(x)$$
$$p(x) \Leftrightarrow q(x) \equiv \forall x, p(x) \leftrightarrow q(x)$$

Implicit quantification $(\Rightarrow, \Leftrightarrow)$

Problem

- $q(n)$: n is a factor of 8; $r(n)$: n is a factor of 4
  $s(n)$: $n < 5$ and $n \neq 3$
  Domain of n is $\mathbb{Z}^+$ (i.e., positive integers)
- What are the relationships between $q(n)$, $r(n)$, and $s(n)$ using symbols $\Rightarrow$ and $\Leftrightarrow$?

Implicit quantification $(\Rightarrow, \Leftrightarrow)$

Problem

- $q(n)$: n is a factor of 8; $r(n)$: n is a factor of 4
  $s(n)$: $n < 5$ and $n \neq 3$
  Domain of n is $\mathbb{Z}^+$ (i.e., positive integers)
- What are the relationships between $q(n)$, $r(n)$, and $s(n)$ using symbols $\Rightarrow$ and $\Leftrightarrow$?

Solution

- Truth set of $q(n) = \{1, 2, 4, 8\}$; Truth set of $r(n) = \{1, 2, 4\}$; Truth set of $s(n) = \{1, 2, 4\}$
- $\forall n$ in $\mathbb{Z}^+, r(n) \rightarrow q(n)$ i.e., $r(n) \Rightarrow q(n)$
  i.e., "n is a factor of 4" $\Rightarrow$ "n is a factor of 8"
- $\forall n$ in $\mathbb{Z}^+, r(n) \leftrightarrow s(n)$ i.e., $r(n) \Leftrightarrow s(n)$
  i.e., "n is a factor of 4" $\Leftrightarrow$ "$n < 5$ and $n \neq 3$"
- $\forall n$ in $\mathbb{Z}^+, s(n) \rightarrow q(n)$ i.e., $s(n) \Rightarrow q(n)$
  i.e., "$n < 5$ and $n \neq 3$" $\Rightarrow$ "n is a factor of 8"

Negation of quantified statements ($\sim$)

Definition

- Formally,

$$\sim (\forall x \in D, p(x)) \equiv \exists x \in D, \sim p(x)$$
$$\sim (\exists x \in D, p(x)) \equiv \forall x \in D, \sim p(x)$$

- Negation of a universal statement (“all are”) is logically equivalent to an existential statement (“there is at least one that is not”)
  Negation of an existential statement (“some are”) is logically equivalent to a universal statement (“all are not”)

Methods

Two methods to avoid errors while finding negations:

1. Write the statements formally and then take negations
2. Ask “What exactly would it mean for the given statement to be false?”

Negation of quantified statements ($\sim$)

Examples

- All mathematicians wear glasses
  Negation (incorrect): No mathematician wears glasses
  Negation (incorrect + ambiguous): All mathematicians do not wear glasses
  Negation (correct): There is at least one mathematician who does not wear glasses
- Some snowflakes are the same
  Negation (incorrect):: Some snowflakes are different
  Negation (correct):: All snowflakes are different

Negation of quantified statements ($\sim$)

Examples

- $\forall$ primes p, p is odd
  Negation: $\exists$ primes p, p is even
- $\exists$ triangle T, sum of angles of T equals 200°
  $\forall$ triangles T, sum of angles of T does not equal 200°
- No politicians are honest
  Formal statement: $\forall$ politicians x, x is not honest
  Formal negation: $\exists$ politician x, x is honest
  Informal negation: Some politicians are honest
- 1357 is not divisible by any integer between 1 and 37
  Formal statement: $\forall n \in [1, 37]$, 1357 is not divisible by n
  Formal negation: $\exists n \in [1, 37]$, 1357 is divisible by n
  Informal negation: 1357 is divisible by some integer between 1 and 37

Negation of universal conditional statements

Definition

- Formally,

$$\begin{aligned} \sim (\forall x, p(x) \to q(x)) &\equiv \exists x, \sim (p(x) \to q(x)) \\ &\equiv \exists x, (p(x) \wedge \sim q(x)) \end{aligned}$$

Examples

- $\forall$ real x, if $x > 10$, then $x^2 > 100$.
  Negation: $\exists$ real x such that $x > 10$ and $x^2 \leq 100$.
- If a computer program has more than 100,000 lines, then it contains a bug.
  Negation: There is at least one computer program that has more than 100,000 lines and does not contain a bug.

Relation between quantifiers $(\forall, \exists)$ and $(\wedge, \vee)$

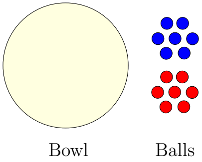
Relation

- Universal statements are generalizations of and statements
  Existential statements are generalizations of or statements
- If $p(x)$ is a predicate and $D = \{x_1, x_2, \ldots, x_n\}$ is the domain of x, then

$$\forall x \in D, p(x) \equiv p(x_1) \wedge p(x_2) \wedge \cdots \wedge p(x_n)$$
$$\exists x \in D, p(x) \equiv p(x_1) \vee p(x_2) \vee \cdots \vee p(x_n)$$

Vacuous truth of universal statements

Problem



- Consider the bowl and the balls
- Consider the statement:
  All the balls in the bowl are blue
- Is the statement true?

Solution

- The statement is false iff its negation is true
- Negation: There exists a ball in the bowl that is not blue
- The negation is false; So, the statement is true, by default

Definition

- A statement of the form

$$\forall x \text{ in } D, \text{ if } p(x), \text{ then } q(x)$$

  is vacuously true or true by default, if and only if $p(x)$ is false for all x in D

Universal conditional statements $(\forall x, p(x) \to q(x))$

Definitions

- Statement: $\forall x$, if $p(x)$ then $q(x)$
- Contrapositive of the statement is $\forall x$, if $\sim q(x)$ then $\sim p(x)$
- Converse of the statement is $\forall x$, if $q(x)$ then $p(x)$
- Inverse of the statement is $\forall x$, if $\sim p(x)$ then $\sim q(x)$

Identities

- Conditional $\equiv$ Contrapositive $\quad \triangleright$ Useful for proofs
- Conditional $\not\equiv$ Converse
- Conditional $\not\equiv$ Inverse
- Converse $\equiv$ Inverse

Formulas

- $\forall x, p(x) \to q(x) \equiv \forall x, \sim q(x) \to \sim p(x)$ $\quad \triangleright$ Useful for proofs
- $\forall x, p(x) \to q(x) \not\equiv \forall x, q(x) \to p(x)$
- $\forall x, p(x) \to q(x) \not\equiv \forall x, \sim p(x) \to \sim q(x)$
- $\forall x, q(x) \to p(x) \equiv \forall x, \sim p(x) \to \sim q(x)$

Universal conditional statement $\forall x, p(x) \rightarrow q(x)$

Definitions

- $\forall x, p(x)$ is a sufficient condition for $q(x)$ means
  $\forall x$, if $p(x)$ then $q(x)$
- $\forall x, p(x)$ is a necessary condition for $q(x)$ means
  $\forall x$, if $\sim p(x)$ then $\sim q(x) \equiv \forall x$, if $q(x)$ then $p(x)$
- $\forall x, p(x)$ only if $q(x)$ means
  $\forall x$, if $\sim q(x)$ then $\sim p(x) \equiv \forall x$, if $p(x)$ then $q(x)$

Example

- For real x, $x = 1$ is a sufficient condition for $x^2 = 1$
  i.e., $\forall x$, if $x = 1$ then $x^2 = 1$ ▷ True
- For real x, $x^2 = 1$ is a necessary condition for $x = 1$
  i.e., $\forall x$, if $x^2 \neq 1$ then $x \neq 1$ ▷ True
- For real x, $x = 1$ only if $x^2 = 1$
  i.e., $\forall x$, if $x^2 \neq 1$ then $x \neq 1$ ▷ True

Statements with Multiple Quantifiers

Statements with multiple quantifiers

Problem

- What is the interpretation for the following statement?
  "There is a person supervising every detail of the production process."

Ambiguous interpretations

1. There is one single person who supervises all the details of the production process.
   $\exists$ person p such that $\forall$ detail d, p supervises d
2. For any particular production detail, there is a person who supervises that detail, but there might be different supervisors for different details.
   $\forall$ detail d, $\exists$ person p such that p supervises d

Statements with multiple quantifiers

Definitions

1. Statement form:

$$\forall x \in D, \exists y \in E \text{ such that } P(x, y)$$

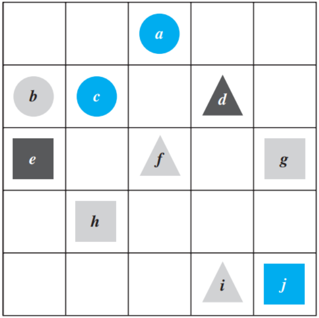
   Interpretation: Allow someone else to pick whatever element x in D they wish. Then, you must find an element y in E that "works" for that particular x.

2. Statement form:

$$\exists x \in D \text{ such that } \forall y \in E, P(x, y)$$

   Interpretation: Your job is to find one particular x in D that will "work" no matter what y in E anyone might choose to challenge you with.

Example: Tarski world



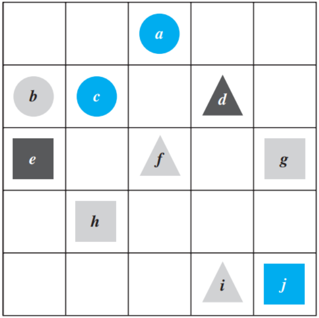
Problem

- For all triangles x, there is a square y such that x and y have the same color. Truth value?

Answer

- True. How?

Example: Tarski world



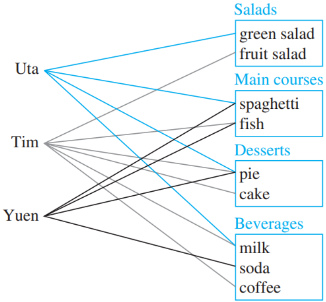
Problem

- There is a triangle x such that for all circles y, x is to the right of y. Truth value?

Answer

- True. How?

Example: College cafeteria



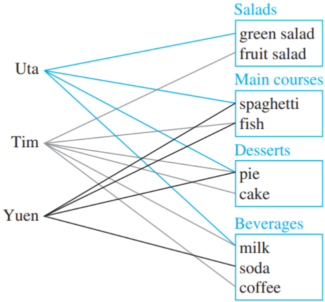
Problem

- $\exists$ an item I such that $\forall$ students S, S chose I.
- Informal statement? Truth value?

Solution

- There is an item that was chosen by every student.
- True. How?

Example: College cafeteria



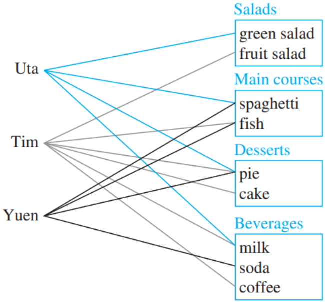
Problem

- $\exists$ a student S such that $\forall$ items I, S chose I.
- Informal statement? Truth value?

Solution

- There is a student who chose every available item.
- False. How?

Example: College cafeteria



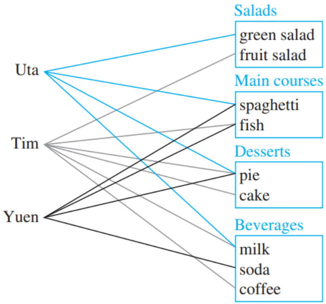
Problem

- $\exists$ a student S such that $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.
- Informal statement? Truth value?

Solution

- There is a student who chose at least one item from every station.
- True. How?

Example: College cafeteria



Problem

- $\forall$ students S and $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.
- Informal statement? Truth value?

Solution

- Every student chose at least one item from every station.
- False. How?

Translating from informal to formal language

Problem

- Every nonzero real number has a reciprocal.
- There is a real number with no reciprocal.
- There is a smallest positive integer.
- There is no smallest positive real number.

Translating from informal to formal language

Problem

- Every nonzero real number has a reciprocal.
- There is a real number with no reciprocal.
- There is a smallest positive integer.
- There is no smallest positive real number.

Solution

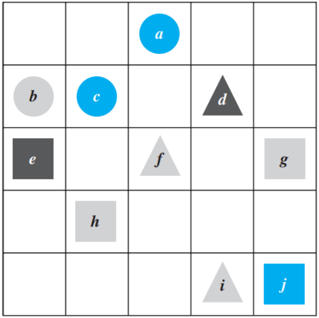
- $\forall$ nonzero real numbers u, $\exists$ a real number v such that $uv = 1$.
- $\exists$ a real number c such that $\forall$ real numbers d, $cd \neq 1$.
- $\exists$ a positive integer m such that $\forall$ positive integers n, $m \leq n$.
- $\forall$ positive real numbers x, $\exists$ a positive real number y such that $y < x$.

Negations of multiply-quantified statements

Definitions

- $\sim (\forall x$ in D, $\exists y$ in E such that $P(x,y))$
  $\equiv \exists x$ in D such that $\sim (\exists y$ in E such that $P(x,y))$
  $\equiv \exists x$ in D such that $\forall y$ in E, $\sim P(x,y)$
- $\sim (\exists x$ in D such that $\forall y$ in E, $P(x,y))$
  $\equiv \forall x$ in D, $\sim (\forall y$ in E, $P(x,y))$
  $\equiv \forall x$ in D, $\exists y$ in E such that $\sim P(x,y)$

Example: Tarski world



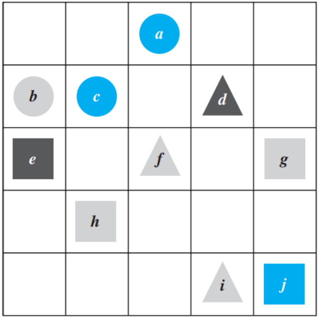
Problem

- For all squares x, there is a circle y such that x and y have the same color. Negation?

Answer

- $\exists$ a square x such that $\forall$ circles y, x and y do not have the same color. True. How?

Example: Tarski world



Problem

- There is a triangle x such that for all squares y, x is to the right of y. Negation?

Answer

- $\forall$ triangles x, $\exists$ a square y such that x is not to the right of y. True. How?

Order of quantifiers

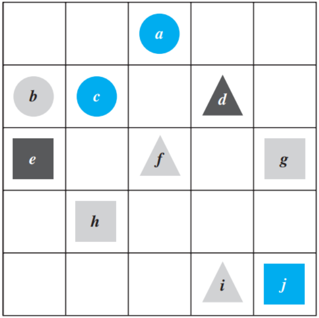
Order

- The order of quantifiers are important when multiple quantifiers are involved

Example

- $\forall$ people x, $\exists$ a person y such that x loves y.
  Quite possible.
- $\exists$ a person y such that $\forall$ people x, x loves y.
  Quite impossible.

Order of quantifiers



Example

- For every square x there is a triangle y such that x and y have different colors ▷ True
- There exists a triangle y such that for every square x, x and y have different colors. ▷ False

Order of quantifiers

Example

Suppose $\mathbb{R}^*$ is a set of nonzero real numbers.

- $\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}^*(xy < 1)$ ▷ True

  Two cases:

  a. For $x \leq 0$, let $y = 1$, then $xy < 1$

  b. For $x > 0$, let $y = 1/(x+1)$, then $xy < 1$

- $\exists y \in \mathbb{R}^*, \forall x \in \mathbb{Z}\ (xy < 1)$ ▷ False

  Two cases:

  a. For $y > 0$, if integer $x \geq 1/y$, then $xy \not< 1$

  b. For $y < 0$, if integer $x \leq 1/y$, then $xy \not< 1$

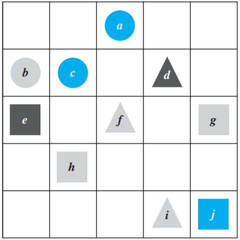
  In both the cases, an adversary can choose an integer that makes the predicate false. Hence, the quantified statement is false.

Formal logical notation

Definitions

- $\forall x$ in D, $P(x)$
  $\equiv \forall x(x \text{ in } D \to P(x))$
- $\exists x$ in D such that $P(x)$
  $\equiv \exists x(x \text{ in } D \land P(x))$

Example: Tarski world



Definitions

- Triangle(x): x is a triangle
- Circle(x): x is a circle
- Square(x): x is a square
- Blue(x): x is blue
- Gray(x): x is gray
- Black(x): x is black
- RightOf(x, y): x is to the right of y
- Above(x, y): x is above y
- SameColor(x, y): x has the same color as y

Example: Tarski world

Problem

- For all circles x, x is above f.
- Formal statement? Formal negation?

Solution

- Formal statement
  $\forall x(\text{Circle}(x) \to \text{Above}(x, f))$
- Formal negation
  $\sim (\forall x(\text{Circle}(x) \to \text{Above}(x, f)))$
  $\equiv \exists x \sim (\text{Circle}(x) \to \text{Above}(x, f))$
  $\equiv \exists x(\text{Circle}(x) \wedge \sim \text{Above}(x, f))$

Example: Tarski world

Problem

- There is a square x such that x is black.
- Formal statement? Formal negation?

Solution

- Formal statement
  $\exists x(\text{Square}(x) \wedge \text{Black}(x))$
- Formal negation
  $\sim (\exists x(\text{Square}(x) \wedge \text{Black}(x)))$
  $\equiv \forall x \sim (\text{Square}(x) \wedge \text{Black}(x))$
  $\equiv \forall x(\sim \text{Square}(x) \vee \sim \text{Black}(x))$

Example: Tarski world

Problem

- For all circles x, there is a square y such that x and y have the same color.
- Formal statement? Formal negation?

Solution

- Formal statement
  $\forall x(\text{Circle}(x) \rightarrow \exists y(\text{Square}(y) \wedge \text{SameColor}(x,y)))$
- Formal negation
  $\sim (\forall x(\text{Circle}(x) \rightarrow \exists y(\text{Square}(y) \wedge \text{SameColor}(x,y))))$
  $\equiv \exists x \sim (\text{Circle}(x) \rightarrow \exists y(\text{Square}(y) \wedge \text{SameColor}(x,y)))$
  $\equiv \exists x(\text{Circle}(x) \wedge \sim (\exists y(\text{Square}(y) \wedge \text{SameColor}(x,y))))$
  $\equiv \exists x(\text{Circle}(x) \wedge \forall y(\sim (\text{Square}(y) \wedge \text{SameColor}(x,y))))$
  $\equiv \exists x(\text{Circle}(x) \wedge \forall y(\sim \text{Square}(y) \vee \sim \text{SameColor}(x,y)))$

Example: Tarski world

Problem

- There is a square x such that for all triangles y, x is to right of y.
- Formal statement? Formal negation?

Solution

- Formal statement
  $\exists x(\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \rightarrow \text{RightOf}(x, y)))$
- Formal negation
  $\sim (\exists x(\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \rightarrow \text{RightOf}(x, y))))$
  $\equiv \forall x \sim (\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \rightarrow \text{RightOf}(x, y)))$
  $\equiv \forall x(\sim \text{Square}(x) \vee \sim (\forall y(\text{Triangle}(y) \rightarrow \text{RightOf}(x, y))))$
  $\equiv \forall x(\sim \text{Square}(x) \vee \exists y(\sim (\text{Triangle}(y) \rightarrow \text{RightOf}(x, y))))$
  $\equiv \forall x(\sim \text{Square}(x) \vee \exists y(\text{Triangle}(y) \wedge \sim \text{RightOf}(x, y)))$

Arguments with Quantified Statements

Universal instantiation

Definition

- If some property is true of everything in a set, then it is true of any particular thing in the set.

Example

- All men are mortal.
  Socrates is a man.
  $\therefore$ Socrates is mortal.

Rule of inference: Universal modus ponens

Definition

- It has the form:
  $\forall x$, if $P(x)$ then $Q(x)$
  $P(a)$ for a particular a
  $\therefore Q(a)$
- Used in direct proofs

Example

- Informal argument
  If an integer is even, then its square is even.
  k is a particular integer that is even.
  $\therefore k^2$ is even
- Formal argument
  $\forall x$, if $E(x)$ then $S(x)$ ▷ $E(x)$? $S(x)$? k?
  $E(k)$ for a particular k
  $\therefore S(k)$

Rule of inference: Universal modus tollens

Definition

- It has the form:
  $\forall x$, if $P(x)$ then $Q(x)$
  $\sim Q(a)$ for a particular a
  $\therefore \sim P(a)$
- Used in proof by contradiction

Example

- Informal argument
  All human beings are mortal.
  Zeus is not mortal.
  $\therefore$ Zeus is not human.
- Formal argument
  $\forall x$, if $H(x)$ then $M(x)$ $\qquad \triangleright$ $H(x)$? $M(x)$? Z?
  $\sim M(Z)$
  $\therefore \sim H(Z)$

Fallacy: Converse and inverse errors

Definition

- Converse error has the form:
  $\forall x$, if $P(x)$ then $Q(x)$
  $Q(a)$ for a particular a
  $\therefore P(a)$
- Inverse error has the form:
  $\forall x$, if $P(x)$ then $Q(x)$
  $\sim P(a)$ for a particular a
  $\therefore \sim Q(a)$

Fallacy: Converse error

Example

- Law

  All the town criminals frequent the Hot Life bar.

  John frequents the Hot Life bar.

  $\therefore$ John is one of the town criminals.

  Suspect John but don't convict him.

- Medicine

  For all x, if x has pneumonia, then x has a fever and chills, coughs deeply, and feels exceptionally tired and miserable.

  John has a fever and chills, coughs deeply, and feels exceptionally tired and miserable.

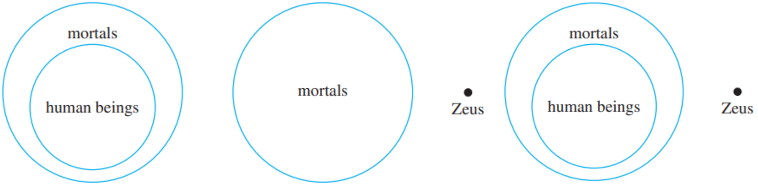
  $\therefore$ John has pneumonia.

  Diagnosis of pneumonia is a strong possibility, though not a certainty.

Using diagrams to test validity: Example 1

Example

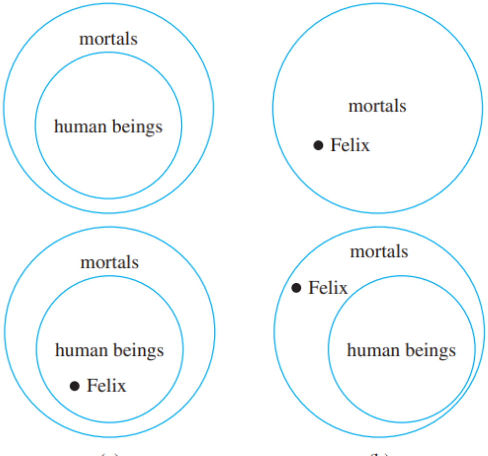
- All human beings are mortal.
  Zeus is not mortal.
  ∴ Zeus is not human. ▷ Valid (Modus tollens)



Using diagrams to test validity: Example 2

Example

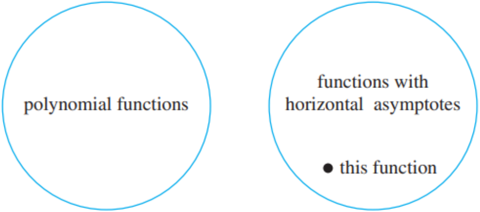
- All human beings are mortal.
  Felix is mortal.
  ∴ Felix is a human being. ▷ Invalid (Converse error)



Using diagrams to test validity: Example 3

Example

- No polynomial functions have horizontal asymptotes.
  This function has a horizontal asymptote.
  $\therefore$ This function is not a polynomial function. $\triangleright$ Valid



Equivalence

- $P(x)$: x is a polynomial function
  $Q(x)$: x does not have a horizontal asymptote
  $\forall x$, if $P(x)$ then $Q(x)$
  $\sim Q(a)$ for a particular a
  $\therefore \sim P(a)$ $\triangleright$ Modus tollens