Problem 1. [12 points]
Rewrite the following statements in if-then form in two ways, one of which is the contrapositive of the other.

(a) [4 points] Either you get to work on time or you are fired.
(b) [4 points] Sam will be allowed on Signe’s racing boat only if he is an expert sailor.
(c) [4 points] A necessary condition for this computer program to be correct is that it does not produce error messages during translation.

Problem 2. [8 points]
Determine whether the statements in (i) and (ii) are logically equivalent.

(a) [4 points] Assume $x$ is a particular real number. (i) $x < 2$ or it is not the case that $1 < x < 3$ and (ii) $x \leq 1$ or either $x < 2$ or $x \geq 3$.
(b) [4 points] Assume $p$ and $q$ are propositions. (i) $p$ unless $q$ and (ii) $q$ unless $p$.

Problem 3. [4 points]
Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

(a) [3 points] If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.
Neither of these two numbers is divisible by 6.
∴ The product of these two numbers is not divisible by 6.
(b) [1 point] If this computer program is correct, then it produces the correct output when run with the test data my teacher gave me.
This computer program produces the correct output when run with the test data my teacher gave me.
∴ This computer program is correct.

Problem 4. [8 points]
An island consists of two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives $A$ and $B$ who speak to you as follows:
$A$ says: Both of us are knights.
$B$ says: $A$ is knave.

What are $A$ and $B$? Please include every tiny step to logically derive the answer with assumptions or definitions or rules of inference used at every step.
Problem 5. [4 points]
Rewrite the following statement in the form: \( \forall --- x, --- \); and write its negation:
No irrational numbers are integers.

Problem 6. [4 points]
Write negations for the following statements:
(a) [2 points] If a function is differentiable, then it is continuous.
(b) [2 points] \( \forall x \in \mathbb{R}, \) if \( x(x + 1) > 0, \) then \( x > 0 \) or \( x < -1. \)

Problem 7. [4 points]
Rewrite the statement in English without using the symbol \( \forall \) or \( \exists \) or variables and expressing your answer as simply as possible, and then write a negation for the statement.
\( \forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+, \) such that \( y > x. \)

Problem 8. [4 points]
Determine the truth or falsity of the following statements. Justify your answers as best as you can.
(a) [2 points] \( \forall x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+, \) such that \( x = y + 1. \)
(b) [2 points] \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \) such that \( xy = 1. \)

Problem 9. [4 points]
Indicate whether the argument is valid or invalid. Support your answers by drawing diagrams.
No college cafeteria food is good.
No good food is wasted.
\( \therefore \) No college cafeteria food is wasted.

Problem 10. [4 points]
Prove the statement. There are real numbers \( a \) and \( b \) such that \( \sqrt{a + b} = \sqrt{a} + \sqrt{b}. \)