Database Design with The Relational Normalization Theory

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Limitations of E-R Designs

- Provides a set of guidelines, does not result in a unique database schema
- Does not provide a way of evaluating alternative schemas
- Normalization theory provides a mechanism for analyzing and refining the schema produced by an E-R design

Redundancy

- Dependencies between attributes cause redundancy
  - Ex. All addresses in the same town have the same zip code

SSN	*Name*	*Town*	*Zip*
1234	Joe	Stony Brook	11790
4321	Mary	Stony Brook	11790
5454	Tom	Stony Brook	11790
	………………………		

Redundancy

Redundancy

Set attributes can also cause redundancy.

In the ER Model:

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	{biking, hiking}

But, they are represented as multiple tuples in the Relational Model:

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	biking
1111	Joe	123 Main	hiking

. . . .

Redundancy

Anomalies

- Redundancy leads to anomalies:
  - **Update anomaly**: A change in *Address* must be made in several places in the example with hobbies
  - **Deletion anomaly**: Suppose a person gives up all hobbies. Do we:
    - Set Hobby attribute to null? No, since *Hobby* is part of key
    - Delete the entire row? No, since we lose other information in the row.
    - **So, we cannot represent this person.**
  - **Insertion anomaly**: *Hobby* value must be supplied for any inserted row since *Hobby* is part of key.
    - So, we cannot inset a person without hobbies.

Decomposition

- **Solution for eliminating redundencies**: we use **<u>two relations</u>** to store Person information
  - Person1(<u>*SSN*</u>, *Name*, *Address*)
  - Hobbies(<u>*SSN*, *Hobby*</u>)
- **The decomposition is more general:** people without hobbies can now be described
- **No update anomalies:**
  - Name and address stored once
  - A hobby can be separately supplied or deleted

Normalization Theory

- The result of E-R analysis need further refinement!
- Appropriate decomposition can solve problems!
- The underlying theory is referred to as *normalization theory* and is based on *functional dependencies* (and other kinds, like *multivalued dependencies*)

Functional Dependencies

- **Definition:** A *functional dependency* (FD) on a relation schema **R** is a **constraint** $X \rightarrow Y$, where X and Y are subsets of attributes of **R.**
- **Definition**: An FD $X \rightarrow Y$ is *satisfied* in an instance **r** of **R** if for every pair of tuples, t and s: if t and s agree on all attributes in X then they must agree on all attributes in Y
  - Key constraint is a special kind of functional dependency: all attributes of relation occur on the right-hand side of the FD:
    - $SSN \rightarrow SSN, Name, Address$

Functional Dependencies

- $Address \rightarrow ZipCode$
  - Stony Brook's ZIP is 11733
- $ArtistName \rightarrow BirthYear$
  - Picasso was born in 1881
- $Autobrand \rightarrow Manufacturer, Engine\ type$
  - Pontiac is built by General Motors with gasoline engine
  - Volt is built by Chevy with electric engine
- $Author, Title \rightarrow PublicationDate$
  - Shakespeare's Hamlet published in 1600

Functional Dependency Running Example

- Consider a brokerage firm that allows multiple clients to share an account, but each account is managed from a single office and a client can have no more than one account in an office.
  - HasAccount(*AcctNum*, *ClientId*, *OfficeId*)
    - keys are: (*AcctNum*, *ClientId*), (*ClientId*, *OfficeId*)
      - *AcctNum*, *ClientId* → *AcctNum*, *ClientId*, *OfficeId*
      - *ClientId*, *OfficeId* → *AcctNum*, *ClientId*, *OfficeId*
    - *AcctNum* → *OfficeId*
      - <u>Thus, attribute values need not depend only on key values</u>

Entailment, Closure, Equivalence

- **Definition**: If $\boldsymbol{F}$ is a set of FDs on schema **R** and f is another FD on **R**, then *$\boldsymbol{F}$ entails f* if every instance **r** of **R** that satisfies every FD in $\boldsymbol{F}$ also satisfies f
  - Example: $\boldsymbol{F} = \{A \to B, B \to C\}$ and f is $A \to C$
    - If $Town \to Zip$ and $Zip \to AreaCode$ then $Town \to AreaCode$
- **Definition**: The *closure* of $\boldsymbol{F}$, denoted $\boldsymbol{F}^+$, is the set of all FDs entailed by $\boldsymbol{F}$
- **Definition**: $\boldsymbol{F}$ and $\boldsymbol{G}$ are *equivalent* if $\boldsymbol{F}$ entails $\boldsymbol{G}$ and $\boldsymbol{G}$ entails $\boldsymbol{F}$

Entailment, Closure, Equivalence

- Satisfaction, entailment, and equivalence are *semantic* concepts – defined in terms of the actual relations in the "real world."
  - They define *what these notions are*, **not** how to compute them
  - **Solution**: find algorithmic, *syntactic* ways to compute these notions
    - *Important*: The syntactic solution must be "correct" with respect to the semantic definitions
    - Correctness has two aspects: *soundness* and *completeness*

Armstrong's Axioms for FDs

- **Reflexivity**: If $Y \subseteq X$ then $X \rightarrow Y$ (trivial FD)
  - $Name, Address \rightarrow Name$
- **Augmentation**: If $X \rightarrow Y$ then $XZ \rightarrow YZ$
  - If $Town \rightarrow Zip$ then $Town, Name \rightarrow Zip, Name$
- **Transitivity**: If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

- The Armstrong's Axioms are the *syntactic* way of computing and testing the various properties of FDs.

Soundness

- Armstrong's axioms are ***sound***: If an FD f: $X \rightarrow Y$ can be derived from a set of FDs $\boldsymbol{F}$ using the axioms, then f holds in every relation that satisfies every FD in $\boldsymbol{F}$.
- Example: Given $X \rightarrow Y$ and $X \rightarrow Z$ then

$$\begin{array}{ll} X \rightarrow XY & \textit{Augmentation by } X \\ YX \rightarrow YZ & \textit{Augmentation by } Y \\ X \rightarrow YZ & \textit{Transitivity} \end{array}$$

  - Thus, $X \rightarrow Y Z$ is satisfied in every relation where both $X \rightarrow Y$ and $X \rightarrow Z$ are satisfied
    - We have derived the *union rule* for FDs: we can take the union of the RHSs of FDs that have the same LHS

Completeness

- Armstrong's Axioms are *complete*: If ***F*** entails *f* , then *f* can be derived from ***F*** using the axioms
- A consequence of completeness is the following (naïve) algorithm to determining if ***F*** entails *f*:
  - Algorithm: Use the axioms in all possible ways to generate $\boldsymbol{F}^+$ (the closure of ***F***, i.e., the set of possible FD's is finite so this can be done) and see if *f* is in $\boldsymbol{F}^+$

Correctness

- The notions of *soundness* and *completeness* link the syntax (Armstrong's axioms) with semantics (the definitions in terms of relational instances)
- This is a precise way of saying that the algorithm for entailment based on the axioms is "*correct*" with respect to the definitions

Generating F^+

F

$AB \rightarrow C$

$A \rightarrow D \xrightarrow{aug} AB \rightarrow BD$

$AB \rightarrow C,\ AB \rightarrow BD \xrightarrow{union} AB \rightarrow BCD$

$D \rightarrow E \xrightarrow{aug} BCD \rightarrow BCDE$

$AB \rightarrow BCD,\ BCD \rightarrow BCDE \xrightarrow{trans} AB \rightarrow BCDE \xrightarrow{decomp} AB \rightarrow CDE$

Thus, $AB \rightarrow BD$, $AB \rightarrow BCD$, $AB \rightarrow BCDE$, and $AB \rightarrow CDE$ are all elements of F^+ (part-of, there are other FDs: $AC \rightarrow CD$, $AE \rightarrow ED$, *etc.*)

Very costly procedure for proving entailment.

Attribute Closure

- Calculating *attribute closure* leads to a more efficient way of checking entailment
- The *attribute closure* of a set of attributes, X, with respect to a set of functional dependencies, $\boldsymbol{F}$, (denoted $X^{+}{}_{\boldsymbol{F}}$) is the set of all attributes, A, such that $X \rightarrow A$ is entailed by $\boldsymbol{F}$
- $X^{+}{}_{\boldsymbol{F1}}$ is not necessarily the same as $X^{+}{}_{\boldsymbol{F2}}$ if $\boldsymbol{F1} \neq \boldsymbol{F2}$
- *Attribute closure and entailment*:
  - Algorithm: Given a set of FDs, $\boldsymbol{F}$, $\boldsymbol{F}$ entails $X \rightarrow Y$ if and only if $X^{+}{}_{\boldsymbol{F}} \supseteq Y$

Computation of the Attribute Closure X^+_F

$closure := X;$ // since $X \subseteq X^+_{\boldsymbol{F}}$

repeat

  $old := closure;$

  if there is an FD $Z \rightarrow V$ in $\boldsymbol{F}$ such that

    $Z \subseteq closure$ **and** $V \cap closure \neq \emptyset$

   then $closure := closure \cup V$

until $old = closure$

<u>Entailment algorithm:</u>

If $T \subseteq X^+_{\boldsymbol{F}}$ then $X \rightarrow T$ is entailed by $\boldsymbol{F}$

Example: Computation of Attribute Closure

Example: Compute the attribute closure of AB with respect to the set of FDs **F**:

$$AB \rightarrow C \quad \text{(a)}$$
$$A \rightarrow D \quad \text{(b)}$$
$$D \rightarrow E \quad \text{(c)}$$
$$AC \rightarrow B \quad \text{(d)}$$

Solution:

Initially: $closure = \{AB\}$

Using (a): $closure = \{ABC\}$

Using (b): $closure = \{ABCD\}$

Using (c): $closure = \{ABCDE\}$

Computing Attribute Closure Examples

$$\begin{aligned} \boldsymbol{F}:\ & AB \to C \\ & A \to D \\ & D \to E \\ & AC \to B \end{aligned}$$

X	$X_{\boldsymbol{F}}^{+}$
A	$\{A, D, E\}$
AB	$\{A, B, C, D, E\}$ (Hence AB is a key)
B	$\{B\}$
D	$\{D, E\}$

Is $AB \to E$ entailed by $\boldsymbol{F}$? *Yes*

Is $D \to C$ entailed by $\boldsymbol{F}$? *No*

Result: $X_{\boldsymbol{F}}^{+}$ allows us to determine FDs of the form $X \to A$ entailed by $\boldsymbol{F}$

Normal Forms

- The *normal forms* are conditions on schemas that guarantees certain properties relating to redundancy and update anomalies
- First normal form (1NF) is the same as the definition of relational model (relations = sets of tuples; each tuple = sequence of atomic values)
- Second normal form (2NF):
  - no non prime attribute is dependent on any proper subset of any candidate key of the table (where a non prime attribute of a table is an attribute that is not a part of any candidate key of the table): every non-prime attribute is either dependent on the whole of a candidate key, or on another non prime attribute.
- The two commonly used normal forms are *third normal form* (3NF) and *Boyce-Codd normal form* (BCNF)

BCNF

- **Definition**: A relation schema **R** is in BCNF if for every FD $X \rightarrow Y$ associated with **R** either
  - $Y \subseteq X$ (i.e., the FD is trivial) or
  - X is a superkey of **R**
    - Remember: a *superkey* is a combination of attributes that can be used to uniquely identify a database record. A table might have many superkeys.
    - Remember: a *candidate* key is a special subset of superkeys that do not have any extraneous information in them: it is a **minimal** superkey.
- **Example**: Person1($\underline{SSN}$, $Name$, $Address$)
  - The only FD is: $SSN \rightarrow Name, Address$
    - Since SSN is a key, Person1 is in BCNF

(non) BCNF Examples

- Person(<u>*SSN*</u>, *Name*, *Address*, <u>*Hobby*</u>)
  - The FD: $SSN \rightarrow Name, Address$ does <u>not</u> satisfy requirements of BCNF
    - **since the (*SSN*) is not a key**
      - **the key is (*SSN*, *Hobby*)**
- HasAccount(*AcctNum*, *ClientId*, *OfficeId*)
  - The FD $AcctNum \rightarrow OfficeId$ does <u>not</u> satisfy BCNF requirements
    - since keys are (*ClientId*, *OfficeId*) and (*AcctNum*, *ClientId*); not *AcctNum*.

What Redundancy?

- Suppose **R** has a FD $A \rightarrow B$, *and* A is not a superkey.
  - If an instance has 2 rows with same value in A, they *must* also have same value in B (=> redundancy, because the B-*value* repeats twice):

$$SSN \rightarrow Name, Address$$

redundancy

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	stamps
1111	Joe	123 Main	coins

  - If A is a superkey, there cannot be two rows with same value of A
  - Hence, **BCNF eliminates redundancy**

Third Normal Form (3NF)

- A relational schema **R** is in 3NF if for every FD $X \rightarrow Y$ associated with **R** either:
  - $Y \subseteq X$ (i.e., the FD is trivial); or
  - X is a superkey of **R**; OR
  - Every $A \in Y$ is part of some key of **R**

BCNF conditions

- 3NF is weaker than BCNF (every schema that is in BCNF is also in 3NF), but not vice-versa.

3NF Example

- HasAccount (*AcctNum*, *ClientId*, *OfficeId*) is in 3NF:
  - *ClientId*, *OfficeId* $\rightarrow$ *AcctNum*
    - OK since LHS is a superkey
  - *AcctNum* $\rightarrow$ *OfficeId*
    - OK since *OfficeId* (RHS) is part of a key (*ClientId*, *OfficeId*)
- HasAccount is in 3NF but it might still contain redundant information due to *AcctNum* → *OfficeId* (which is not allowed by BCNF)

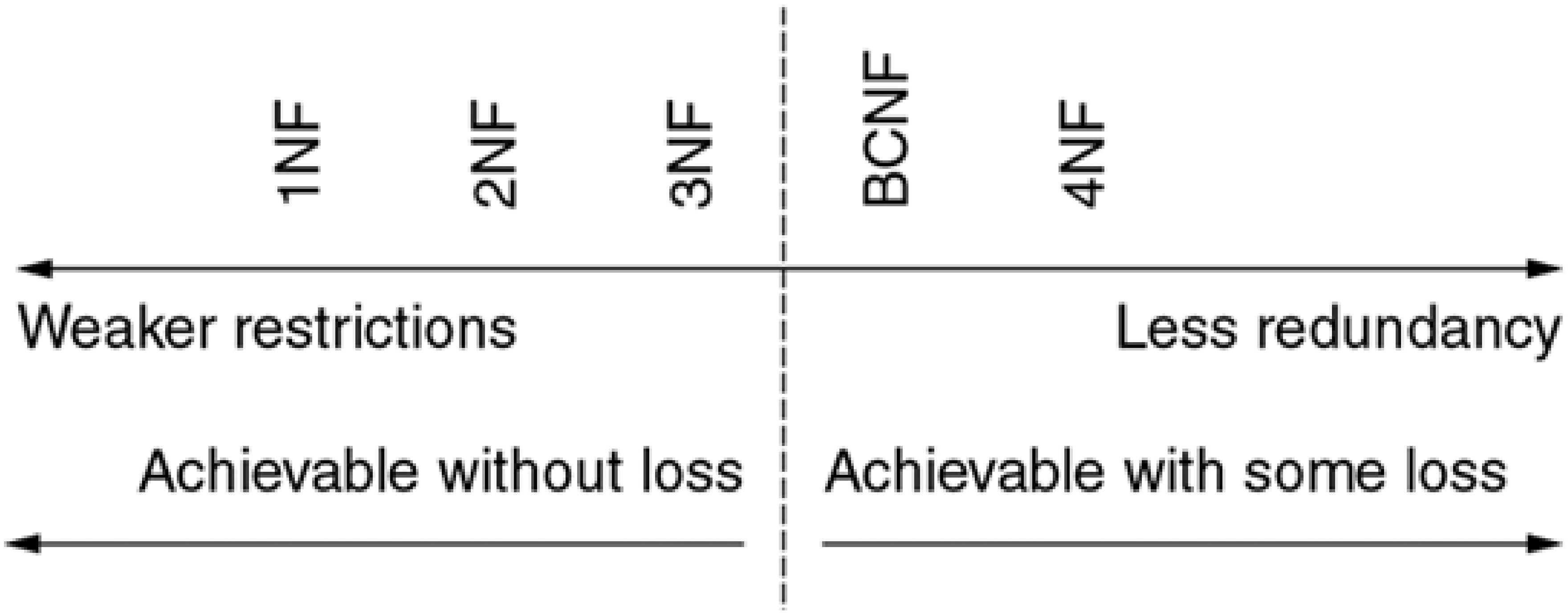
3NF (Non)-Example

- **Person (*SSN*, *Name*, *Address*, *Hobby*):**
  - (*SSN*, *Hobby*) is the only key
  - $SSN \rightarrow Name$ violates 3NF conditions since:
    - it is not a trivial FD,
    - *SSN* (LHS) is not a superkey, and
    - *Name* (RHS) is not part of a key.

Decompositions

- **Goal**: Eliminate redundancy by decomposing a relation into several relations in a higher normal form
- Decomposition MUST be *lossless*: it must be possible to reconstruct the original relation from the relations in the decomposition.

Normal Forms



Decomposition

- Consider a relation schema: $\mathbf{R} = (R, \boldsymbol{F})$
  - R is set a of attributes
  - $\boldsymbol{F}$ is a set of functional dependencies over R
    - Each key is described by a FD
- The *decomposition of the (relation) schema* **R** is a collection of (relation) schemas $\mathbf{R}_{\mathrm{i}} = (R_i, \boldsymbol{F}_i)$ where
  - $R = \cup_i R_i$ for all i (*no new attributes*)
  - $\boldsymbol{F}_i$ is a set of functional dependences involving only attributes of R_i
  - $\boldsymbol{F}$ entails $\boldsymbol{F}_i$ for all i (*no new FDs*)
- The *decomposition of an instance*, **r**, of **R** is a set of relations $\mathbf{r}_i = \pi_{R_i}(\mathbf{r})$ for all i

Example Decomposition

Schema (R, F) where

$R = \{\underline{SSN}, Name, Address, \underline{Hobby}\}$

$\boldsymbol{F} = \{SSN \rightarrow Name, Address\}$

can be decomposed into:

$R_1 = \{\underline{SSN}, Name, Address\}$

$\boldsymbol{F}_1 = \{SSN \rightarrow Name, Address\}$

and

$R_2 = \{\underline{SSN, Hobby}\}$

$\boldsymbol{F}_2 = \{\ \}$

Lossless Schema Decomposition

- A decomposition should not lose information
- A decomposition ($\mathbf{R}_1, \ldots, \mathbf{R}_n$) of a schema, $\mathbf{R}$, is *lossless* if every valid instance, $\mathbf{r}$, of $\mathbf{R}$ can be reconstructed from its components:

$$\mathbf{r} = \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots \bowtie \mathbf{r}_n$$

where each $\mathbf{r}_i = \pi_{\mathbf{R}_i}(\mathbf{r})$

Lossy Decomposition

The following is always the case:

$$\mathbf{r} \subseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots \bowtie \mathbf{r}_n$$

*But the following is **not always true**:*

$$\mathbf{r} \supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots \bowtie \mathbf{r}_n$$

Example: $\mathbf{r} \not\supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$

r

SSN	*Name*	*Address*
1111	Joe	1 Pine
2222	Alice	2 Oak
3333	Alice	3 Pine

r₁

SSN	*Name*
1111	Joe
2222	Alice
3333	Alice

r₂

Name	*Address*
Joe	1 Pine
Alice	2 Oak
Alice	3 Pine

The tuples (2222, Alice, 3 Pine) and (3333, Alice, 2 Oak) *are in the join, but not in the original*

Lossy Decompositions: *What is Actually Lost?*

- In the previous example, the tuples (2222, Alice, 3 Pine) and (3333, Alice, 2 Oak) were *gained*, not lost!
  - Why do we say that the decomposition was lossy?
- What was lost is *information*:
  - That 2222 lives at 2 Oak:

    In the decomposition, 2222 can live at either 2 Oak or 3 Pine
  - That 3333 lives at 3 Pine:

    In the decomposition, 3333 can live at either 2 Oak or 3 Pine

Testing for Losslessness

- A (binary) decomposition of $\mathbf{R} = (R, \boldsymbol{F})$ into $\mathbf{R}_1 = (R_1, \boldsymbol{F}_1)$ and $\mathbf{R}_2 = (R_2, \boldsymbol{F}_2)$ is ***lossless*** if and only if :
  - either the FD
    - $(R_1 \cap R_2) \rightarrow R_1$ is in $\boldsymbol{F}^+$
  - or the FD
    - $(R_1 \cap R_2) \rightarrow R_2$ is in $\boldsymbol{F}^+$

Testing for Losslessness Example

Consider the schema (R, F) where

$$R = \{\underline{SSN}, Name, Address, \underline{Hobby}\}$$

$$\boldsymbol{F} = \{SSN \rightarrow Name, Address\}$$

It can be decomposed into

$$R_1 = \{\underline{SSN}, Name, Address\}$$

$$\boldsymbol{F}_1 = \{SSN \rightarrow Name, Address\}$$

and

$$R_2 = \{\underline{SSN, Hobby}\}$$

$$\boldsymbol{F}_2 = \{\ \}$$

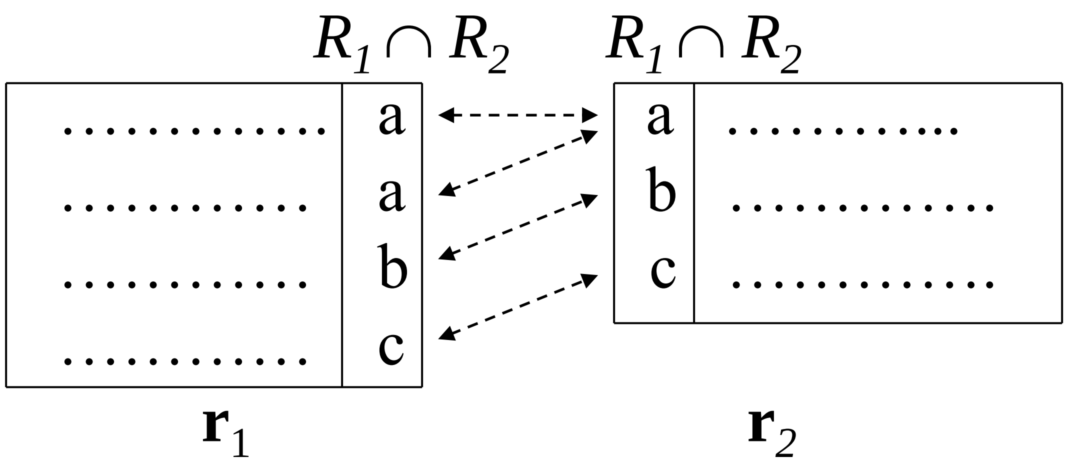
$R_1 \cap R_2 = SSN$ and

$SSN \rightarrow \{SSN, Name, Address\} = R_1$

=> the decomposition is lossless!

Intuition Behind the Test for Losslessness

- Suppose $R_1 \cap R_2 \rightarrow R_2$.
- Then a row of $\mathbf{r}_1$ can combine with exactly one row of $\mathbf{r}_2$ in the natural join (since in $\mathbf{r}_2$ a particular set of values for the attributes in $R_1 \cap R_2$ defines a unique row):



- The join will have exactly the number of tuples in $\mathbf{r}_1$ and **r**

Proof of Lossless Condition

- $r \subseteq r_1 \bowtie r_2$ – *this is true for any decomposition by definition of a decomposition*
- $\mathbf{r} \supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$ – *we need to prove this for lossless*

If $R_1 \cap R_2 \rightarrow R_2$ then

$$card\ (\mathbf{r}_1 \bowtie r_2) = card\ (\mathbf{r}_1)$$

(since each row of r_1 joins with exactly one row of r_2)

But $card\ (\mathbf{r}) \geq card\ (\mathbf{r}_1)$ *(since $\mathbf{r}_1$ is a projection of $\mathbf{r}$)*

and therefore $card\ (\mathbf{r}) \geq card\ (\mathbf{r}_1 \bowtie \mathbf{r}_2)$

From the join (Cartesian product) we have:

$card\ (\mathbf{r}) \leq card\ (\mathbf{r}_1 \bowtie \mathbf{r}_2)$

Hence $\mathbf{r} = \mathbf{r}_1 \bowtie \mathbf{r}_2$ must be true

Dependency Preservation

- Consider a decomposition of $\mathbf{R} = (R, \boldsymbol{F})$ into $\mathbf{R}_1 = (R_1, \boldsymbol{F}_1)$ and $\mathbf{R}_2 = (R_2, \boldsymbol{F}_2)$
  - An FD $X \to Y$ of $\boldsymbol{F}^+$ is in $\boldsymbol{F}_i$ iff $X \cup Y \subseteq R_i$ (all the attributes of the functional dependency are in R_i)
  - An FD, $f \in \boldsymbol{F}^+$ may be in neither $\boldsymbol{F}_1$, nor $\boldsymbol{F}_2$, nor even $(\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - Checking that f is true in $\mathbf{r}_1$ or $\mathbf{r}_2$ is (relatively) easy
    - Checking f in $\mathbf{r}_1 \bowtie \mathbf{r}_2$ is harder – requires a join
    - *Ideally*: want to check FDs locally, in $\mathbf{r}_1$ and $\mathbf{r}_2$, and have a guarantee that every $f \in F$ holds in $\mathbf{r}_1 \bowtie \mathbf{r}_2$
- The decomposition is ***dependency preserving*** iff the FD sets $\boldsymbol{F}$ and $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$ are equivalent: $\boldsymbol{F}^+ = (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
  - Then checking all FDs in $\boldsymbol{F}$, as $\mathbf{r}_1$ and $\mathbf{r}_2$ are updated, can be done by checking $\boldsymbol{F}_1$ in $\mathbf{r}_1$ and F_2 in $\mathbf{r}_2$

Dependency Preservation

- If f is an FD in $\boldsymbol{F}$, but f is not in $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$, there are two possibilities:
  - $f \in (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - If the constraints in $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are maintained, f will be maintained automatically.
  - $f \notin (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - f can be checked only by first taking the join of $\mathbf{r}_1$ and $\mathbf{r}_2$.

Example 1

Schema (R, $\boldsymbol{F}$) where

$$R = \{SSN, Name, Address, Hobby\}$$

$$\boldsymbol{F} = \{SSN \rightarrow Name, Address\}$$

can be decomposed into

$$R_1 = \{SSN, Name, Address\}$$

$$\boldsymbol{F}_1 = \{SSN \rightarrow Name, Address\}$$

and

$$R_2 = \{SSN, Hobby\}$$

$$\boldsymbol{F}_2 = \{\ \}$$

Since $\boldsymbol{F} = \boldsymbol{F}_1 \cup \boldsymbol{F}_2$ the decomposition is dependency preserving

Example 2

- Schema: $(ABC;\ F)$, $\boldsymbol{F} = \{A \rightarrow B, B \rightarrow C, C \rightarrow B\}$
- Decomposition:
  - $(AC, \boldsymbol{F}_1)$, $\boldsymbol{F}_1 = \{A \rightarrow C\}$
    - Note: $\mathrm{A} \rightarrow \mathrm{C} \notin \boldsymbol{F}$, but in $\boldsymbol{F}^+$
  - $(BC, \boldsymbol{F}_2)$, $\boldsymbol{F}_2 = \{B \rightarrow C, C \rightarrow B\}$
- $A \rightarrow B \notin (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)$, **but** $A \rightarrow B \in (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
  - So $\boldsymbol{F}^+ = (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$ and thus the decomposition is still dependency preserving

Example 3

- HasAccount $(AcctNum, ClientId, OfficeId)$

  f_1: $AcctNum \rightarrow OfficeId$

  f_2: $ClientId, OfficeId \rightarrow AcctNum$

- Decomposition:

  $R_1 = (AcctNum, OfficeId; \ \{AcctNum \rightarrow OfficeId\})$

  $R_2 = (AcctNum, ClientId; \ \{\})$

- Decomposition is lossless:

  $R_1 \cap R_2 = \{AcctNum\}$ and $AcctNum \rightarrow AcctNum, OfficeId = R_1$

- This decomposition is in BCNF (we showed that before).
- But it is Not dependency preserving: $f_2 \notin (F_1 \cup F_2)^+$
- HasAccount *does not* have BCNF decompositions that are both lossless and dependency preserving! (Check, eg, by enumeration of all decompositions)
- Hence: BCNF+lossless+dependency preserving decompositions are not always achievable!

BCNF Decomposition Algorithm

Input: $\mathbf{R} = (R; \boldsymbol{F})$

Decomp := {**R**}

while there is $\mathbf{S} = (S; \boldsymbol{F}') \in$ *Decomp* and **S** not in BCNF **do**

Find $X \rightarrow Y \in \boldsymbol{F}'$ that violates BCNF // *X isn't a superkey in* **S**

Replace **S** in *Decomp* with

$\mathbf{S_1} = (XY; \boldsymbol{F}_1)$ and

$\mathbf{S_2} = (S - (Y - X); \boldsymbol{F}_2)$

where $\boldsymbol{F}_1$ = *all FDs of* $\boldsymbol{F}'$ *involving only attributes of* XY

and $\boldsymbol{F}_2$ = *all FDs of* $\boldsymbol{F}'$ *involving only attributes of* $S - (Y - X)$

end

return *Decomp*

Simple Example

- HasAccount :

$(ClientId,\ \ OfficeId,\ \ AcctNum)$
$Keys\text{: } (ClientId,OfficeId)$ and $(ClientId,AcctNum)$
$ClientId,OfficeId \rightarrow\ AcctNum$
$AcctNum \rightarrow\ OfficeId$

- Decompose using $AcctNum \rightarrow\ OfficeId$:

$(OfficeId,\ \underline{AcctNum})$

FD: $AcctNum \rightarrow\ OfficeId$
is in BCNF: $AcctNum$ is key

$(\underline{ClientId\ ,\ \ AcctNum})$
Is in BCNF (only trivial FDs)

A Larger Example

Given: $\mathbf{R} = (R;\ \boldsymbol{F})$ where $R = ABCDEGHK$ and

$\boldsymbol{F} = \{ABH \rightarrow C,\ A \rightarrow DE,\ BGH \rightarrow K,\ K \rightarrow ADH,\ BH \rightarrow GE\}$

step 1: Find a FD that violates BCNF

Not $ABH \rightarrow C$ since $(ABH)^+$ includes all attributes

(BH is a key (minimal superkey))

$A \rightarrow DE$ violates BCNF since A is not a superkey ($A^+ = ADE$)

step 2: Split **R** into:

$\mathbf{R_1} = (ADE,\ \boldsymbol{F}_1 = \{A \rightarrow DE\})$

$\mathbf{R_2} = (ABCGHK;\ \boldsymbol{F}_1 = \{ABH \rightarrow C,\ BGH \rightarrow K,\ K \rightarrow AH,\ BH \rightarrow G\})$

Note 1: $\mathbf{R_1}$ is in BCNF

Note 2: Decomposition <u>is</u> *lossless* since A is a key of $\mathbf{R_1}$.

Note 3: FDs $K \rightarrow D$ and $BH \rightarrow E$ are not in $\boldsymbol{F}_1$ or $\boldsymbol{F}_2$. But both can be derived from $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$

(*E.g.*, $K \rightarrow A$ and $A \rightarrow D$ implies $K \rightarrow D$)

Hence, the decomposition <u>*is*</u> *dependency preserving*.

Is $\mathbf{R_2}$ in BCNF?

A Larger Example (con't)

Given: $\mathbf{R_2}$ **=** $(ABCGHK; \{ABH \rightarrow C, BGH \rightarrow K, K \rightarrow AH, BH \rightarrow G\})$

step 1: Find a FD that violates BCNF.

Not $ABH \rightarrow C$ or $BGH \rightarrow K$, since BH is a key of $\mathbf{R_2}$

$K \rightarrow AH$ violates BCNF since K is not a superkey ($K^+ = AH$)

step 2: Split $\mathbf{R_2}$ into:

$\mathbf{R_{21}}$ **=** $(KAH, \boldsymbol{F}_{21} = \{K \rightarrow AH\})$

$\mathbf{R_{22}}$ **=** $(BCGK; \boldsymbol{F}_{22} = \{\})$

Note 1: Both $\mathbf{R_{21}}$ and $\mathbf{R_{22}}$ are in BCNF.

Note 2: The decomposition is *lossless* (since K is a key of $\mathbf{R_{21}}$)

Note 3: FDs $ABH \rightarrow C$, $BGH \rightarrow K$, $BH \rightarrow G$ are not in $\boldsymbol{F}_{21}$ or $\boldsymbol{F}_{22}$, and they can't be derived from $\boldsymbol{F}_1 \cup \boldsymbol{F}_{21} \cup \boldsymbol{F}_{22}$.

Hence the decomposition is *not* dependency-preserving

Properties of BCNF Decomposition Algorithm

- Let $X \rightarrow Y$ violate BCNF in $\mathbf{R} = (R, \boldsymbol{F})$.
- $\mathbf{R_1} = (R_1, \boldsymbol{F}_1)$ and $\mathbf{R_2} = (R_2, \boldsymbol{F}_2)$ is the resulting decomposition. Then:
  - There are *fewer violations* of BCNF in $\mathbf{R_1}$ and $\mathbf{R_2}$ than there were in $\mathbf{R}$
  - $X \rightarrow Y$ implies X is a key of $\mathbf{R_1}$
    - Hence $X \rightarrow Y \in \boldsymbol{F}_1$ does not violate BCNF in $\mathbf{R_1}$ and, since $X \rightarrow Y \notin \boldsymbol{F}_2$, does not violate BCNF in $\mathbf{R_2}$ either
- Suppose f is $X' \rightarrow Y'$ and $f \in \boldsymbol{F}$ doesn't violate BCNF in $\mathbf{R}$. If $f \in \boldsymbol{F}_1$ or $\boldsymbol{F}_2$ it does not violate BCNF in $\mathbf{R_1}$ or $\mathbf{R_2}$ either since X' is a superkey of $\mathbf{R}$ and hence also of $\mathbf{R_1}$ and $\mathbf{R_2}$.

Properties of BCNF Decomposition Algorithm

- A BCNF decomposition is *not necessarily* dependency preserving
- But *always* lossless:

  since $R_1 \cap R_2 = X, \quad X \rightarrow Y,$ and $\quad R_1 = XY$
- BCNF+lossless+dependency preserving is sometimes unachievable

Third Normal Form

- The Third Normal Form is the Compromise = Not all redundancy removed, but dependency preserving decompositions are <u>always</u> possible (and, of course, lossless)
- 3NF decomposition is based on a *minimal cover*

Minimal Cover

- A *minimal cover* of a set of functional dependencies F is a set of dependencies U such that:
  - U is equivalent to F (i.e., $F^+ = U^+$)
  - All FDs in U have the form $X \rightarrow A$ where A is a single attribute
  - It is not possible to make U smaller (while preserving equivalence) by
    - Deleting an FD
    - Deleting an attribute from an FD (either from LHS or RHS)
  - FDs and attributes that can be deleted in this way are called *redundant*

Computing the Minimal Cover

- **Example**: $\boldsymbol{F} = \{ABH \rightarrow CK, A \rightarrow D, C \rightarrow E,$
  $BGH \rightarrow L, L \rightarrow AD, E \rightarrow L, BH \rightarrow E\}$
- **step 1**: Make RHS of each FD into a single attribute:
  - $ABH \rightarrow CK$ is replaced by $ABH \rightarrow C$ and $ABH \rightarrow K$
  - $L \rightarrow AD$ is replaced by $L \rightarrow A$ and $L \rightarrow D$
- **step 2**: Eliminate redundant attributes from LHS:
  - *Algorithm*: If FD $XB \rightarrow A \in \boldsymbol{F}$ (where B is a single attribute) and $X \rightarrow A$ is entailed by $\boldsymbol{F}$, then B was unnecessary
  - Example: Can an attribute be deleted from $ABH \rightarrow C$?
    - Compute $AB^{+}{}_{\boldsymbol{F}}$, $AH^{+}{}_{\boldsymbol{F}}$, $BH^{+}{}_{\boldsymbol{F}}$.
    - Since $C \in (BH)^{+}{}_{\boldsymbol{F}}$, $BH \rightarrow C$ is entailed by $\boldsymbol{F}$ and A is redundant in $ABH \rightarrow C$.

Computing the Minimal Cover

- **step 3**: Delete redundant FDs from $\boldsymbol{F}$
  - *Algorithm*: If $\boldsymbol{F} - \{f\}$ entails f, then f is redundant
    - Alternative: If f is $X \rightarrow A$ then check if A $\in X^{+}{}_{\boldsymbol{F}\text{-}\{f\}}$
  - Example: $BGH \rightarrow L$ is entailed by $E \rightarrow L$, $BH \rightarrow E$, so it is redundant.

Synthesizing a 3NF Schema

Starting with a schema $\mathbf{R} = (R, \boldsymbol{F})$

- **step 1**: Compute a minimal cover, $\boldsymbol{U}$, of $\boldsymbol{F}$ (the decomposition is based on $\boldsymbol{U}$, but since $\boldsymbol{U}^+ = \boldsymbol{F}^+$ the same functional dependencies will hold)
  - A minimal cover for

$$\boldsymbol{F} = \{ABH \rightarrow CK,\ A \rightarrow D,\ C \rightarrow E,\ BGH \rightarrow L,\ L \rightarrow AD,\ E \rightarrow L,\ BH \rightarrow E\}$$

is

$$\boldsymbol{U} = \{BH \rightarrow C,\ BH \rightarrow K,\ A \rightarrow D,\ C \rightarrow E,\ L \rightarrow A,\ E \rightarrow L\}$$

Synthesizing a 3NF schema (con't)

- The minimal cover was:

$$\boldsymbol{U} = \{BH \rightarrow C,\ BH \rightarrow K,\ A \rightarrow D,\ C \rightarrow E,\ L \rightarrow A,\ E \rightarrow L\}$$

- **step 2**: Partition $\boldsymbol{U}$ into sets $\boldsymbol{U}_1, \boldsymbol{U}_2, \ldots \boldsymbol{U}_n$ such that the LHS of all elements of $\boldsymbol{U}_i$ are the same

$$\boldsymbol{U}_1 = \{BH \rightarrow C, BH \rightarrow K\}$$

$$\boldsymbol{U}_2 = \{A \rightarrow D\}$$

$$\boldsymbol{U}_3 = \{C \rightarrow E\}$$

$$\boldsymbol{U}_4 = \{L \rightarrow A\}$$

$$\boldsymbol{U}_5 = \{E \rightarrow L\}$$

Synthesizing a 3NF schema (con't)

$\boldsymbol{U}_1 = \{BH \to C, BH \to K\}$, $\boldsymbol{U}_2 = \{A \to D\}$,

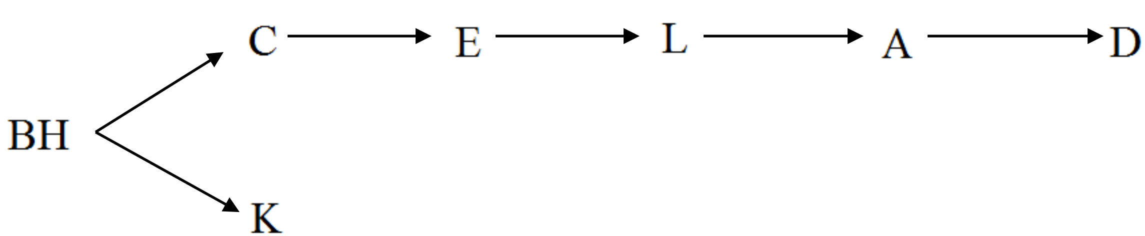
$\boldsymbol{U}_3 = \{C \to E\}$, $\boldsymbol{U}_4 = \{L \to A\}$, $\boldsymbol{U}_5 = \{E \to L\}$

- **step 3**: For each $\boldsymbol{U}_i$ form a schema $\mathbf{R_i} = (R_i, \boldsymbol{U}_i)$, where R_i is the set of all attributes mentioned in $\boldsymbol{U}_i$
  - Each FD of $\boldsymbol{U}$ will be in some $\mathbf{R_i}$. Hence the decomposition is *dependency preserving:*

    $\mathbf{R_1} = (BHCK;\ BH \to C, BH \to K)$, $\mathbf{R_2} = (AD;\ A \to D)$,

    $\mathbf{R_3} = (CE;\ C \to E)$, $\mathbf{R_4} = (AL;\ L \to A)$,

    $\mathbf{R_5} = (EL;\ E \to L)$
  - Unify relations that have the same set of attributes.
  - Add to each $\mathbf{R_i}$ all dependencies *f* entailed by the original set *F* where all the attributes are in $\boldsymbol{R}_i$



Synthesizing a 3NF schema (con't)

- **step 4**: If no R_i is a superkey of **R**, add schema $\mathbf{R_0} = (R_0, \{\})$ where R_0 is a key of **R**.
  - $\mathbf{R_0} = (BGH, \{\})$
    - $\mathbf{R_0}$ might be needed when not all attributes are necessarily contained in $R_1 \cup R_2 \ldots \cup R_n$
      - A missing attribute, A, must be part of all keys

        (since it's not in any FD of U, deriving a key constraint from U involves the augmentation axiom)
    - $\mathbf{R_0}$ might be needed even if all attributes are accounted for in $R_1 \cup R_2 \ldots \cup R_n$
      - Example: $(ABCD; \{A \rightarrow B, C \rightarrow D\})$.

        Step 3 decomposition: $R_1 = (AB; \{A \rightarrow B\})$, $R_2 = (CD; \{C \rightarrow D\})$.

        Lossy! Need to add (AC; { }), for losslessness
  - Step 4 guarantees **lossless** decomposition.

BCNF Design Strategy

- The resulting decomposition, $\mathbf{R_0, R_1, \ldots R_n}$, is
  - Dependency preserving (since every FD in *U* is a FD of some schema)
  - Lossless
  - In 3NF
- Strategy for decomposing a relation:
  - Use 3NF decomposition first to get lossless, dependency preserving decomposition
  - If any resulting schema is not in BCNF, split it using the BCNF algorithm (but this may yield a non-dependency preserving result)

Normalization Drawbacks

- By limiting redundancy, normalization helps maintain consistency and saves space
- But performance of querying can suffer because related information that was stored in a single relation is now distributed among several
- **Example**: A join is required to get the names and grades of all students taking CSE305 in F2016.

```
SELECT  S.Name, T.Grade
FROM  Student S, Transcript T
WHERE  S.Id = T.StudId  AND
       T.CrsCode = 'CSE305'  AND  T.Semester = 'F2016'
```

Denormalization

- **Tradeoff**: *Judiciously* introduce redundancy to improve performance of certain queries
- **Example**: Add attribute *Name* to Transcript

SELECT T.*Name*, T.*Grade*
FROM Transcript′ T
WHERE T.*CrsCode* = 'CSE305' AND T.*Semester* = 'F2016'

  - Join is avoided
  - If queries are asked more frequently than Transcript is modified, added redundancy might improve average performance
  - But, Transcript′ is no longer in BCNF since key is (*StudId*, *CrsCode*, *Semester*) and *StudId* $\rightarrow$ *Name*

Fourth Normal Form

SSN	*PhoneN*	*ChildSSN*
111111	123-4444	222222
111111	123-4444	333333
111111	321-5555	222222
111111	321-5555	333333
222222	987-6666	444444
222222	777-7777	444444
222222	987-6666	555555
222222	777-7777	555555

Person

redundancy

- Relation has redundant data
- Yet it is in BCNF (since there are no non-trivial FDs)
- Redundancy is due to set valued attributes (in the E-R sense)

Multi-Valued Dependency

- **Problem**: multi-valued (or binary join) dependency
  - **Definition**: If every instance of schema **R** can be (losslessly) decomposed using attribute sets (X,Y) such that:

$$\mathbf{r} = \pi_X(\mathbf{r}) \bowtie \pi_Y(\mathbf{r})$$

then a ***multi-valued dependency***

$$\mathbf{R} = \pi_X(\mathbf{R}) \bowtie \pi_Y(\mathbf{R})$$

holds in **r**

Ex: $\text{Person} = \pi_{SSN,PhoneN}(\text{Person}) \bowtie \pi_{SSN,ChildSSN}(\text{Person})$

Fourth Normal Form (4NF)

- A schema is in *fourth normal form* (4NF) if for every multi-valued dependency

$$R = X \bowtie Y$$

in that schema, either:

- $X \subseteq Y$ or $Y \subseteq X$ (trivial case); or
- $X \cap Y$ is a superkey of R (*i.e.*, $X \cap Y \rightarrow R$)

Fourth Normal Form (Cont'd)

- *Intuition*: if $X \cap Y \rightarrow R$, there is a unique row in relation **r** for each value of $X \cap Y$ (hence no redundancy)
  - Ex: *SSN* does not uniquely determine *PhoneN* or *ChildSSN*, thus Person is not in 4NF.
- *Solution*: Decompose *R* into *X* and *Y*
  - Decomposition is lossless – but not necessarily dependency preserving (since 4NF implies BCNF – next)

4NF Implies BCNF

- Suppose R is in 4NF and $X \rightarrow Y$ is an FD.
  - $R_1 = XY,\ R_2 = R - Y$ is a lossless decomposition of R
  - Thus R has the multi-valued dependency:

$$R = R_1 \bowtie R_2$$

– Since R is in 4NF, one of the following must hold :

– $XY \subseteq R - Y$ (an impossibility)

– $R - Y \subseteq XY$ (i.e., $R = XY$ and X is a superkey) or

– $XY \cap R - Y$ $(= X)$ is a superkey

Hence $X \rightarrow Y$ satisfies BCNF condition