Sorting

Paul Fodor

CSE260, Computer Science B: Honors

Stony Brook University

http://www.cs.stonybrook.edu/~cse260

Objectives

- To study and analyze time complexity of various sorting algorithms
  - To design, implement, and analyze ***bubble sort***
  - To design, implement, and analyze ***merge sort***
  - To design, implement, and analyze ***quick sort***
  - To design and implement a ***binary heap***
  - To design, implement, and analyze ***heap sort***
  - To design, implement, and analyze ***bucket sort*** and ***radix sort***
  - To design, implement, and analyze ***external sort*** for files that have a large amount of data

What data to sort?

- The data to be sorted might be integers, doubles, characters, Strings or any objects that are comparable.
- For simplicity, we assume:
  - data to be sorted are ***integers***
  - data is sorted in ***ascending*** order
  - data is stored in an ***array***
    - The programs can be easily modified to sort other types of data, to sort in descending order, or to sort data in an **`ArrayList`** or a **`LinkedList`**.
      - **`LinkedList`** might be more efficient for some operations (no shifting required to insert/delete an element at any position of the array), but overall complexity stays the same
- The Java API contains several overloaded sort methods for sorting primitive type values and objects in the **`java.util.Arrays`** and **`java.util.Collections`** class.

Bubble Sort

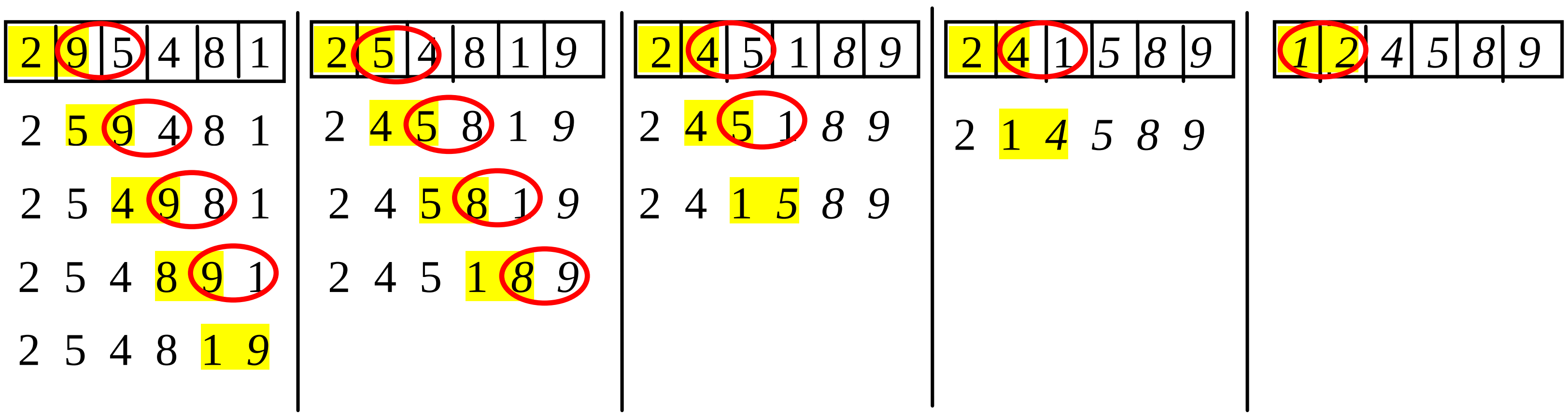
Repeatedly steps through the list to be sorted, compares each **pair of adjacent items** and swaps them if they are in the wrong order

```
for (int k = 1; k < list.length; k++) {
  // Perform the kth pass
  for (int i = 0; i < list.length - k; i++) {
    if (list[i] > list[i + 1])
      // swap list[i] with list[i + 1];
      ...
  }
}
```

- After the first pass, the last element becomes the largest in the array
- After the second pass, the second-to-last element becomes the second largest in the array
  - It is called "***bubble***" ***sort*** because the large values gradually "bubble" to the top (end of the array)
  - It is also called "***sinking sort***" because the smaller values gradually "***sink***" their way to the bottom (beginning of the array)

Bubble Sort Example

- Example:



(a) 1st pass (b) 2nd pass (c) 3rd pass (d) 4th pass (e) 5th pass

Bubble Sort Optimization

- The pass through the list is repeated until no swaps are needed, which indicates that the list is sorted
  - If no swap takes place in a pass, there is no need to perform the next pass, because all the elements are already sorted!
  - We can use this property to improve the previous algorithm:

```
boolean needNextPass = true;
for (int k = 1; k < list.length && needNextPass; k++) {
  // Array may be sorted and next pass not needed
  needNextPass = false;
  // Perform the kth pass
  for (int i = 0; i < list.length - k; i++)
    if (list[i] > list[i + 1]) {
      // swap list[i] with list[i + 1];
      needNextPass = true; // Next pass still needed
    }
}
```

Bubble Sort Analysis

- In the **best case**, the bubble sort algorithm needs just the first pass to find that the array is already sorted—no next pass is needed.
  - Since the number of comparisons is ***n - 1*** in the first pass, the best-case time for a bubble sort is ***O*(*n*)**.

Bubble Sort Analysis

- **Time complexity (i.e., Worse case) :**

$$(n-1)+(n-2)+\ldots+2+1=\frac{n^2}{2}-\frac{n}{2}$$

time: $\boldsymbol{O}(\mathbf{n}^2)$

```java
public class BubbleSort {
  public static void bubbleSort(int[] list) {
    boolean needNextPass = true;
    for (int k = 1; k < list.length && needNextPass; k++) {
      // Array may be sorted and next pass not needed
      needNextPass = false;
      // Perform the kth pass
      for (int i = 0; i < list.length - k; i++) {
        if (list[i] > list[i + 1]) {
          int temp = list[i];
          list[i] = list[i + 1];
          list[i + 1] = temp;
          needNextPass = true; // Next pass still needed
        }
      }
    }
  }
  public static void main(String[] args) {
    int size = 100000;
    int[] a = new int[size];
    randomInitiate(a);
    long startTime = System.currentTimeMillis();
    bubbleSort(a);
    long endTime = System.currentTimeMillis();
    System.out.println((endTime - startTime) + "ms");
  }
  private static void randomInitiate(int[] a) {
    for (int i = 0; i < a.length; i++)
      a[i] = (int) (Math.random() * a.length);
  }
}
```

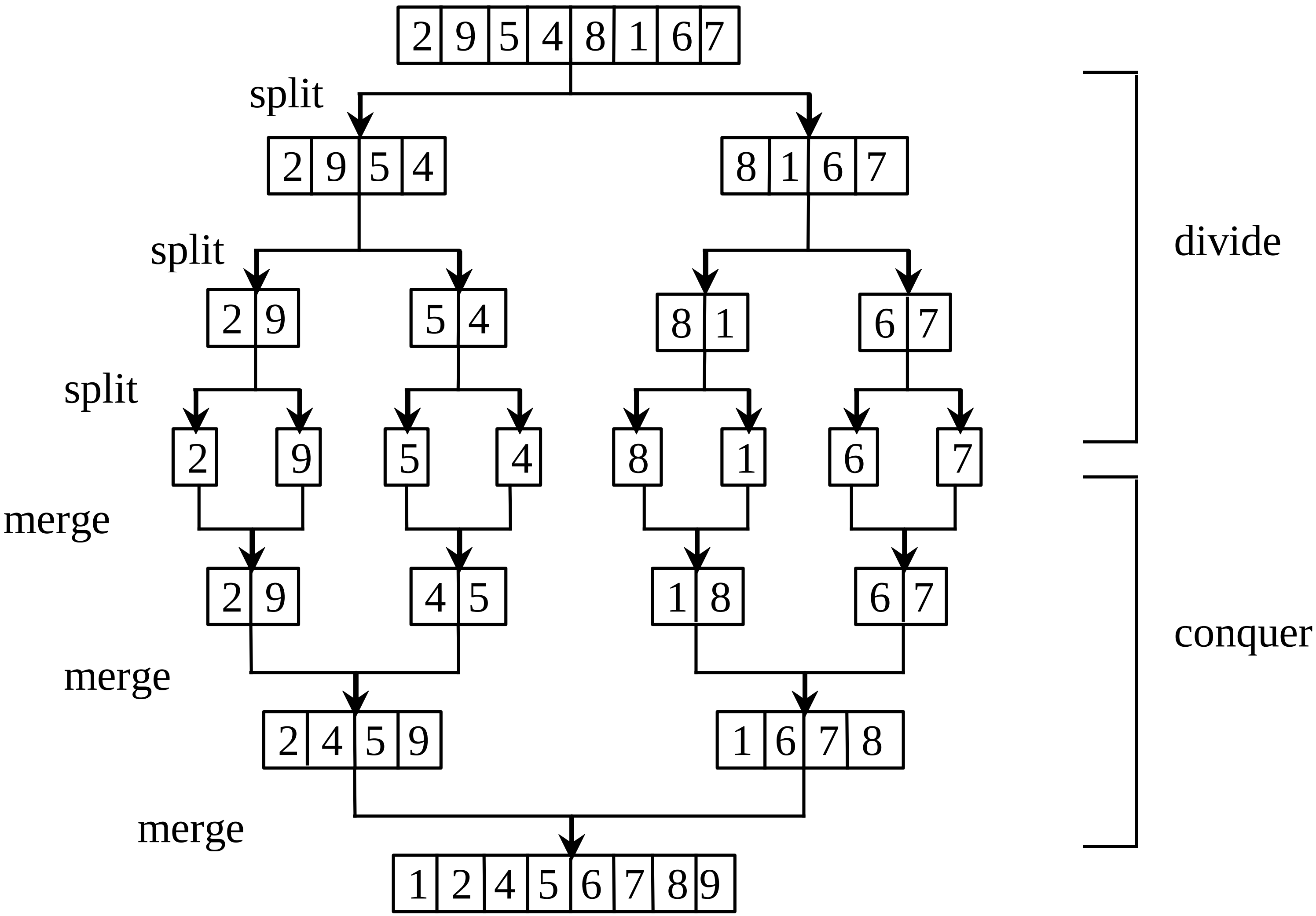
14650ms

Merge Sort

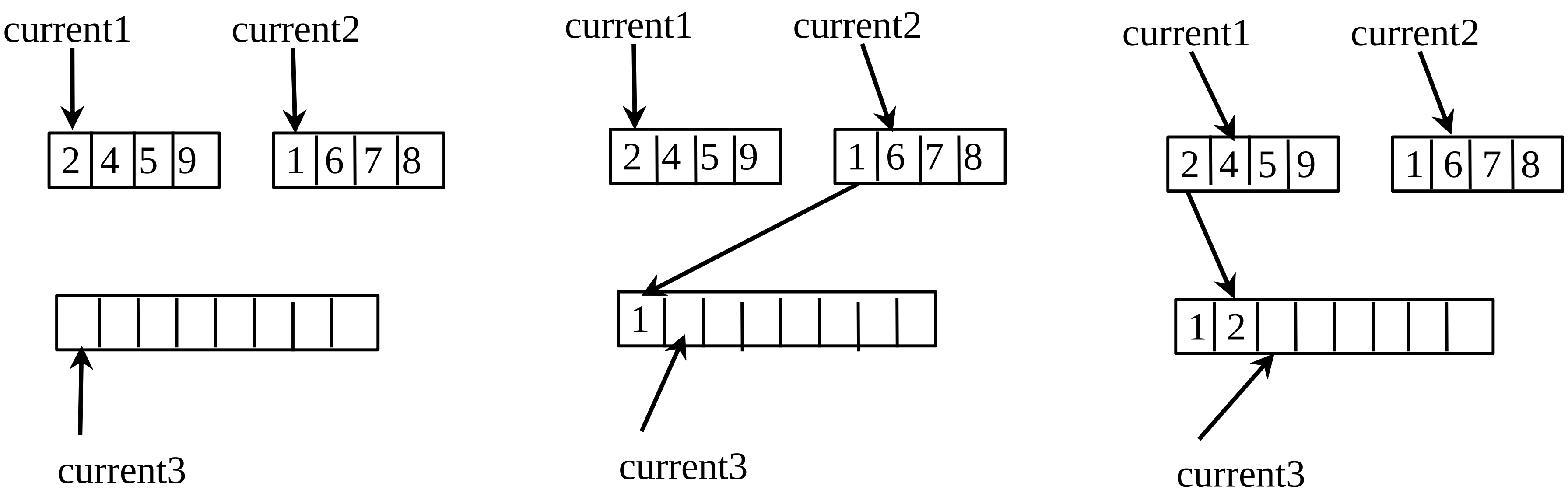
John von Neumann

- ***Merge sort*** is a **divide and conquer algorithm** invented by John von Neumann in 1945
  - Divide the unsorted list into **`n`** sublists, each containing 1 element (a list of 1 element is considered sorted)
  - Repeatedly **`merge`** sorted sublists to produce new sorted sublists until there is only 1 sublist remaining
    - This will be the sorted list.

Merge Sort is a divide&conquer algorithm

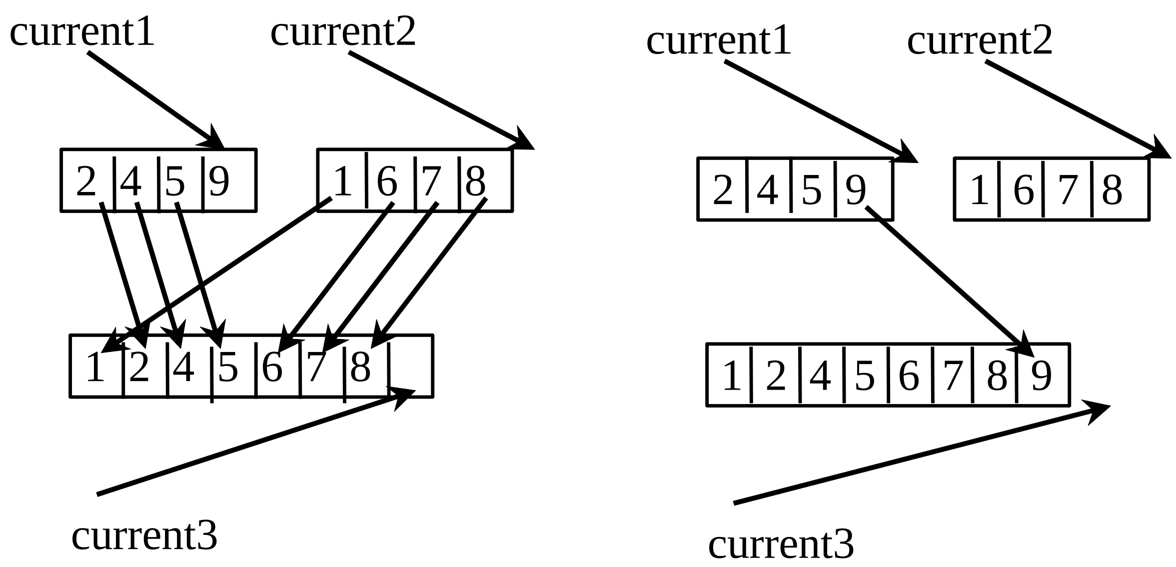


How to Merge Two Sorted Lists



(a) Beginning state (b) After moving 1 to temp (c) After moving 2 to temp

How to Merge Two Sorted Lists



After moving all the elements in list2 to temp

After moving the rest of list1 to temp

```java
/** Merge two sorted lists */
public static void merge(int[] list1, int[] list2, int[] temp){
  int current1 = 0; // Current index in list1
  int current2 = 0; // Current index in list2
  int current3 = 0; // Current index in temp

  while (current1 < list1.length && current2 < list2.length) {
    if (list1[current1] < list2[current2])
      temp[current3++] = list1[current1++];
    else
      temp[current3++] = list2[current2++];
  }

  while (current1 < list1.length)
    temp[current3++] = list1[current1++];

  while (current2 < list2.length)
    temp[current3++] = list2[current2++];
}
```

```java
public static void mergeSort(int[] list) {
 if (list.length > 1) {
  // Merge sort the first half
  int[] firstHalf = new int[list.length / 2];
  System.arraycopy(list,0,firstHalf,0,list.length / 2);
  mergeSort(firstHalf);

  // Merge sort the second half
  int secondHalfLength = list.length - list.length / 2;
  int[] secondHalf = new int[secondHalfLength];
  System.arraycopy(list,list.length / 2,secondHalf,0,
      secondHalfLength);
  mergeSort(secondHalf);

  // Merge firstHalf with secondHalf into list
  merge(firstHalf, secondHalf, list);
 }
}
```

Example:

```
public static void main(String[] args) {
   int[] list = {14,12,2,3,2,-2,1,3,6,5};
   mergeSort(list);
   for (int x:list)
      System.out.print(x + " ");
}

-2 1 2 2 3 3 5 6 12 14
```

```java
public class MergeSortTest {
  public static void mergeSort(int[] list) {
    if (list.length > 1) {
      // Merge sort the first half
      int[] firstHalf = new int[list.length / 2];
      System.arraycopy(list, 0, firstHalf, 0, list.length / 2);
      mergeSort(firstHalf);
      // Merge sort the second half
      int secondHalfLength = list.length - list.length / 2;
      int[] secondHalf = new int[secondHalfLength];
      System.arraycopy(list, list.length / 2, secondHalf, 0, secondHalfLength);
      mergeSort(secondHalf);
      // Merge firstHalf with secondHalf into list
      merge(firstHalf, secondHalf, list);
    }
  }
  public static void merge(int[] list1, int[] list2, int[] temp) {
    int current1 = 0; // Current index in list1
    int current2 = 0; // Current index in list2
    int current3 = 0; // Current index in temp
    while (current1 < list1.length && current2 < list2.length) {
      if (list1[current1] < list2[current2])
        temp[current3++] = list1[current1++];
      else
        temp[current3++] = list2[current2++];
    }
    while (current1 < list1.length)
      temp[current3++] = list1[current1++];
    while (current2 < list2.length)
      temp[current3++] = list2[current2++];
  }
```

```java
  public static void main(String[] args) {
    int size = 100000;
    int[] a = new int[size];
    randomInitiate(a);
    long startTime = System.currentTimeMillis();
    mergeSort(a);
    long endTime = System.currentTimeMillis();
    System.out.println((endTime - startTime) + "ms");
  }

  private static void randomInitiate(int[] a) {
    for (int i = 0; i < a.length; i++)
      a[i] = (int) (Math.random() * a.length);
  }

}
```

```
16ms
```

Merge Sort Time Complexity

- Let ***T*(*n*)** denote the time required for sorting an array of ***n*** elements using merge sort.
  - The merge sort algorithm splits the array into two subarrays, sorts the subarrays using the same algorithm recursively, and then merges the subarrays

$$T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + mergetime$$

    - The first $T(n/2)$ is the time for sorting the first half of the array and the second $T(n/2)$ is the time for sorting the second half

Merge Sort Time

- To merge two sorted subarrays, it takes at most ***n-1*** comparisons to compare the elements from the two subarrays and ***n*** moves to move elements to the temporary array

$$T(n) = 2T(\frac{n}{2}) + 2n - 1 = 2(2T(\frac{n}{4}) + 2\frac{n}{2} - 1) + 2n - 1$$

$$= 2^2 T(\frac{n}{2^2}) + 2n - 2 + 2n - 1$$

$$= 2^k T(\frac{n}{2^k}) + 2n - 2^{k-1} + \ldots + 2n - 2 + 2n - 1$$

$$= 2^{\log n} T(\frac{n}{2^{\log n}}) + 2n - 2^{\log n - 1} + \ldots + 2n - 2 + 2n - 1$$

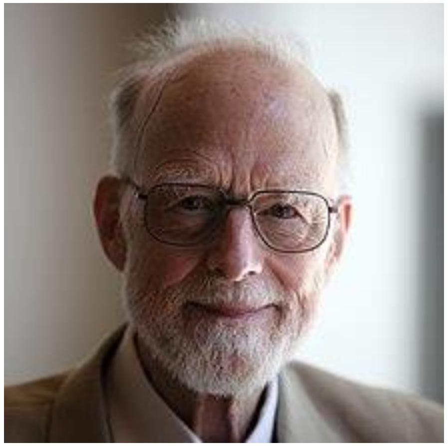
$$= n + 2n \log n - 2^{\log n} + 1$$

$$= 2n \log n + 1$$

$$= O(n \log n)$$

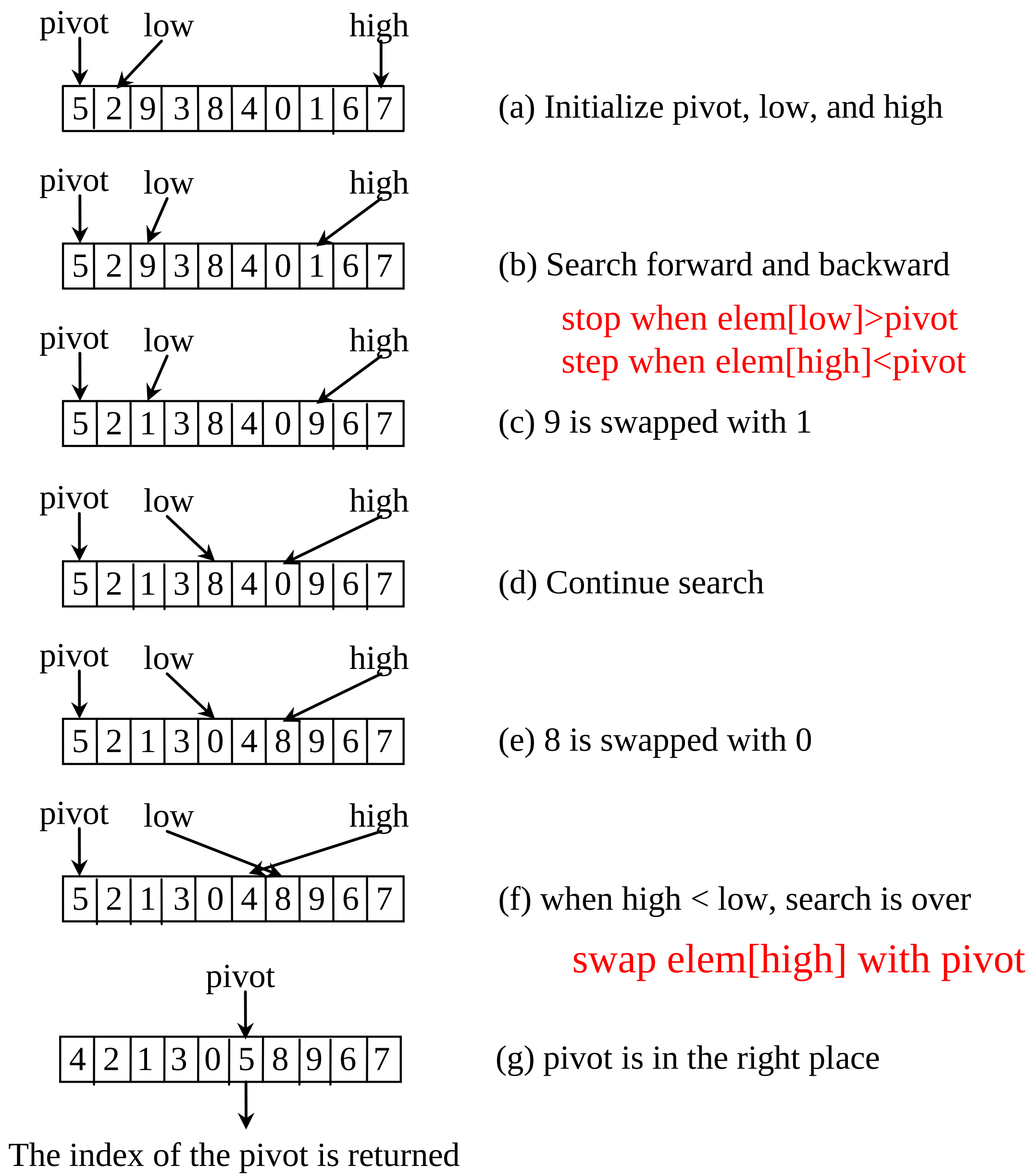
Quick Sort

- ***Quick sort***, developed by C. A. R. Hoare in 1962, works as follows:
  - The algorithm selects an element, called the ***pivot***, in the array
    - could be just the first element
  - **Divides/partitions** the array into two parts such that all the elements in the first part are **less** than or equal to the pivot and all the elements in the second part are **greater** than the pivot
    - **This can be done in the same array (see how next slide)**
  - Recursively apply the quick sort algorithm to the first part and then the second part
  - Concatenate the first sorted part, the pivot and the second sorted part into the final sorted list

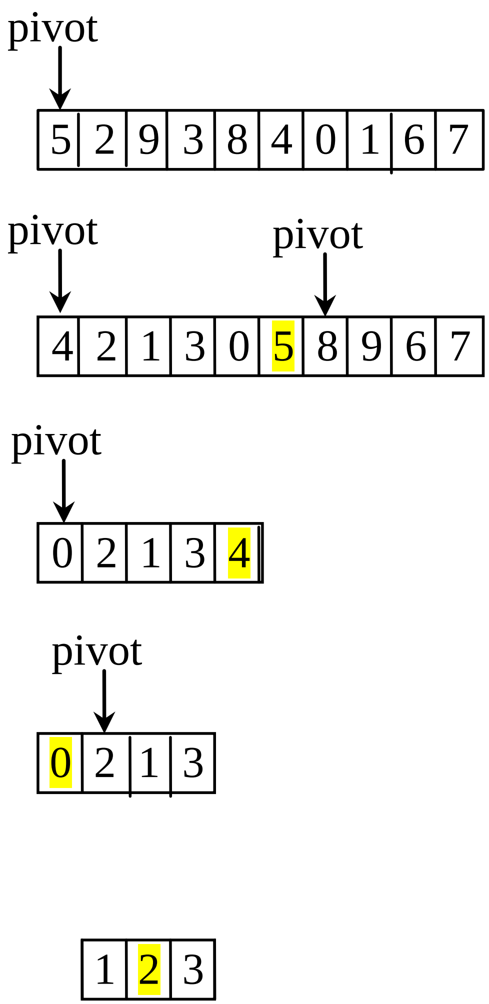


C. A. R. Hoare

Partition with forward and backward search



Quick Sort Example Steps



(a) The original array

(b)The original array is partitioned

(c) The partial array (4 2 1 3 0) is partitioned

(d) The partial array (0 2 1 3) is partitioned

(e) The partial array (2 1 3) is partitioned

```java
public static void quickSort(int[] list) {
  quickSort(list, 0, list.length - 1);
}

public static void quickSort(int[] list, int first, int last) {
  if (last > first) {
    int pivotIndex = partition(list, first, last);
    quickSort(list, first, pivotIndex - 1);
    quickSort(list, pivotIndex + 1, last);
  }
}
```

```java
public static int partition(int[] list, int first, int last) {
  int pivot = list[first]; // Choose the first element as pivot
  int low = first + 1; // Index for forward search
  int high = last; // Index for backward search
  while (high > low) {
    // Search forward from left
    while (low <= high && list[low] <= pivot)
      low++;
    // Search backward from right
    while (low <= high && list[high] > pivot)
      high--;
     // Swap two elements in the list
     if (high > low) {
       int temp = list[high];
       list[high] = list[low];
       list[low] = temp;
     }
   }
   // Account for duplicated elements:
   while (high > first && list[high] >= pivot)
     high--;
   // Swap pivot with list[high]
   if (pivot > list[high]) {
     list[first] = list[high];
     list[high] = pivot;
     return high;
   } else
     return first;
}
```

```
public class QuickSortTest {
  public static void quickSort(int[] list) {
    quickSort(list, 0, list.length - 1);
  }
  public static void quickSort(int[] list, int first, int last) {
    if (last > first) {
      int pivotIndex = partition(list, first, last);
      quickSort(list, first, pivotIndex - 1);
      quickSort(list, pivotIndex + 1, last);
    }
  }
  public static int partition(int[] list, int first, int last) {
    int pivot = list[first]; // Choose the first element as pivot
    int low = first + 1; // Index for forward search
    int high = last; // Index for backward search
    while (high > low) {
      // Search forward from left
      while (low <= high && list[low] <= pivot)
        low++;

      // Search backward from right
      while (low <= high && list[high] > pivot)
        high--;
      // Swap two elements in the list
      if (high > low) {
        int temp = list[high];
        list[high] = list[low];
        list[low] = temp;
      }
    }
    // Account for duplicated elements:
    while (high > first && list[high] >= pivot)
      high--;
```

```
    // Swap pivot with list[high]
    if (pivot > list[high]) {
      list[first] = list[high];
      list[high] = pivot;
      return high;
    } else
      return first;
  }

  public static void main(String[] args) {
    int size = 100000;
    int[] a = new int[size];
    randomInitiate(a);

    long startTime = System.currentTimeMillis();
    quickSort(a);
    long endTime = System.currentTimeMillis();

    System.out.println((endTime - startTime) + "ms");
  }
  private static void randomInitiate(int[] a) {
    for (int i = 0; i < a.length; i++)
      a[i] = (int) (Math.random() * a.length);
  }

}
```

16ms

Quick Sort Time Complexity

- To **`partition`** an array of ***n*** elements, it takes ***`n-1`*** comparisons and ***n*** moves in the worst case
- So, the time required for partition is ***O*(*n*)**
- In the **worst case**, each time the pivot divides the array into one big subarray with the other empty
  - The size of the big subarray is one less than the one before divided
  - Therefore, the algorithm requires:

$$(n-1)+(n-2)+\ldots+2+1=O(n^2)$$

Best-Case Time Complexity

- In the best case, each time the pivot divides the array into two parts of about the same size

$$T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + n = O(n \log n)$$

Average-Case Time

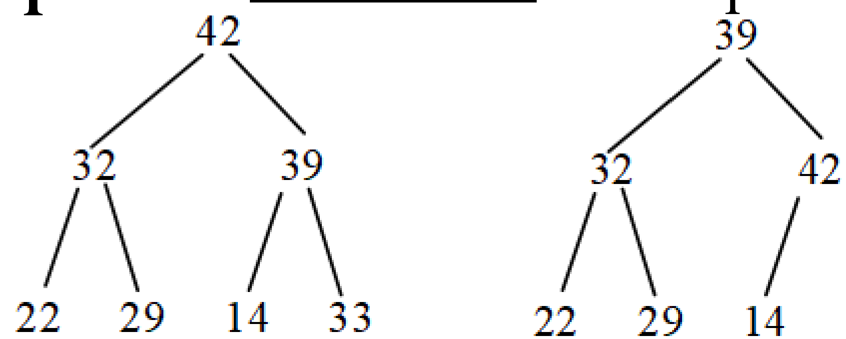
- On average, each time the pivot will not divide the array into one empty part nor two parts of the same size
  - However, statistically, the sizes of the two parts are very close => the average time is also ***O(n log n)***
- Both merge sort and quick sort employ the divide-and-conquer approach
  - For merge sort, the bulk of the work is to merge two sublists
    - Merge sort is more efficient than quick sort in the worst case, but the two are equally efficient in the average case
  - Merge sort requires a temporary array for sorting two subarrays
    - Quick sort does not need additional array space. **Thus, quick sort is more space efficient than merge sort.**

Heap Sort: Binary tree

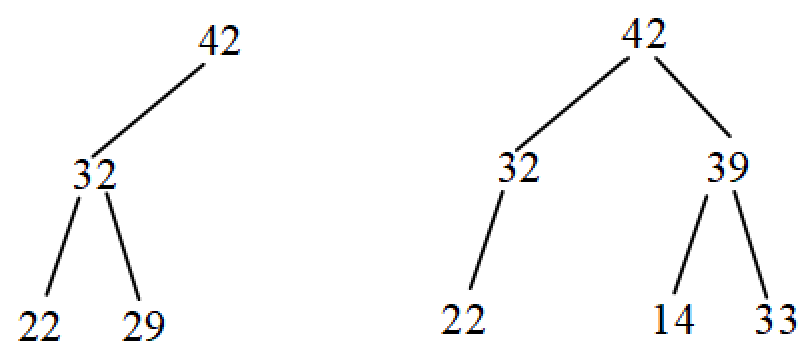
- A ***binary tree*** is a hierarchical structure: it either is empty or it consists of an element, called the ***root***, and two distinct binary trees, called the ***left subtree*** and ***right subtree***
  - The ***length*** of a path is the number of the edges in the path
  - The ***depth*** of a node is the length of the path from the root to that node

Complete Binary Tree

- A binary tree is ***complete*** if every level of the tree is full except that the **last level may not be full** and **all the leaves on the last level are placed <u>left-most</u>**. Examples of complete binary trees:

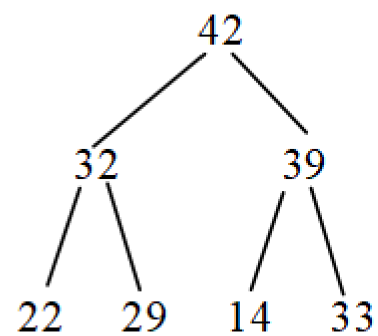


  - Not complete:

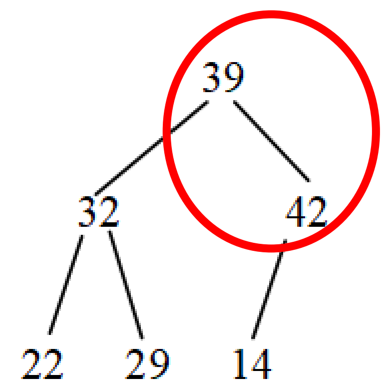


Binary Heap

- A ***binary heap*** is a binary tree with the following properties:
  - It is a complete binary tree, and
  - Each node is greater than or equal to any of its children
    - Example heap:



    - Example <u>not a heap</u>, because the root (39) is less than its right child (42)



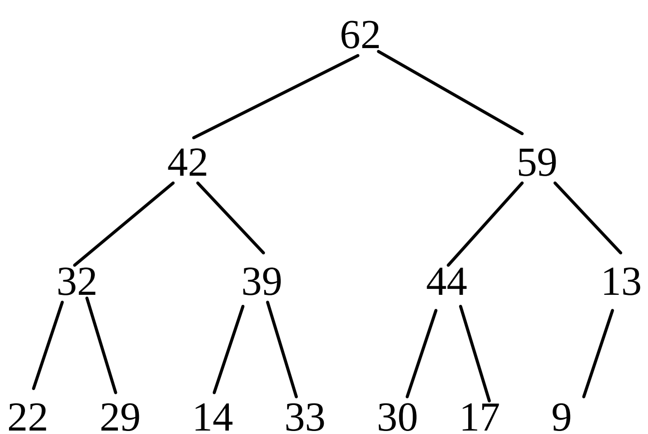
Heap Sort

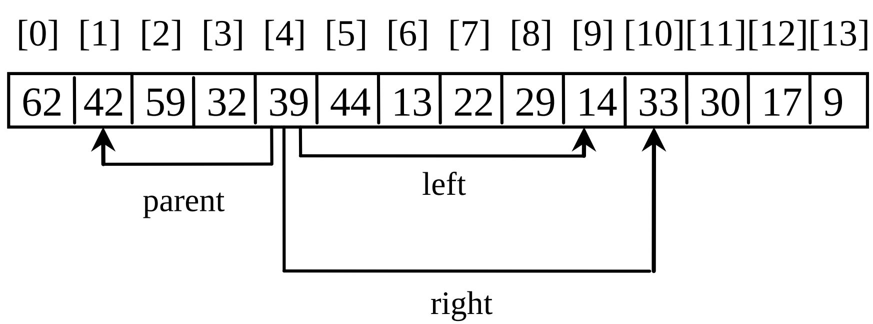
- Heaps are a useful data structures for designing efficient sorting algorithms and priority queues

- ***Heap sort*** uses a binary heap: it first adds all the elements to a heap and then removes the largest elements successively to obtain a sorted list

Storing a Heap

- A heap can be stored in an **`ArrayList`** or an array if the heap size is known in advance
  - For a node at position **`i`**, its left child is at position **`2i+1`** and its right child is at position **`2i+2`**, and its parent is at index **`(i-1)/2`** (integer division)
    - For example: the root is at position **`0`**, and its two children are at positions **`1`** and **`2`**
      - The node for element **`39`** is at position **`4`**, so its left child (element **`14`**) is at **`9`** because **`(2*4+1)`**, its right child (element **`33`**) is at **`10`** because **`(2*4+2)`**, and its parent (element **`42`**) is at **`1`** because **`((4-1)/2)`**





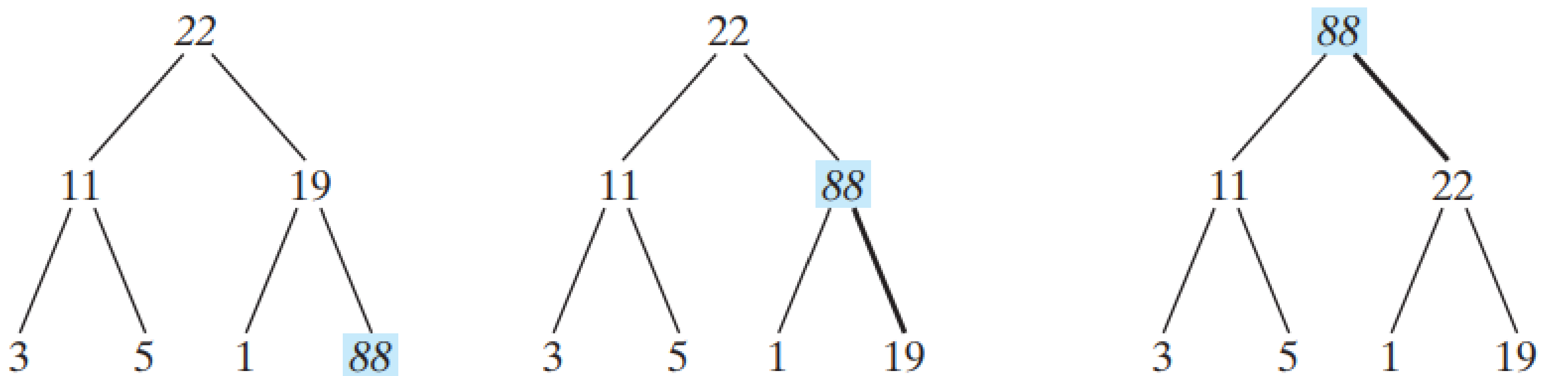
Adding Elements to a Heap

- To add a new node to a heap, first add it to the end of the heap and then rebuild the tree with this algorithm:

```
Let the last node be the current node;
while (the current node is greater than its parent) {
  Swap the current node with its parent;
  Now the current node is one level up;
}
```

Adding Elements to the Heap

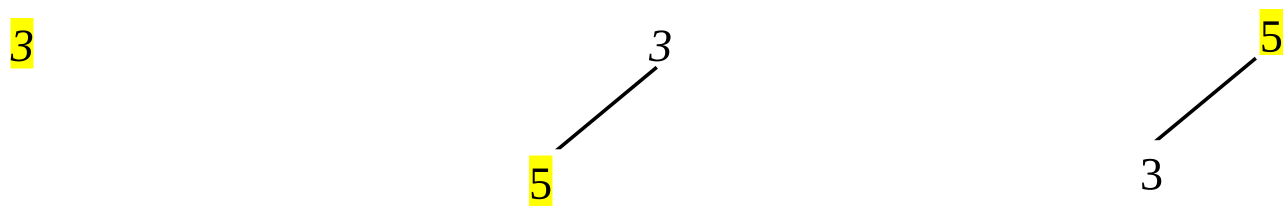
- Adding 88 in a heap:
  - Place the new node 88 at the end of the tree
  - Swap 88 with 19
  - Swap 88 with 22



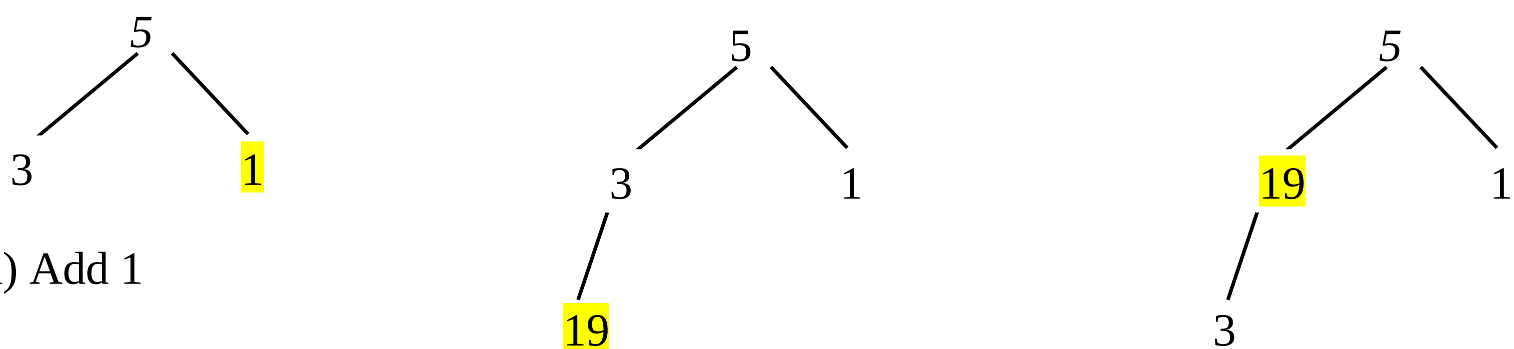
(a) Add 88 to a heap (b) After swapping 88 with 19 (c) After swapping 88 with 22

Adding Elements to the Heap

- Suppose a heap is initially empty. after adding numbers 3, 5, 1, 19, 11, and 22 **in this order**



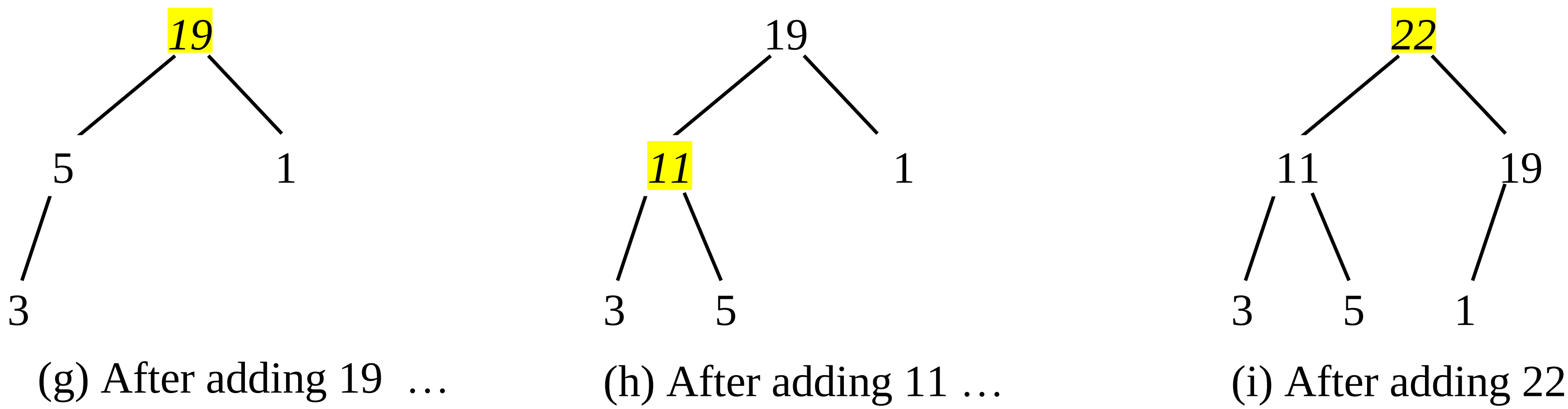
(a) After adding 3 (b) Add 5 (c) After adding 5



(d) Add 1 (e) Add 19 (f) Swap 19 with 3

Adding Elements to the Heap

- Suppose a heap is initially empty. after adding numbers 3, 5, 1, 19, 11, and 22 **<u>in this order</u>**



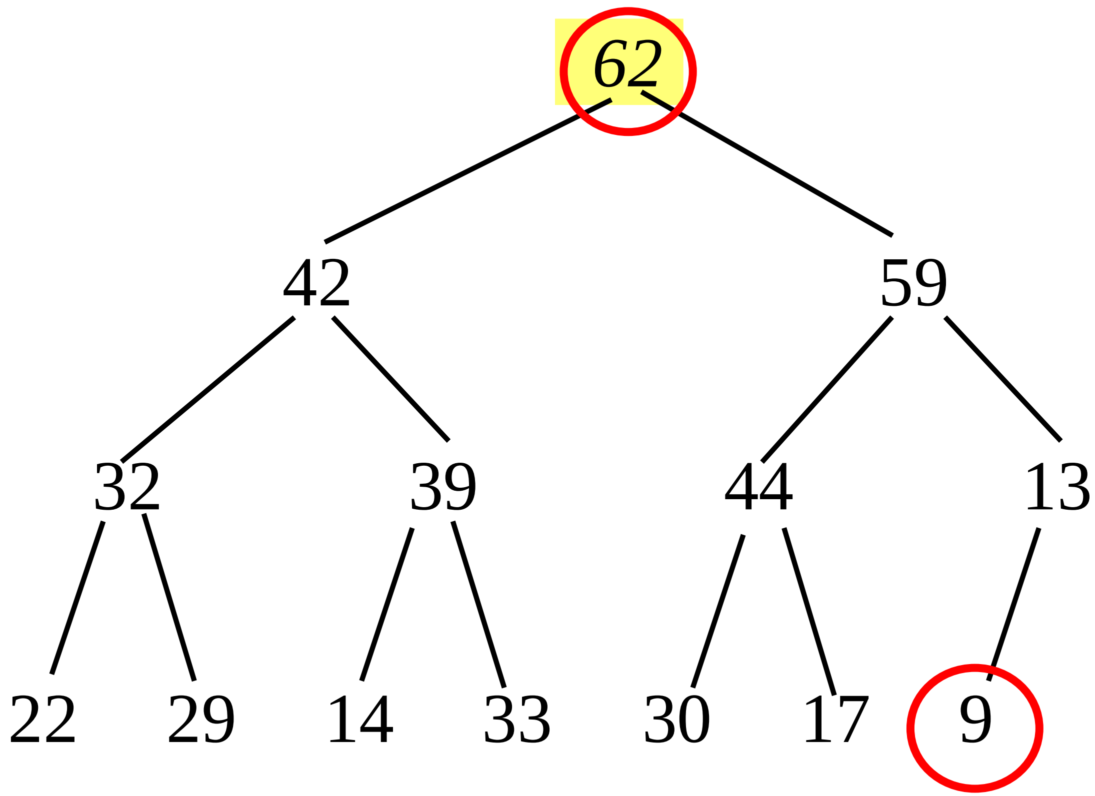
Removing the Root and Rebuild the Heap

- Often we need to remove the maximum element, which is the root in a heap
  - After the root is removed, the tree must be rebuilt to maintain the heap property using this algorithm:

```
Move the last node to replace the root;
Let the root be the current node;
while (the current node has children and the
current node is smaller than one of its children) {
  Swap the current node with the larger of its
    children;
  Now the current node is one level down;
}
```

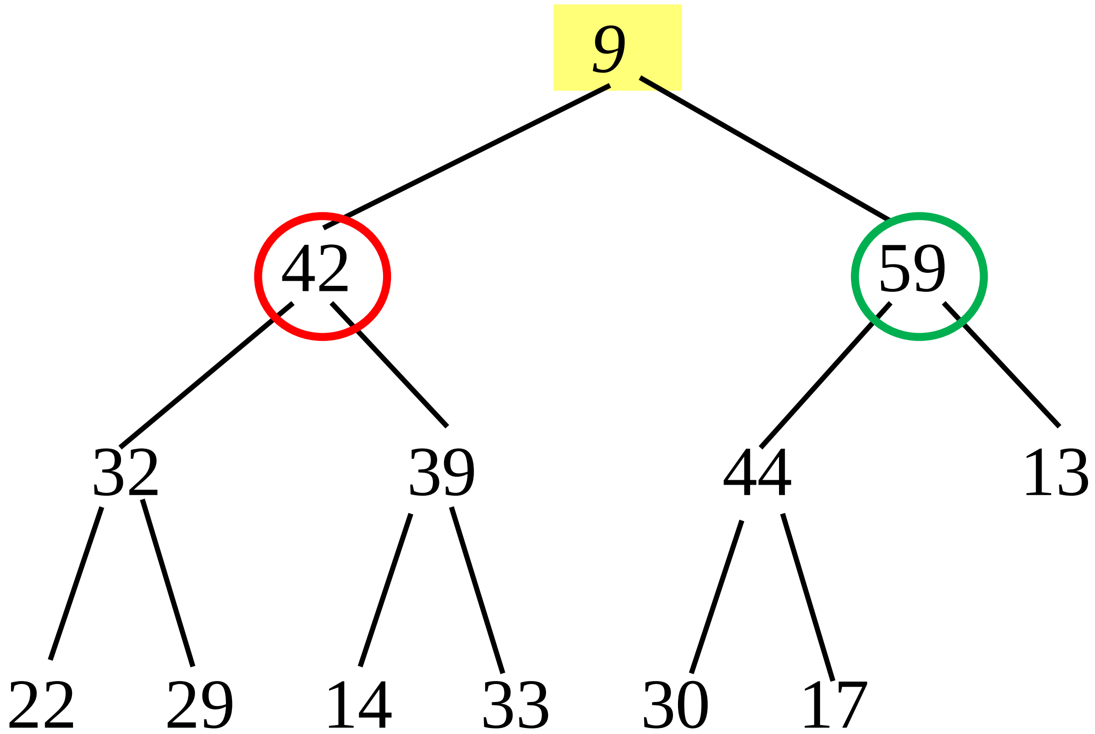
Removing the Root and Rebuild the Heap

- Removing root 62 from the heap (replaces it with the last node in the heap: 9)

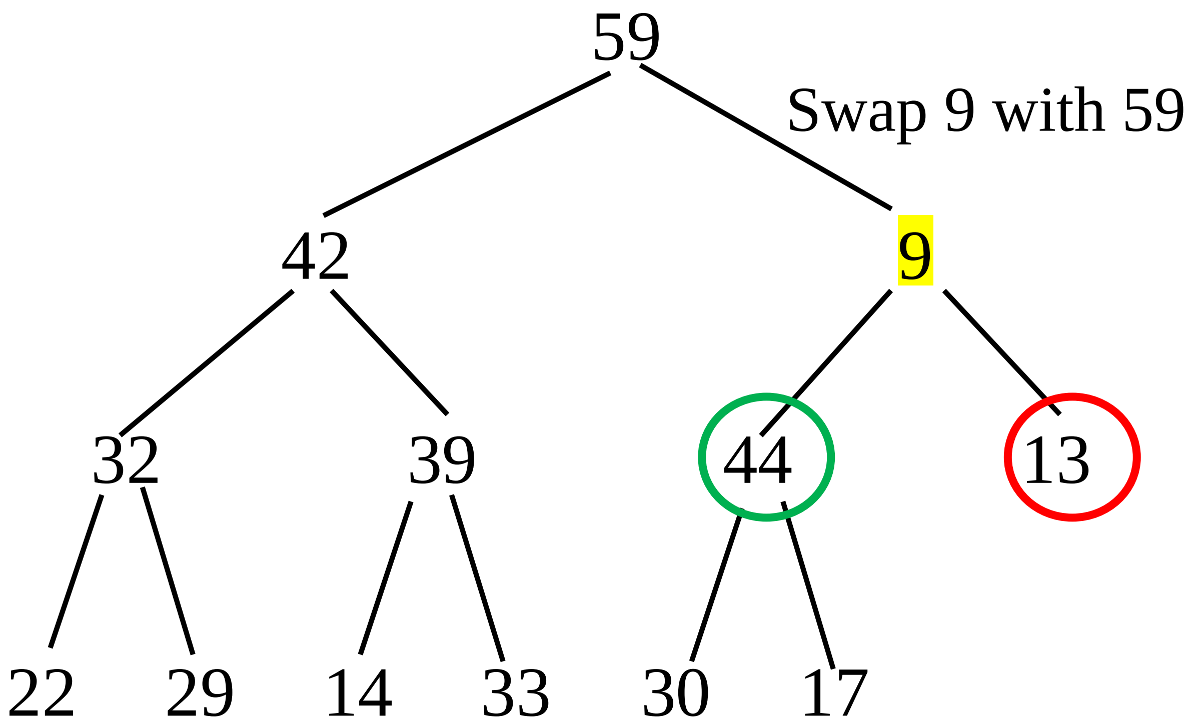


Removing the Root and Rebuild the Heap

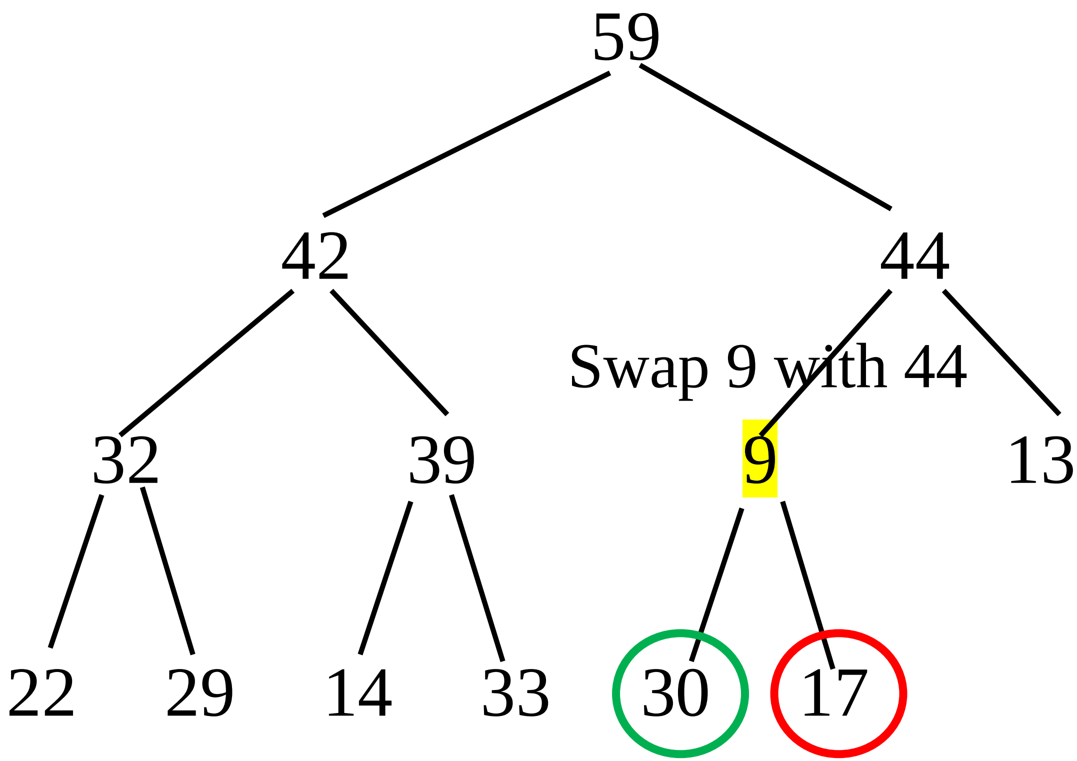
Move 9 to root and compare it with its children



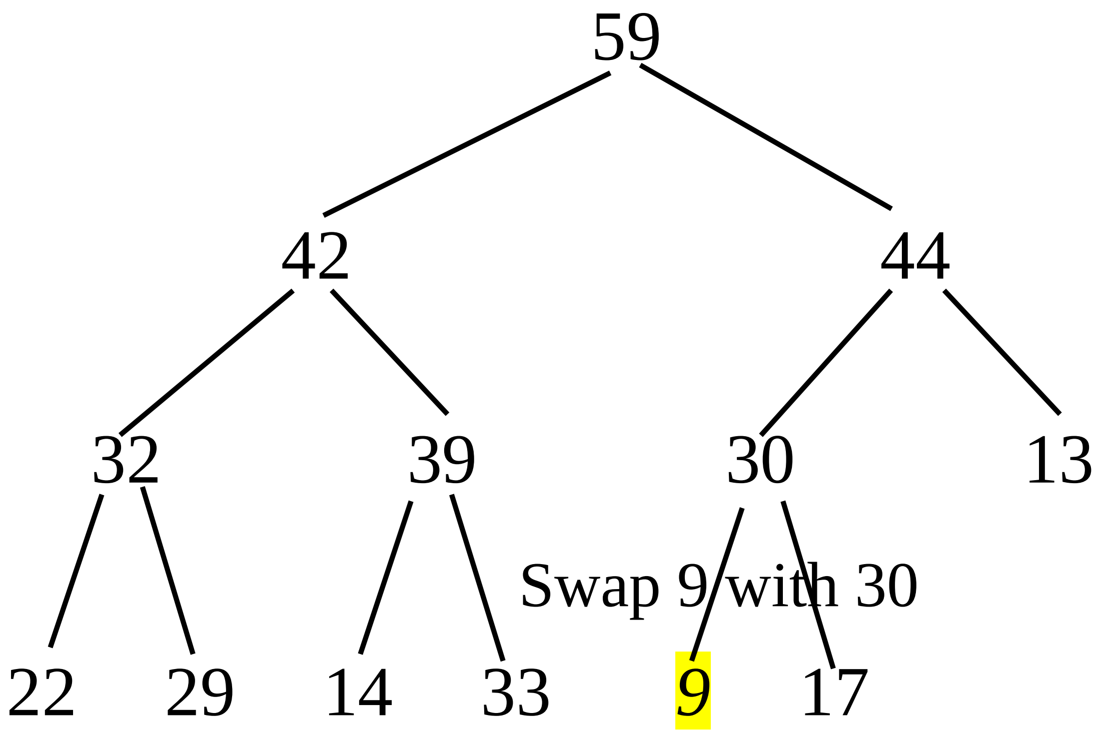
Removing the Root and Rebuild the Heap



Removing the Root and Rebuild the Heap



Removing the Root and Rebuild the Heap



The Heap Class

Heap<E extends Comparable<E>>	
`-list: java.util.ArrayList<E>`	
`+Heap()`	Creates a default empty heap.
`+Heap(objects: E[])`	Creates a heap with the specified objects.
`+add(newObject: E): void`	Adds a new object to the heap.
`+remove(): E`	Removes the root from the heap and returns it.
`+getSize(): int`	Returns the size of the heap.

```java
public class Heap<E extends Comparable> {
  private java.util.ArrayList<E> list = new java.util.ArrayList<E>();
  /** Create a default heap */
  public Heap() {
  }
  /** Create a heap from an array of objects */
  public Heap(E[] objects) {
    for (int i = 0; i < objects.length; i++)
      add(objects[i]);
  }
  /** Add a new object into the heap */
  public void add(E newObject) {
    list.add(newObject); // Append to the end of the heap
    int currentIndex = list.size() - 1; // The index of the last node
    while (currentIndex > 0) {
      int parentIndex = (currentIndex - 1) / 2;
      // Swap if the current object is greater than its parent
      if (list.get(currentIndex).compareTo(
          list.get(parentIndex)) > 0) {
        E temp = list.get(currentIndex);
        list.set(currentIndex, list.get(parentIndex));
        list.set(parentIndex, temp);
      } else
        break; // the tree is a heap now
      currentIndex = parentIndex;
    }
  }
```

```java
/** Remove the root from the heap */
public E remove() {
  if (list.size() == 0) return null;

  E removedObject = list.get(0);
  list.set(0, list.get(list.size() - 1));
  list.remove(list.size() - 1);

  int currentIndex = 0;
  while (currentIndex < list.size()) {
    int leftChildIndex = 2 * currentIndex + 1;
    int rightChildIndex = 2 * currentIndex + 2;

    // Find the maximum between two children
    if (leftChildIndex >= list.size())
      break; // The tree is a heap
    int maxIndex = leftChildIndex;
    if (rightChildIndex < list.size())
      if (list.get(maxIndex).compareTo(
          list.get(rightChildIndex)) < 0)
        maxIndex = rightChildIndex;
```

```java
      // Swap if the current node is less than the maximum
      if (list.get(currentIndex).compareTo(
          list.get(maxIndex)) < 0) {
        E temp = list.get(maxIndex);
        list.set(maxIndex, list.get(currentIndex));
        list.set(currentIndex, temp);
        currentIndex = maxIndex;
      }
      else
        break; // The tree is a heap
    }
    return removedObject;
  }

  /** Get the number of nodes in the tree */
  public int getSize() {
    return list.size();
  }
}
```

Heap Sort

```
public class HeapSort {
  public static <E extends Comparable> void heapSort(E[] list) {
    // Create a Heap of E
    Heap<E> heap = new Heap<E>();

    // Add elements to the heap
    for (int i = 0; i < list.length; i++)
      heap.add(list[i]);

    // Remove the highest elements from the heap
    for (int i = list.length - 1; i >= 0; i--)
      list[i] = heap.remove();
  }

  /** A test method */
  public static void main(String[] args) {
    Integer[] list = {2, 3, 2, 5, 6, 1, -2, 3, 14, 12};
    heapSort(list);
    for (int i = 0; i < list.length; i++)
      System.out.print(list[i] + " ");
  }
}
```

```java
public class HeapSortTest {
  public static void main(String[] args) {
    int size = 100000;
    int[] a = new int[size];
    randomInitiate(a);
    Integer[] b = new Integer[a.length];
    for(int i=0; i<b.length; i++)
      b[i] = a[i];

    long startTime = System.currentTimeMillis();
    HeapSort.heapSort(b);
    long endTime = System.currentTimeMillis();

    System.out.println((endTime - startTime) + "ms");
  }

  private static void randomInitiate(int[] a) {
    for (int i = 0; i < a.length; i++)
      a[i] = (int) (Math.random() * a.length);
  }
}
76ms
```

Heap Sort Time Complexity

- Let *h* denote the height for a heap of *n* elements. Since a heap is a complete binary tree, the first level has 1 node, the second level has 2 nodes, the *k*th level has $2^{(k-1)}$ nodes, the *(h-1)*th level has $2^{(h-2)}$ nodes, and the *h*th level has at least one node and at most $2^{(h-1)}$ nodes. Therefore, the number of nodes ***n*** is:

$$1+2+\ldots+2^{h-2} < n \le 1+2+\ldots+2^{h-2}+2^{h-1}$$

$$2^{h-1}-1 < n \le 2^{h}-1$$

$$2^{h-1} < n+1 \le 2^{h}$$

$$\log 2^{h-1} < \log(n+1) \le \log 2^{h}$$

$$h-1 < \log(n+1) \le h$$

- Thus, $log(n + 1) \le h < log(n + 1) + 1$
- Hence, the height of the heap is **O(log n)**

Heap Add and Remove Time

- Since the add method traces a path from a leaf to a root, it takes at most **h = log n** steps to add a new element to the heap.
  - Thus, the total time for constructing an initial heap is **O(n log n)** for an array of **n** elements.
- Since the remove method traces a path from a root to a leaf, it takes at most **h = log n** steps to rebuild a heap after removing the root from the heap.
  - Since the remove method is invoked **n** times, the total time for producing a sorted array from a heap is **O(n log n)**

Heap Sort Time Complexity

- Heap Sort Time: **O(n log n)**

- Merge sort requires a temporary array for merging two subarrays; a heap sort does not need additional array space.
  - Therefore, a heap sort is more space efficient than a merge sort.

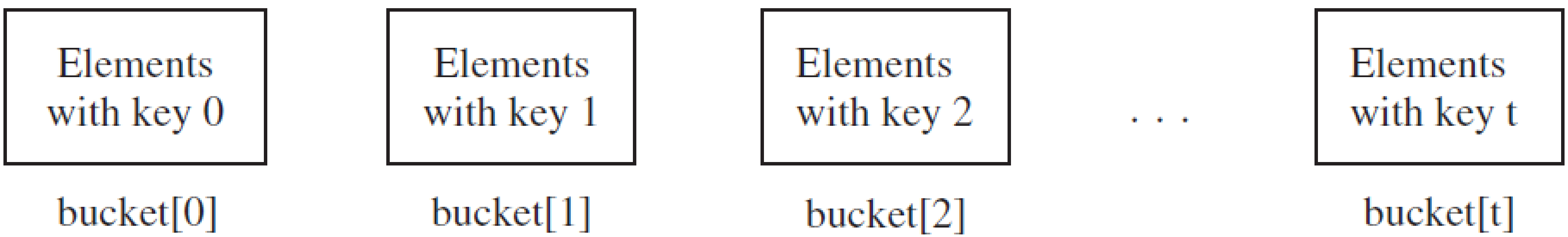
Bucket Sort and Radix Sort

- All sort algorithms discussed so far are general sorting algorithms that work for **any types of *keys*** (e.g., integers, strings, and any comparable objects)
  - These algorithms sort the elements by comparing their keys
  - The lower bound for general sorting algorithms is ***O(n logn)***
  - So, no sorting algorithms based on comparisons can perform better than ***O(n logn)***

- However, if the keys are "small" integers, you can use bucket sort without having to compare the keys

Bucket Sort

- The ***bucket sort*** algorithms:
  - Assume the keys are in the range from **`0`** to **`t`**
  - We need **`t+1`** buckets labeled **`0`**, **`1`**, …, and **`t`**
  - If an element's key is **`i`**, the element is put into the bucket **`i`**
    - Each bucket holds the elements with the same key value



- You can use an **`ArrayList`** to implement each bucket element

Bucket Sort

```
static int t;
public static <E> void bucketSort(E[] list) {
  java.util.ArrayList<E>[] bucket = new java.util.ArrayList[t+1];
  // Distribute the elements from list to buckets
  for (int i = 0; i < list.length; i++) {
    // Assume element has the getKey() method
    int key = getKey(list[i]); // list[i].getKey()
    if (bucket[key] == null)
      bucket[key] = new java.util.ArrayList<E>();
    bucket[key].add(list[i]);
  }
  // Now move the elements from the buckets back to list
  int k = 0; // k is an index for list
  for (int i = 0; i < bucket.length; i++)
    if (bucket[i] != null)
      for (int j = 0; j < bucket[i].size(); j++)
        list[k++] = bucket[i].get(j);
}
```

Bucket Sort

- Takes ***O(n + t)*** time to sort the list and uses ***O(n + t)*** space, where ***n*** is the list size and ***t*** is the number of buckets
- Bucket sort is ***stable***, meaning that if two elements in the original list have the same key value, their order is not changed in the sorted list.
  - That is, if element **`e1`** and element **`e2`** have the same key and **`e1`** precedes **`e2`** in the original list, **`e1`** still precedes **`e2`** in the sorted list
- For sorting positive integers, if ***t*** is too large, using the bucket sort is not desirable
  - Instead, you can use a radix sort
  - The radix sort is based on the bucket sort, but a radix sort uses only <u>**ten**</u> buckets

Radix Sort

- Assume that the keys are positive integers
- The idea for the radix sort is to divide the keys into subgroups based on their radix/digits positions
  - It applies a bucket sort repeatedly for the key values on radix positions, starting from the least-significant position

Radix Sort

- Bucket sort 33**1**, 45**4**, 23**0**, 3**4**, 34**3**, 4**5**, 5**9**, 45**3**, 34**5**, 23**1**, **9**

bucket[0]	bucket[1]	bucket[2]	bucket[3]	bucket[4]	bucket[5]	bucket[6]	bucket[7]	bucket[8]	bucket[9]
230	331		343	454	45				59
	231		453	34	345				9

- Remove elements from the buckets and bucket sort by 2nd digit:

2**3**0, 3**3**1, 2**3**1, 3**4**3, 4**5**3, 4**5**4, **3**4, **4**5, 3**4**5, **5**9, **0**9

bucket[0]	bucket[1]	bucket[2]	bucket[3]	bucket[4]	bucket[5]	bucket[6]	bucket[7]	bucket[8]	bucket[9]
9			230	343	453				
			331	45	454				
			231	345	59				
			34						

009, **2**30, **3**31, **2**31, **0**34, **3**43, **0**45, **3**45, **4**53, **4**54,59

bucket[0]	bucket[1]	bucket[2]	bucket[3]	bucket[4]	bucket[5]	bucket[6]	bucket[7]	bucket[8]	bucket[9]
9		230	331	453					
34		231	343	454					
45			345						
59									

9, 34, 45, 59, 230, 231, 331, 343, 345, 453, 454

Radix Sort

- Radix sort takes $O(d*n)$ time to sort n elements with integer keys, where d is the maximum number of the radix positions among all keys

```java
public class RadixSort {
  static int t = 10;
  public static void bucketSort(int[] list) {
    java.util.ArrayList<Integer>[] bucket =new java.util.ArrayList[t+1];
    // Distribute the elements from list to buckets
    for (int i = 0; i < list.length; i++) {
      // Assume element has the getKey() method
      int key = getKey(list[i]);
      if (bucket[key] == null)
        bucket[key] = new java.util.ArrayList<Integer>();
      bucket[key].add(list[i]);
    }
    // Now move the elements from the buckets back to list
    int k = 0; // k is an index for list
    for (int i = 0; i < bucket.length; i++) {
      if (bucket[i] != null) {
        for (int j = 0; j < bucket[i].size(); j++)
          list[k++] = (int)(bucket[i].get(j));
      }
    }
  }
  static int radix = 0;
  public static int getKey(int n) { // get the n'th digit from right
    for(int i=0; i<radix; i++)
      n = n / 10;
    return n % 10;
  }
```

```java
  public static void main(String[] args) {
    int size = 100000;
    int[] a = new int[size];
    randomInitiate(a);
    long startTime = System.currentTimeMillis();
    // radix sort
    for(int i=0; i<6; i++) { // run bucketSort for 6 digits
      radix = i;             //    starting from right
      bucketSort(a);
    }
    long endTime = System.currentTimeMillis();
    System.out.println((endTime - startTime) + "ms");
  }
  private static void randomInitiate(int[] a) {
    for (int i = 0; i < a.length; i++)
      a[i] = (int) (Math.random() * a.length);
  }
}
```

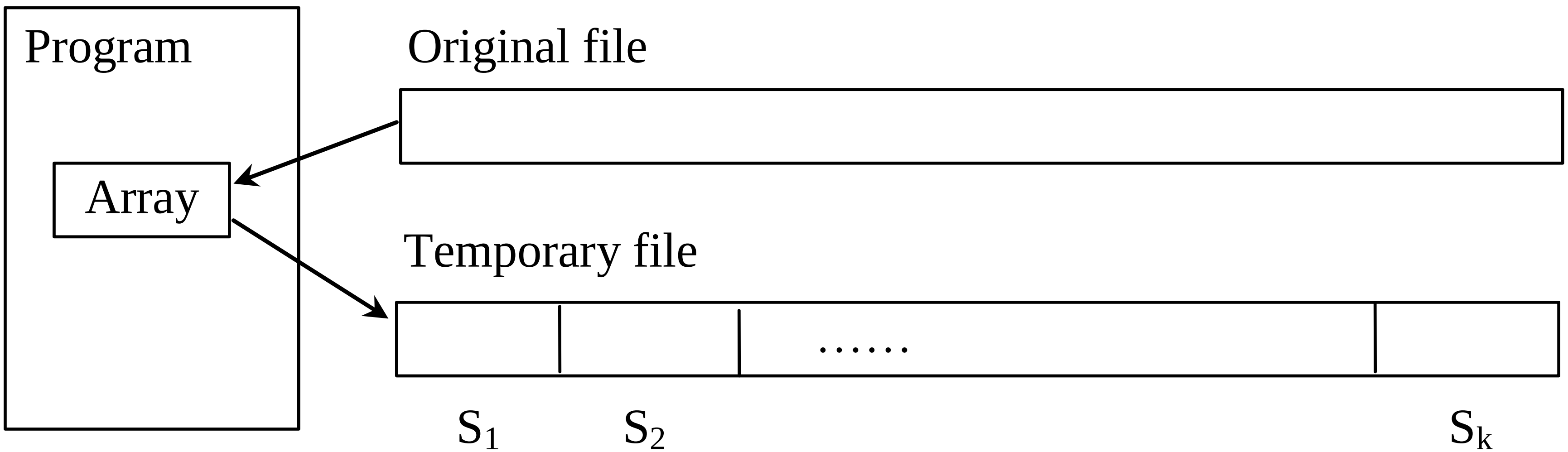
433ms

External Sort

- All the sort algorithms discussed in the preceding sections assume that all data to be sorted is available at one time in internal memory such as an array
- To sort data stored in an external file, you may first bring data to the memory, then sort it internally.
- However, if the file is too large, all data in the file cannot be brought to memory at one time

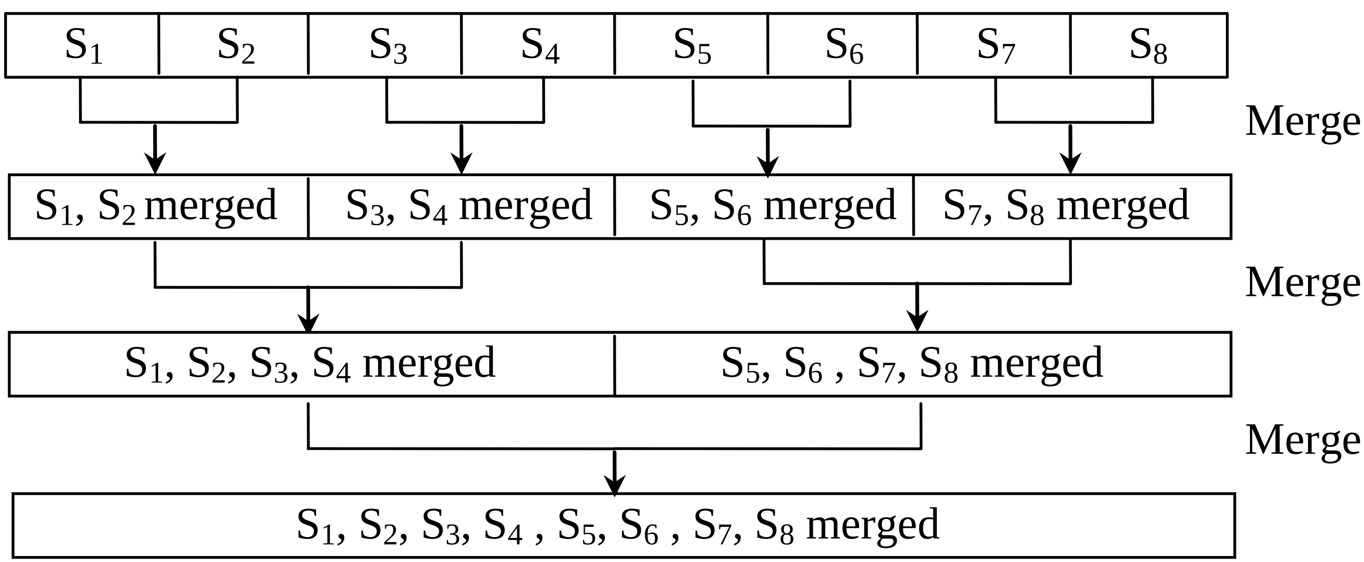
Phase I

- Repeatedly bring partial data from the file to an array, sort the array using an internal sorting algorithm, and output the data from the array to a temporary file



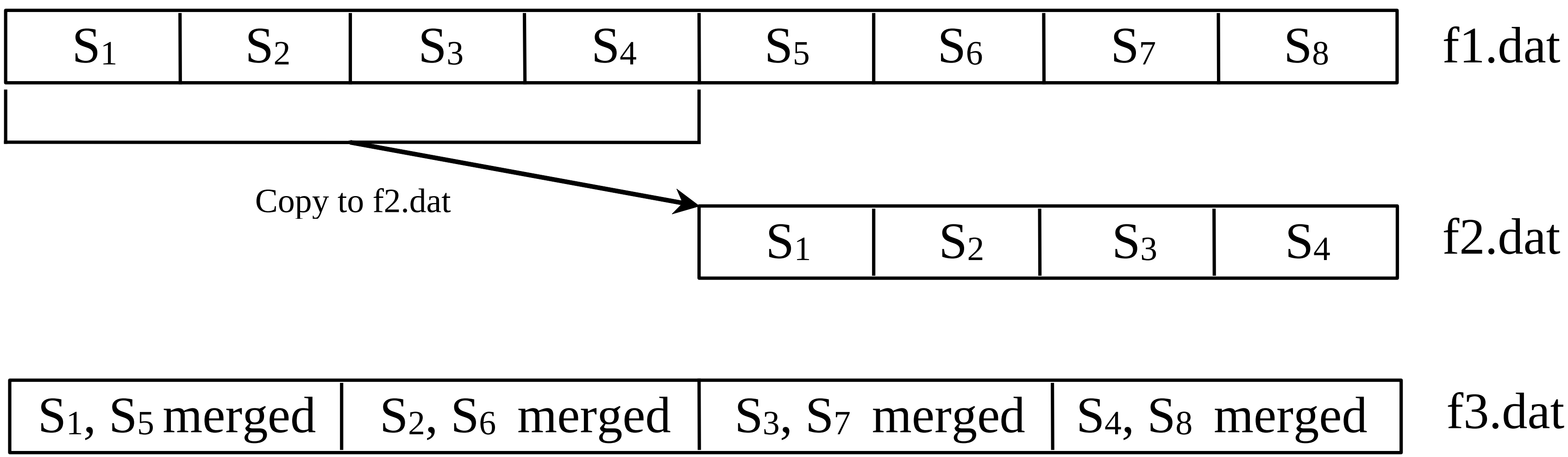
Phase II

- **Merge** pairs of sorted segments (e.g., S1 with S2, S3 with S4, ..., and so on) into a larger sorted segment and save the new segment into a new temporary file
- Continue the same process until one sorted segment results



Implementing Phase II

- A segment is too large to be brought to an array in memory
  - To implement a merge step, copy half number of segments from file f1.dat to a temporary file f2.dat.



  - Then merge the first remaining segment in f1.dat with the first segment in f2.dat into a temporary file named f3.dat.

External Sort Complexity

- In the external sort, the dominating cost is that of I/O.
- Assume **`n`** is the number of elements to be sorted in the file
  - In Phase I, **`n`** number of elements are read from the original file and output to a temporary file, therefore, the I/O for Phase I is **`O(n)`**.

External Sort Complexity

- In Phase II, before the first merge step, the number of sorted segments is **`n/c`**, where **`c`** is **`MAX_ARRAY_SIZE`**
  - Each merge step reduces the number of segments by half
    - After the first merge step, the number of segments is

`n/c * 1/2 = n/2c`

  - After the second merge step, the number of segments is

$\texttt{n/2c * 1/2 = n/(2}^{2}\texttt{c)}$

  - After the third merge step the number of segments is **$\texttt{n/2}^{3}\texttt{c}$**
  - After **`log(n/c)`** merge steps, the number of segments is reduced to 1
  - Therefore, the total number of merge steps is **`log(n/c)`**

External Sort Complexity

- In each merge step, half the number of segments are read from file f1 and then written into a temporary file f2 - the remaining segments in f1 are merged with the segments in f2
  - The number of I/Os in each merge step is **`O(n)`**
- Since the total number of merge steps is **`log(n/c)`**, the total number of I/Os in Step 2 is:

  `O(n)*log(n/c) = O(n log n)`

- Therefore, the complexity of the external sort is

`O(n log n)` in I/Os

```java
import java.io.*;

public class CreateLargeFile {
  public static void main(String[] args) throws Exception {
    DataOutputStream output = new DataOutputStream(
      new BufferedOutputStream(new FileOutputStream("largedata.dat")));
    for (int i = 0; i < 800004; i++)
      output.writeInt((int)(Math.random() * 1000000));
    output.close();

    // Display first 100 numbers
    DataInputStream input =
      new DataInputStream(new FileInputStream("largedata.dat"));
    for (int i = 0; i < 100; i++)
      System.out.print(input.readInt() + " ");
    input.close();
  }
}
```

```java
import java.io.*;

public class SortLargeFile {
  public static final int MAX_ARRAY_SIZE = 43;
  public static final int BUFFER_SIZE = 100000;

  public static void main(String[] args) throws Exception {
    // Sort largedata.dat to sortedfile.dat
    sort("largedata.dat", "sortedfile.dat");

    // Display the first 100 numbers in the sorted file
    displayFile("sortedfile.dat");
  }

  /** Sort data in source file and into target file */
  public static void sort(String sourcefile, String targetfile)
      throws Exception {
    // Implement Phase 1: Create initial segments
    int numberOfSegments =
      initializeSegments(MAX_ARRAY_SIZE, sourcefile, "f1.dat");

    // Implement Phase 2: Merge segments recursively
    merge(numberOfSegments, MAX_ARRAY_SIZE,
      "f1.dat", "f2.dat", "f3.dat", targetfile);
  }
```

```java
/** Sort original file into sorted segments */
private static int initializeSegments
    (int segmentSize, String originalFile, String f1)
    throws Exception {
  int[] list = new int[segmentSize];
  DataInputStream input = new DataInputStream(
    new BufferedInputStream(new FileInputStream(originalFile)));
  DataOutputStream output = new DataOutputStream(
    new BufferedOutputStream(new FileOutputStream(f1)));
  int numberOfSegments = 0;
  while (input.available() > 0) {
    numberOfSegments++;
    int i = 0;
    for ( ; input.available() > 0 && i < segmentSize; i++) {
      list[i] = input.readInt();
    }
    // Sort an array list[0..i-1]
    java.util.Arrays.sort(list, 0, i);
    // Write the array to f1.dat
    for (int j = 0; j < i; j++) {
      output.writeInt(list[j]);
    }
  }
  input.close();
  output.close();
  return numberOfSegments;
}
```

```
private static void merge(int numberOfSegments, int segmentSize,
    String f1, String f2, String f3, String targetfile)
    throws Exception {
  if (numberOfSegments > 1) {
    mergeOneStep(numberOfSegments, segmentSize, f1, f2, f3);
    merge((numberOfSegments + 1) / 2, segmentSize * 2,
      f3, f1, f2, targetfile);
  }
  else { // Rename f1 as the final sorted file
    File sortedFile = new File(targetfile);
    if (sortedFile.exists()) sortedFile.delete();
    new File(f1).renameTo(sortedFile);
  }
}

private static void mergeOneStep(int numberOfSegments,
    int segmentSize, String f1, String f2, String f3)
    throws Exception {
  DataInputStream f1Input = new DataInputStream(
    new BufferedInputStream(new FileInputStream(f1), BUFFER_SIZE));
  DataOutputStream f2Output = new DataOutputStream(
    new BufferedOutputStream(new FileOutputStream(f2), BUFFER_SIZE));

  // Copy half number of segments from f1.dat to f2.dat
  copyHalfToF2(numberOfSegments, segmentSize, f1Input, f2Output);
  f2Output.close();
```

```
  // Merge remaining segments in f1 with segments in f2 into f3
  DataInputStream f2Input = new DataInputStream(
    new BufferedInputStream(new FileInputStream(f2), BUFFER_SIZE));
  DataOutputStream f3Output = new DataOutputStream(
    new BufferedOutputStream(new FileOutputStream(f3), BUFFER_SIZE));

  mergeSegments(numberOfSegments / 2,
    segmentSize, f1Input, f2Input, f3Output);

  f1Input.close();
  f2Input.close();
  f3Output.close();
}

/** Copy first half number of segments from f1.dat to f2.dat */
private static void copyHalfToF2(int numberOfSegments,
    int segmentSize, DataInputStream f1, DataOutputStream f2)
    throws Exception {
  for (int i = 0; i < (numberOfSegments / 2) * segmentSize; i++) {
    f2.writeInt(f1.readInt());
  }
}
```

```java
/** Merge all segments */
private static void mergeSegments(int numberOfSegments,
    int segmentSize, DataInputStream f1, DataInputStream f2,
    DataOutputStream f3) throws Exception {
  for (int i = 0; i < numberOfSegments; i++) {
    mergeTwoSegments(segmentSize, f1, f2, f3);
  }

  // If f1 has one extra segment, copy it to f3
  while (f1.available() > 0) {
    f3.writeInt(f1.readInt());
  }
}

/** Merges two segments */
private static void mergeTwoSegments(int segmentSize,
  DataInputStream f1, DataInputStream f2,
  DataOutputStream f3) throws Exception {
  int intFromF1 = f1.readInt();
  int intFromF2 = f2.readInt();
  int f1Count = 1;
  int f2Count = 1;
```

```java
    while (true) {
      if (intFromF1 < intFromF2) {
        f3.writeInt(intFromF1);
        if (f1.available() == 0 || f1Count++ >= segmentSize) {
          f3.writeInt(intFromF2);
          break;
        } else {
          intFromF1 = f1.readInt();
        }
      } else {
        f3.writeInt(intFromF2);
        if (f2.available() == 0 || f2Count++ >= segmentSize) {
          f3.writeInt(intFromF1);
          break;
        } else {
          intFromF2 = f2.readInt();
        }
      }
    }
    while (f1.available() > 0 && f1Count++ < segmentSize) {
      f3.writeInt(f1.readInt());
    }
    while (f2.available() > 0 && f2Count++ < segmentSize) {
      f3.writeInt(f2.readInt());
    }
}
```

```java
  /** Display the first 100 numbers in the specified file */
  public static void displayFile(String filename) {
    try {
      DataInputStream input =
        new DataInputStream(new FileInputStream(filename));
      for (int i = 0; i < 100; i++)
        System.out.print(input.readInt() + " ");
      input.close();
    }
    catch (IOException ex) {
      ex.printStackTrace();
    }
  }
}
```