Normalized cuts and image segmentation

Department of EE
University of Washington

Yeping Su
Xiaodan Song

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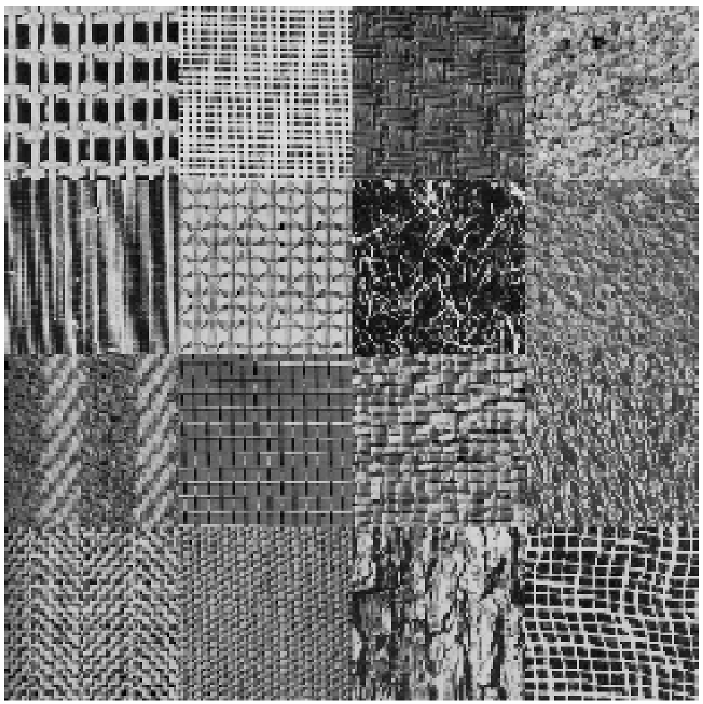
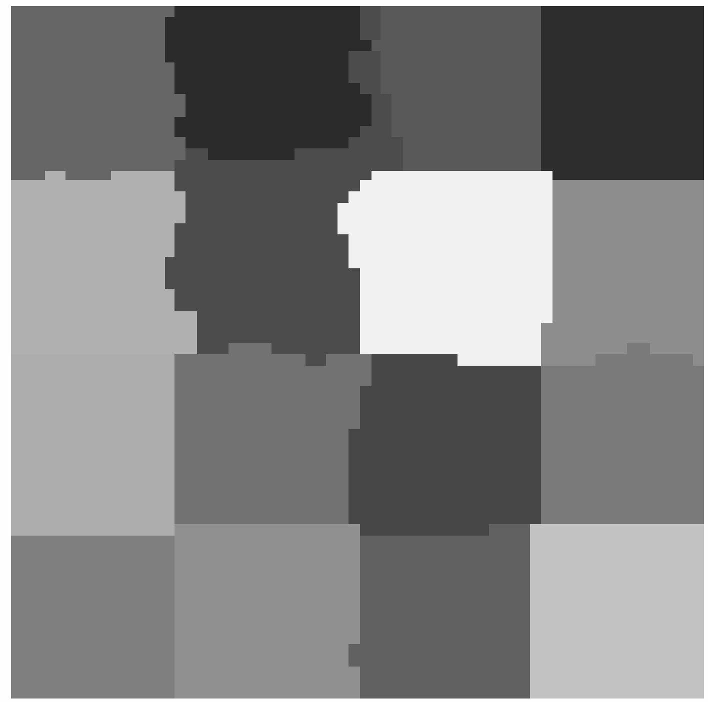
Outline

1. Image segmentation methods overview
   i. Definition
   ii. Complete & partial segmentation
   iii. Gestalt laws of perceptual organization
   iv. Image segmentation methods
2. Normalized cut
   i. Basic idea
   ii. Grouping method
   iii. Experiment
   iv. Comparison
3. Conclusion

What is image segmentation?

Image segmentation is the process of dividing an image into parts that have a strong correlation with objects or areas of the real world.

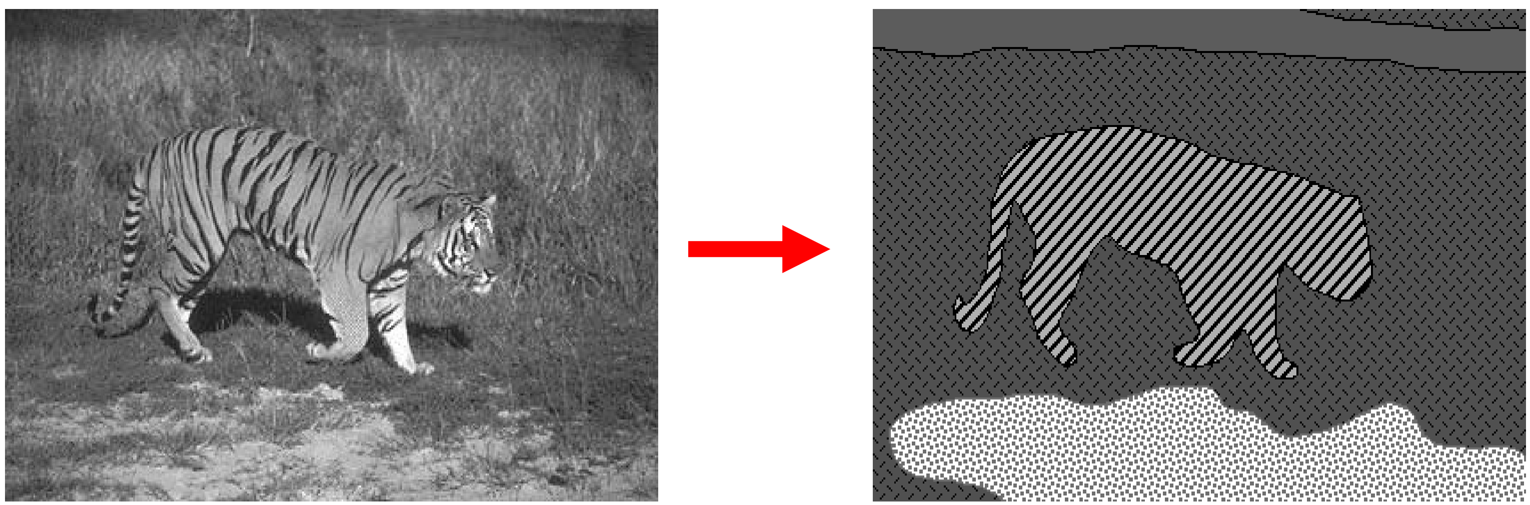




Complete vs. partial segmentation

Complete segmentation - divides an image into non-overlapping regions that match to the real world objects.

Cooperation with higher processing levels which use specific knowledge of the problem domain is necessary.



Complete vs. partial segmentation

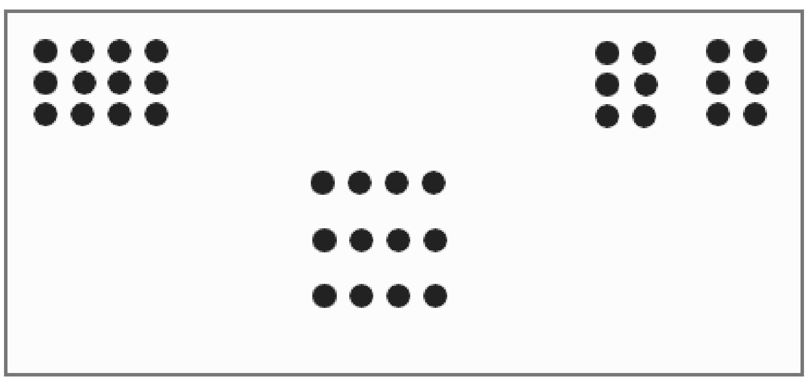
Partial segmentation- in which regions do not correspond directly with image objects.

Image is divided into separate regions that are homogeneous with respect to a chosen property such as brightness, color, texture, etc.

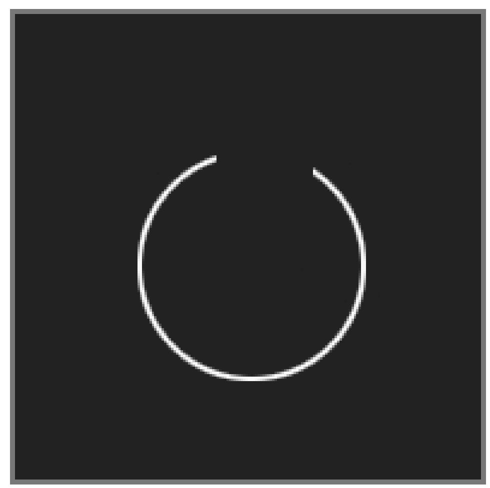
Gestalt laws of perceptual organization

The emphasis in the Gestalt approach was on the **configuration** of the elements.

Proximity: Objects that are closer to one another tend to be grouped together.



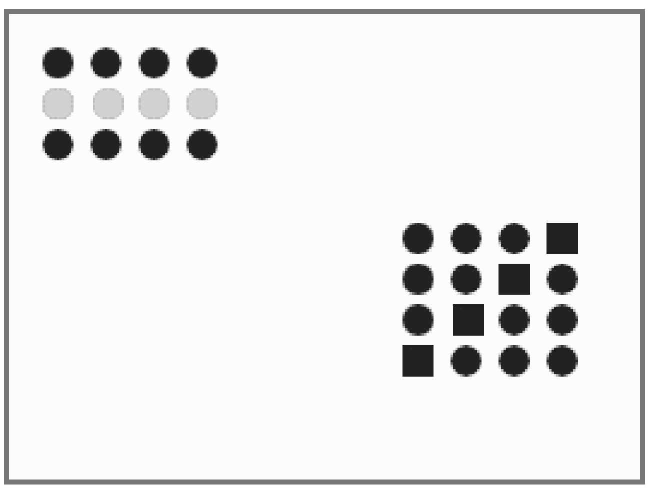
Closure: Humans tend to enclose a space by completing a contour and ignoring gaps.



Gestalt laws of perceptual organization

Similarity: Elements that look similar will be perceived as part of the same form. (color, shape, texture, and motion).

Continuation: Humans tend to continue contours whenever the elements of the pattern establish an implied direction.



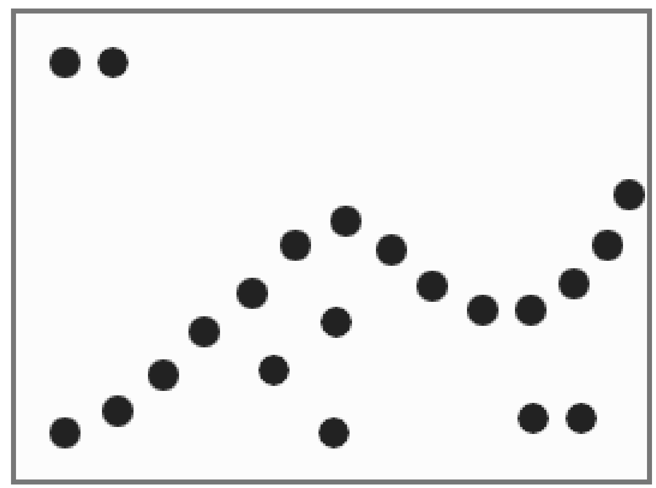


Image segmentation methods

1. Local filtering approaches

2. Active contour methods

3. Region growing and splitting techniques

4. Global optimization approaches based on energy functional

Local filtering approaches

Such as canny edge detector

original image (Lena)

norm of the gradient

thresholding

thinning

Local filtering approaches

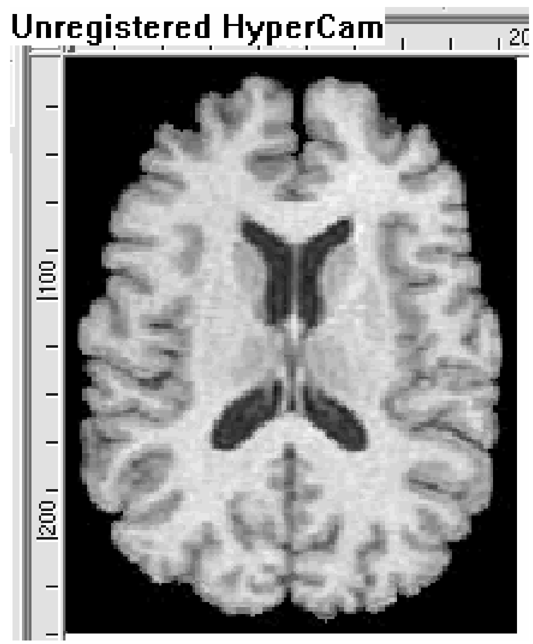
Only makes use of local information and cannot guarantee continuous closed edge contours

Active contour method (Snakes)

$$E(C) = \alpha \int_0^1 \left|C'(s)\right|^2 ds + \beta \int_0^1 \left|C''(s)\right|^2 ds - \lambda \int_0^1 \left|\nabla I(C(s))\right| ds$$

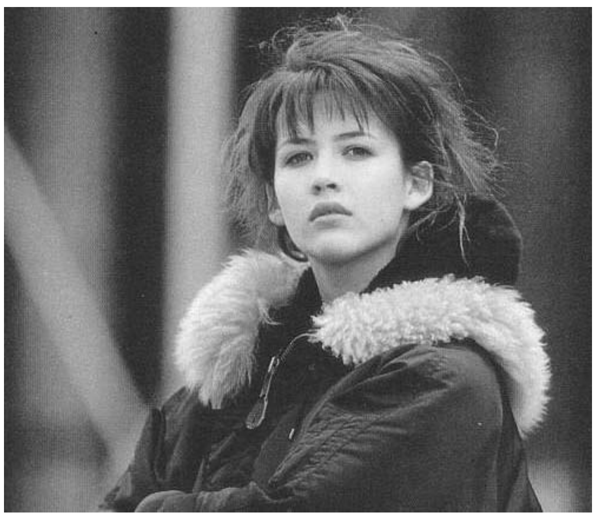
The internal energies emanate from the shape of the snakes-smoothness

The external energy attracts the snake to contours with large image gradients.



Region growing and splitting techniques

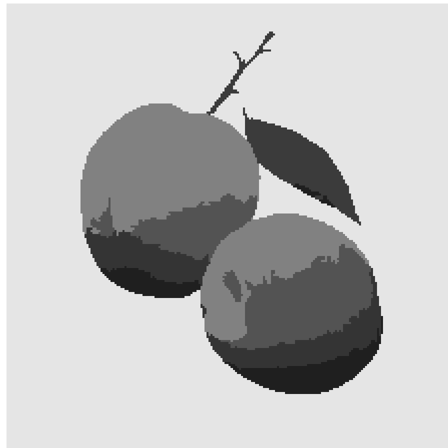
Such as K-means, mean shift, EM algorithms



Region based methods

An advantage of region based methods is that it tests the statistics inside the region

However it often generates irregular boundaries and small holes.



Global optimization approaches based on energy functional

Suppose that we are given a blurred image g:

$$g = Aw + n$$

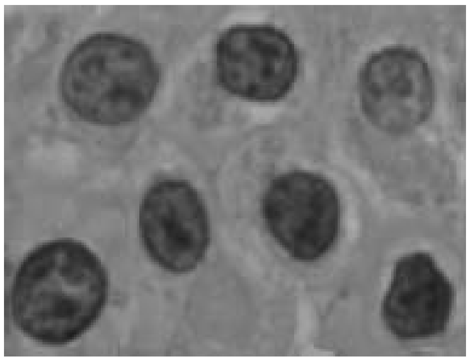
where A is a blurring operator, w is the unblurred image, n is the noise.

Mumford-Shah energy functional:

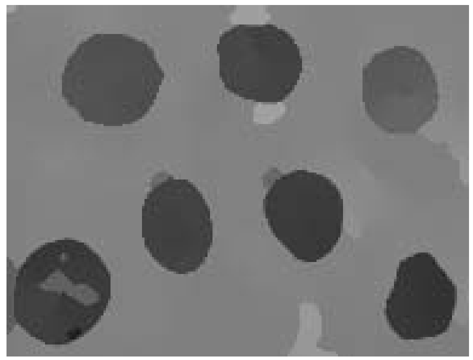
$$E(w,\Gamma) = \beta \iint_{\Omega} (g-w)^2 dxdy + \alpha \iint_{\Omega/\Gamma} \|\nabla w\|^2 dxdy + \gamma \|\Gamma\|$$

– data fidelity
– region smoothness
– boundary compactness

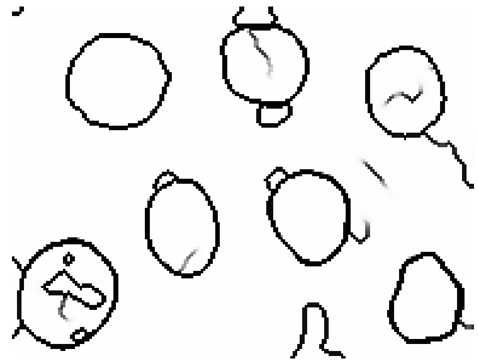
Mumford-Shah energy segmentation



Original



segmented results



boundaries

Normalized cut method

Goal:

Perceptually significant groups are detected first while small variations and details are treated later.

Different image features: intensity, color, texture, contour continuity, motion are treated in one uniform framework

Normalized cut method

Basic idea:

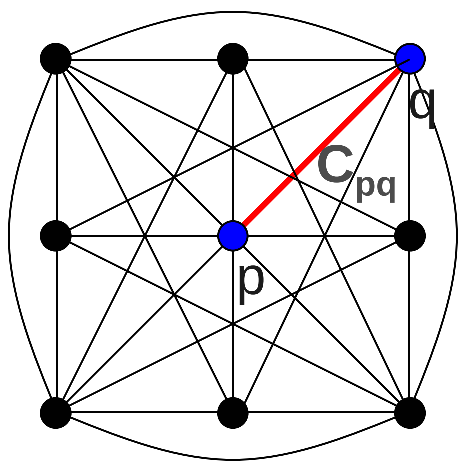
Propose a new graph-theoretic criterion for measuring the goodness of an image partition

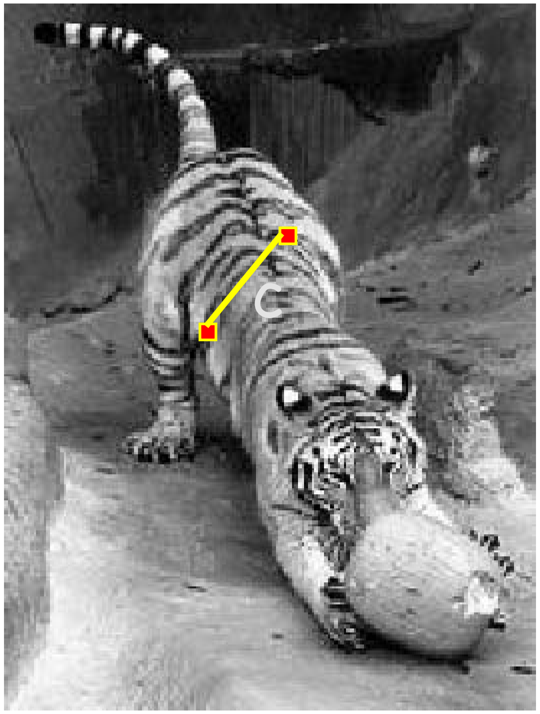
The minimization of this criterion can be formulated/approximated as a generalized eigenvalue problem.

Two questions:

1. What is a precise criterion for a good partition?
2. How can such a partition be computed efficiently?

Images as graphs

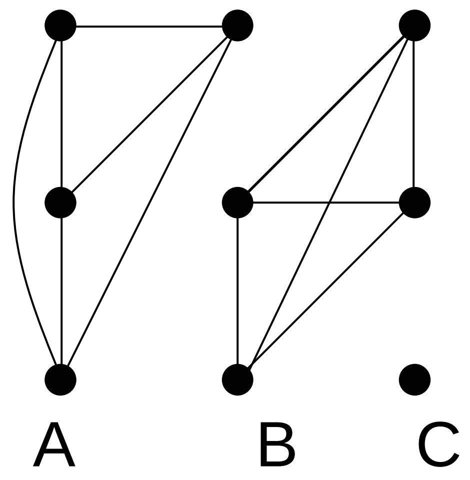




Fully-connected graph

- node for every pixel
- link between *every* pair of pixels, **p**,**q**
- cost $\mathbf{c}_{pq}$ for each link
  - $\mathbf{c}_{pq}$ measures *similarity*
    - » similarity is *inversely proportional* to difference in color, position and some other features

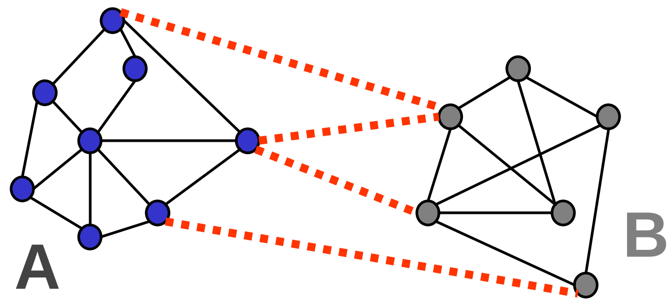
Segmentation by Graph Cuts



Break Graph into Segments

- Delete links that cross between segments
- Easiest to break links that have low cost
  - similar pixels should be in the same segments
  - dissimilar pixels should be in different segments

Cuts in a graph



Link Cut

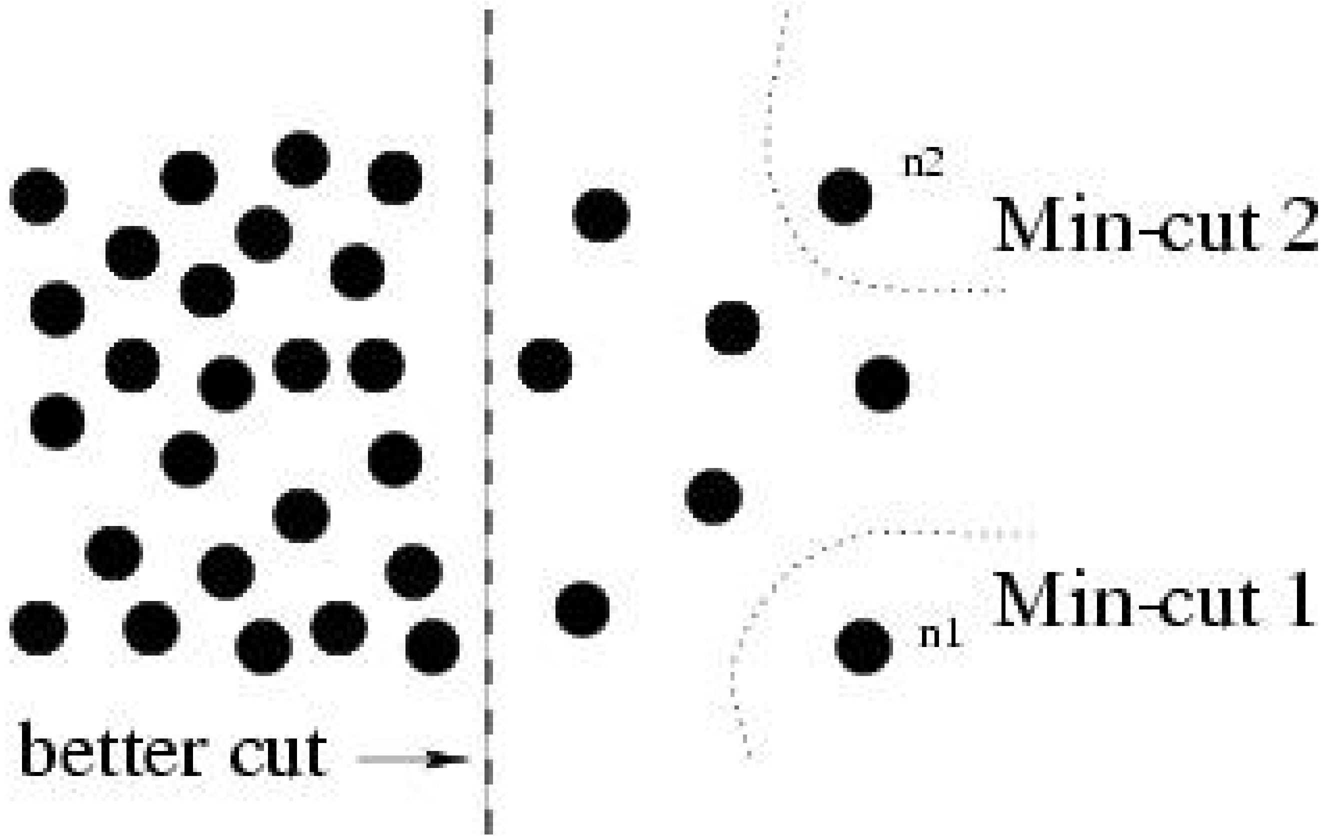
- set of links whose removal makes a graph disconnected
- cost of a cut:

$$cut(A, B) = \sum_{p \in A, q \in B} c_{p,q}$$

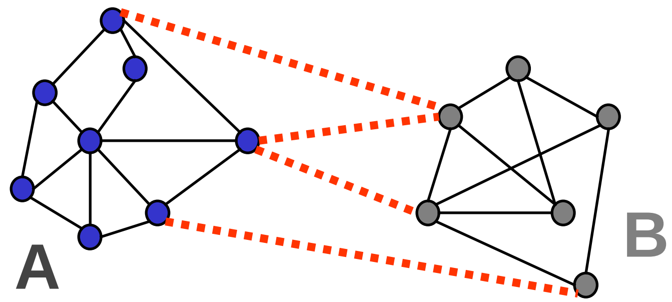
Find minimum cut

Well studied problem, and efficient algorithms exist

But min cut is not always the best cut...



Cuts in a graph



Normalized Cut

- a cut penalizes large segments
- fix by normalizing for size of segments

$$Ncut(A,B) = \frac{cut(A,B)}{volume(A)} + \frac{cut(A,B)}{volume(B)}$$

- Volume(A) = sum of costs of all edges that touch A

$$assoc(A,V) = \sum_{u \in A, t \in V} c(u,t)$$

Normalize Cut in Matrix Form

$\mathbf{W}$ is the cost matrix : $\mathbf{W}(i, j) = c_{i,j}$;

$\mathbf{D}$ is the sum of costs from node i : $\mathbf{D}(i,i) = \sum_j \mathbf{W}(i, j)$; $\mathbf{D}(i,j) = 0$

Can write normalized cut as: (NP-complete)

$$Ncut(A,B) = \frac{\mathbf{y}^{\mathbf{T}}(\mathbf{D}-\mathbf{W})\mathbf{y}}{\mathbf{y}^{\mathbf{T}}\mathbf{D}\mathbf{y}}, \quad \text{with } \mathbf{y}_i \in \{1, -b\}, \mathbf{y}^T\mathbf{D}1 = 0.$$

- Solution given by “generalized” eigenvalue problem:

$$(\mathbf{D}-\mathbf{W})\mathbf{y} = \lambda \mathbf{D}\mathbf{y}$$

- Solved by converting to standard eigenvalue problem:

$$\mathbf{D}^{-\frac{1}{2}}(\mathbf{D}-\mathbf{W})\mathbf{D}^{-\frac{1}{2}}\mathbf{z} = \lambda \mathbf{z}, \quad where\ \mathbf{z} = \mathbf{D}^{\frac{1}{2}}\mathbf{y}$$

- Solution of the continuous version problem corresponds to second smallest eigenvector
  - Not optimal!

Recursive two-way Ncut grouping algorithm

1. Given an image or image sequence, set up a weighted graph **G=(V, E)** and set the weight on the edge connection two nodes to be a measure of the similarity between the two nodes.
2. Solve $(\mathbf{D}-\mathbf{W})\mathbf{x}=\lambda\mathbf{D}\mathbf{x}$ for eigenvectors with the smallest eigenvalues.
3. Use the eigenvector with the second smallest eigenvalue to bipartition the graph.
4. Decide if the current partition should be subdivided and recursively repartition the segmented parts if necessary.

   Decide if the current partition should be subdivided by checking the stability of the cut, and make sure Ncut is below the pre-specified value

Simultaneous K-way cut

Use all of the top eigenvectors to simultaneously obtain a K-way partition.

1. A simple clustering method, such as k-means, is used to obtain an over-segmentation of the image into *k'* groups
2. (a) Greedy pruning: iteratively merge two segments as a time until only k segments are left, minimizing the k-way Ncut criterion

$$Ncut_k = \frac{cut(A_1, V - A_1)}{assoc(A_1, V)} + \frac{cut(A_2, V - A_2)}{assoc(A_2, V)} + \ldots + \frac{cut(A_k, V - A_k)}{assoc(A_k, V)}$$

(b) Global recursive cut

From the initial segments, build a condensed graph. Based on this graph, recursively bipartition according to Ncut criterion, either with the generalized eigenvector system or with exhaustive search in discrete domain.

Example: Brightness image

1. Using the brightness value of the pixels and their spatial location as the weight:

$$w_{ij} = e^{\frac{-\left|F_{(i)} - F_{(j)}\right|}{\sigma_I^2}} * \begin{cases} e^{\frac{-\left|X_{(i)} - X_{(j)}\right|}{\sigma_X^2}} & \text{if } \left\| X(i) - X(j) \right\|_2 < r \\ 0 & \text{otherwise} \end{cases}$$

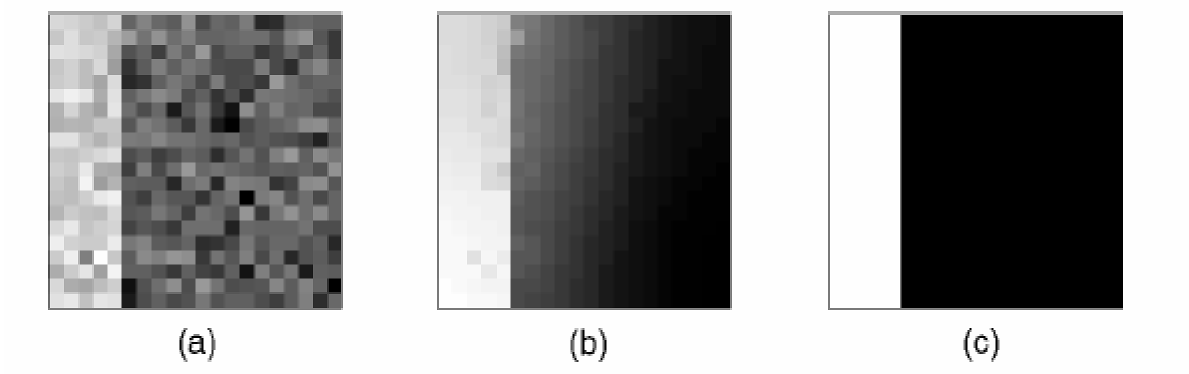
2. Can represent different type of segmentation

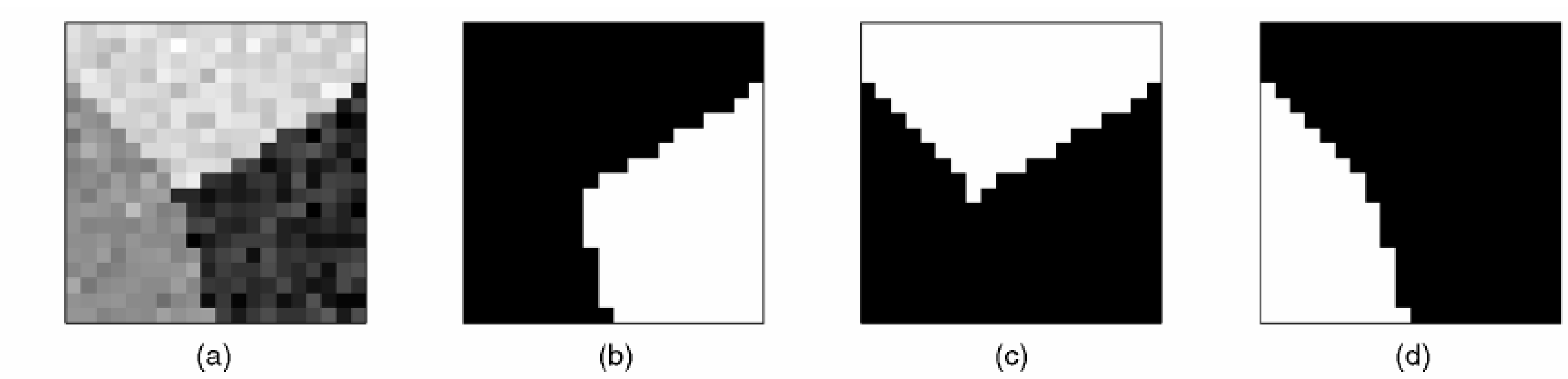
3. Solve for the eigenvectors for the second smallest eigenvector:

$$D^{-\frac{1}{2}}(D - W)D^{-\frac{1}{2}}x = \lambda x$$

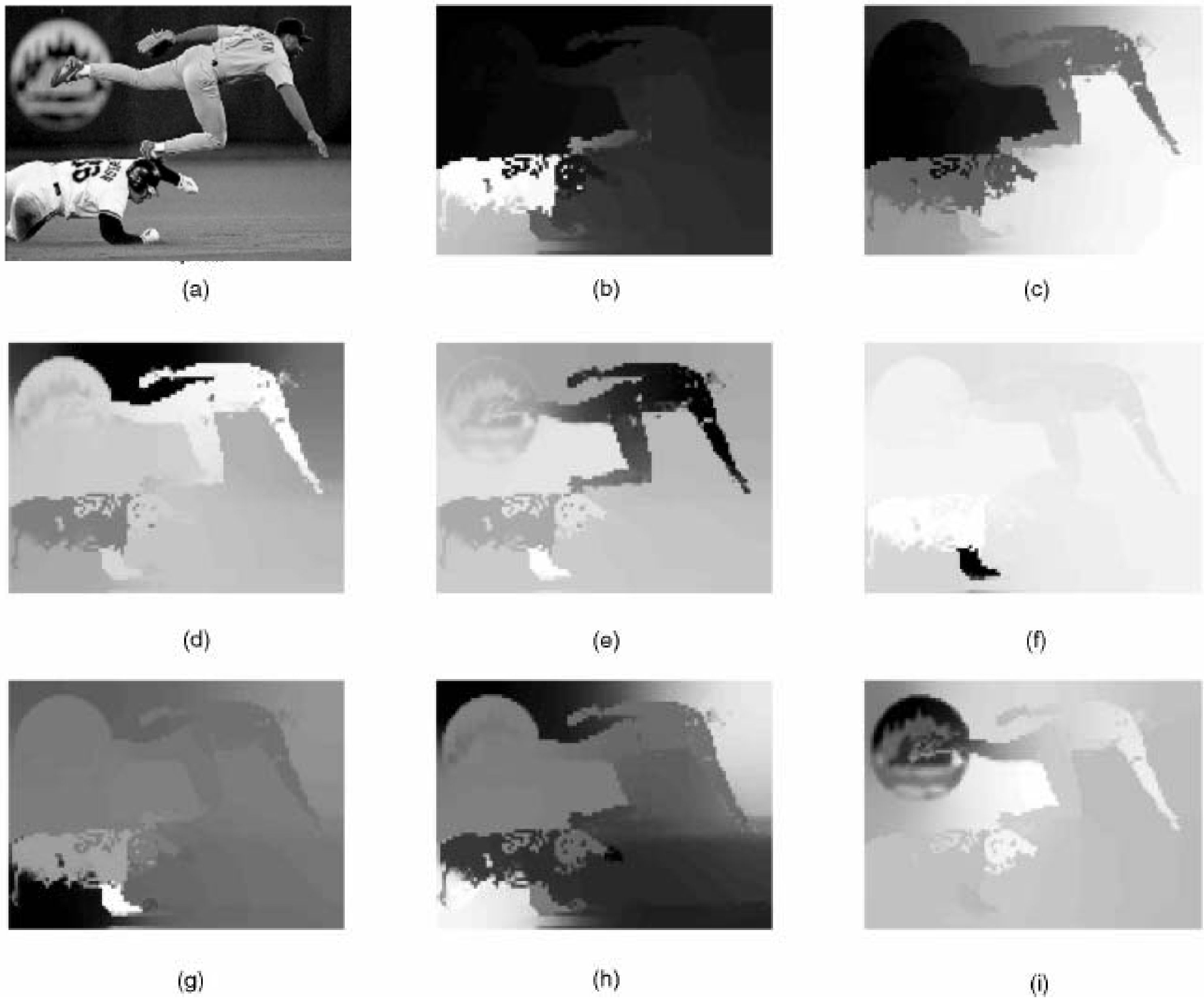
4. Somehow choose a splitting point to discretize the eigenvector to get the bipartition

Experiments – simple synthetic images



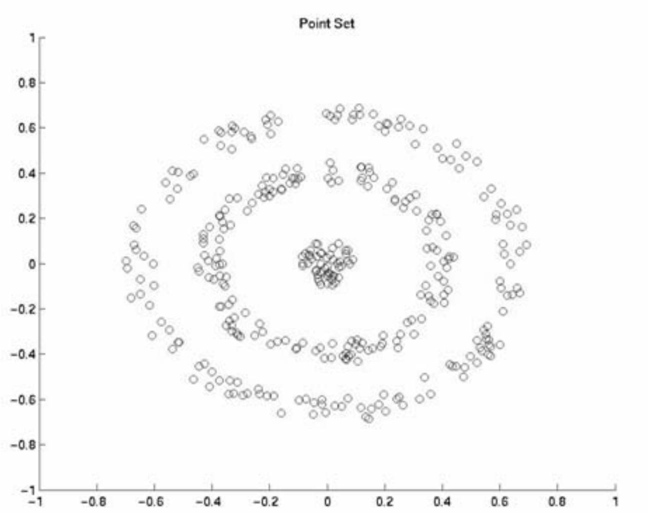


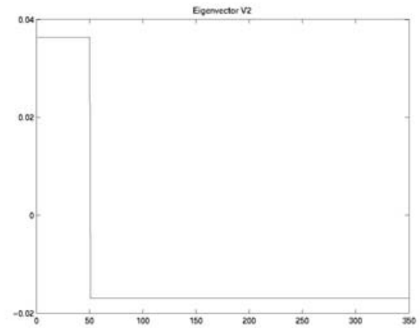
Experiments: gray image

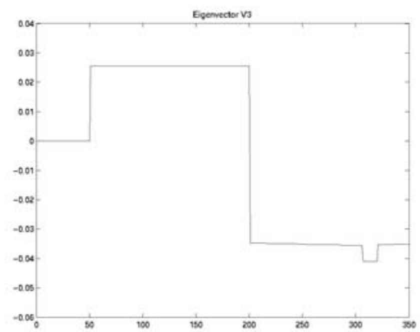

(a) (b) (c) (d) (e) (f) (g) (h) (i)

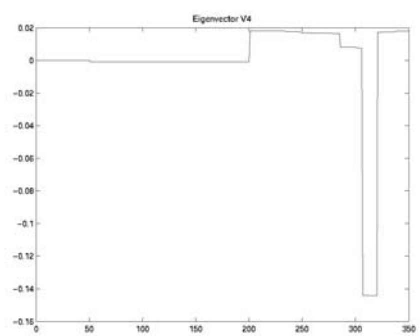
More Experiments

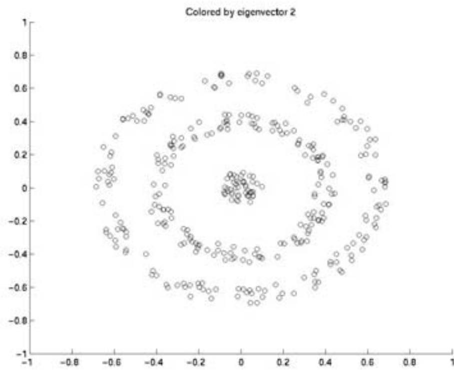
1. Spatial point set

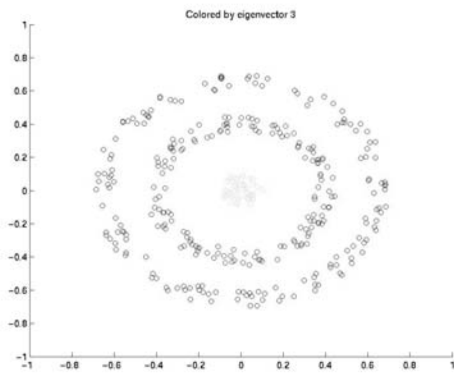


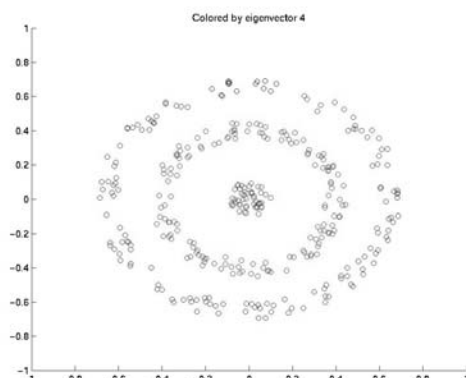




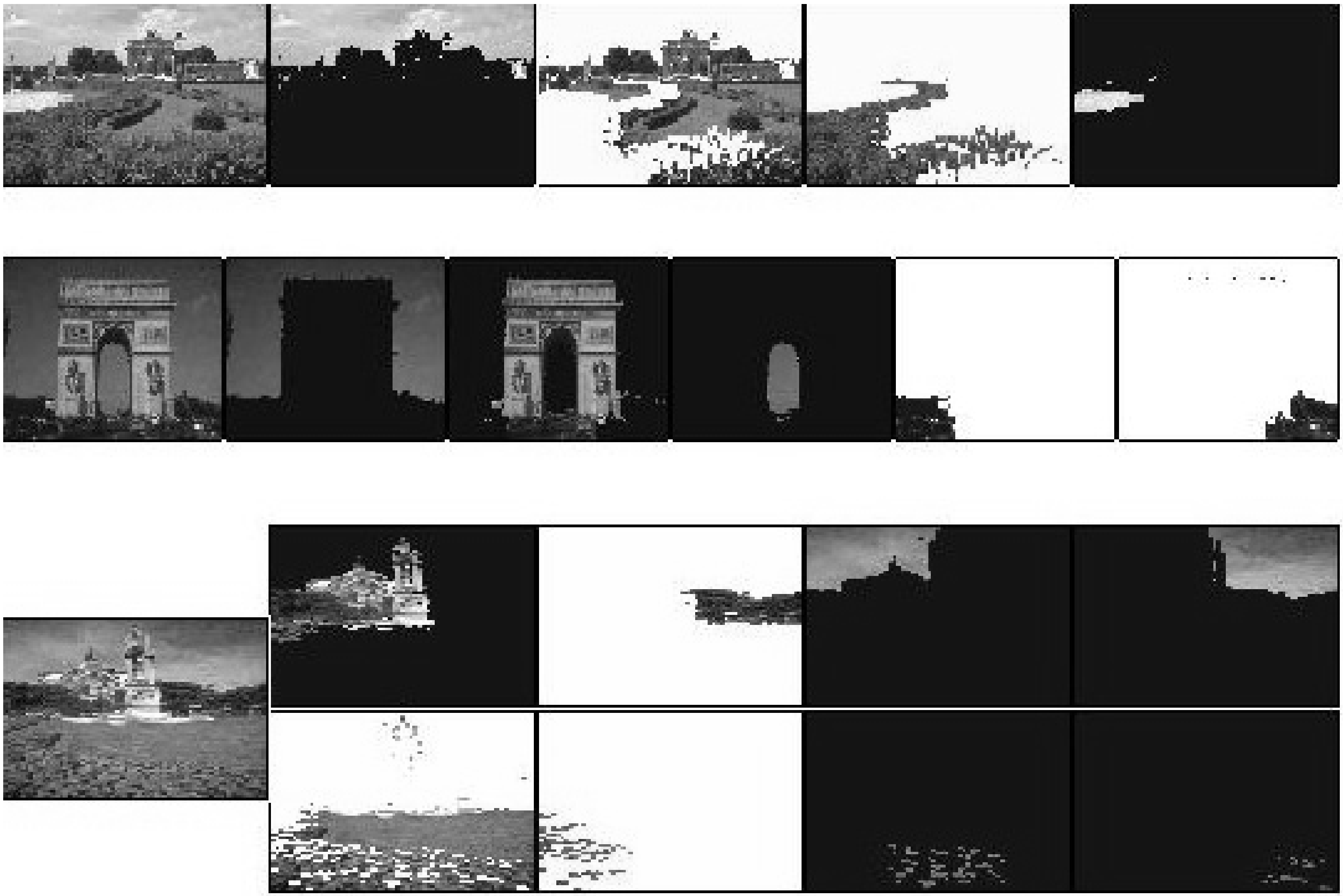








Color Image Segmentation



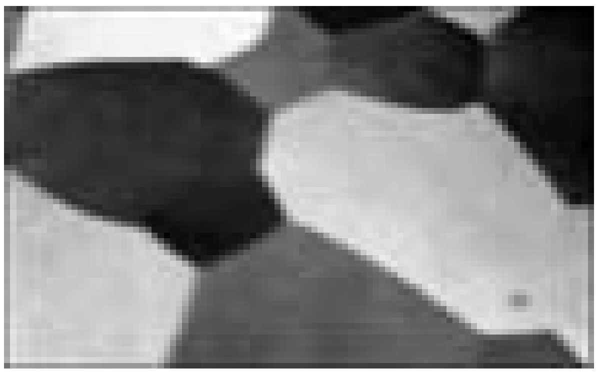
Other experiments & issues

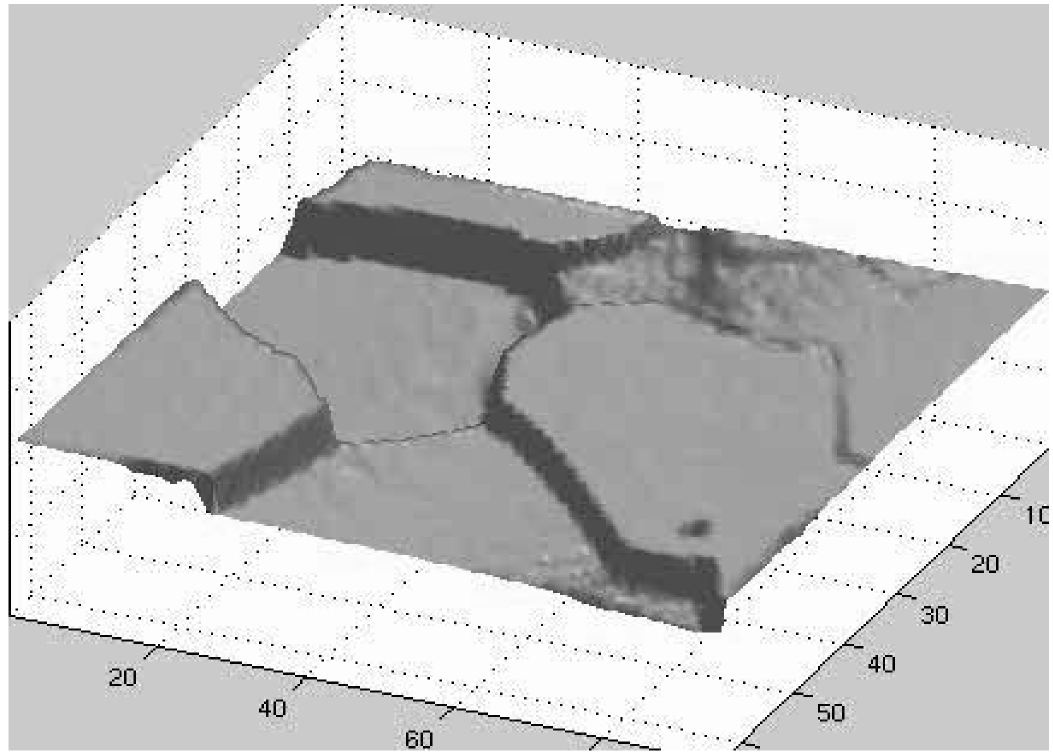
1. Can also handle:
   - Texture segmentation
   - Motion segmentation

   Need proper similarity definition

2. General problem of defining feature similarity incorporating many cues/features is non-trivial

Interpretation as a Dynamical System

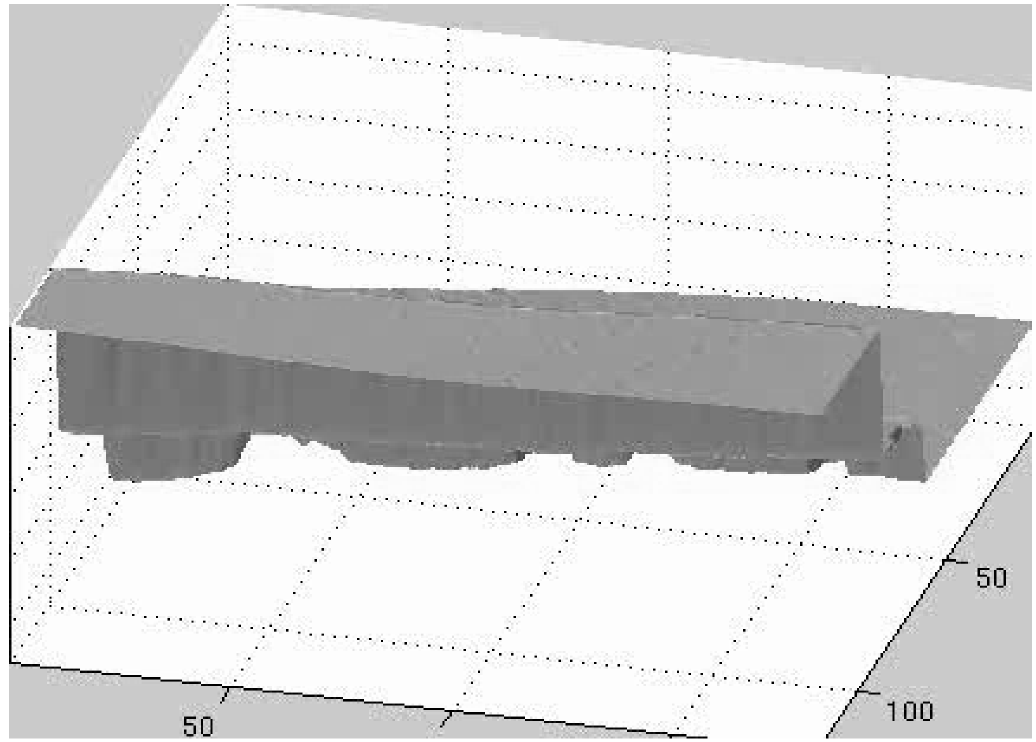




Treat the links as springs and shake the system

- elasticity proportional to cost
- vibration “modes” correspond to segments

Interpretation as a Dynamical System



Results based on a photo

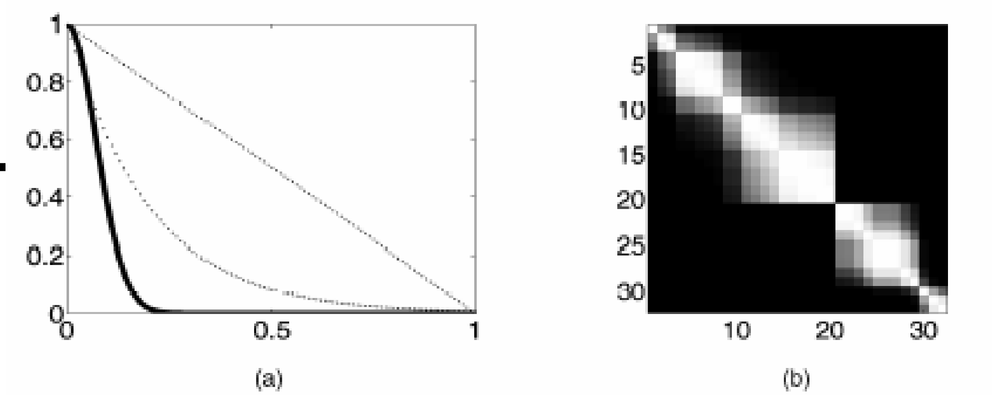
Relationship to other graph theoretic approaches

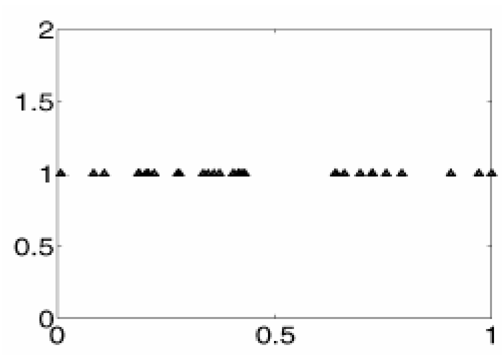
Finding Clumps ⟷ Finding Splits

	Average Association	Normalized Cut	Average Cut
Discrete Formulation	Assoc(A,A)/|A| + Assoc(B,B)/|B|	Cut(A,B)/assoc(A,V)+ Cut(A,B)/assoc(B,V) = 2 – (assoc(A,A)/assoc(A,V) + assoc(B,B)/assoc(B,V))	Cut(A,B)/|A| + Cut(A,b)/|B|
Continuous Formulation	$Wx=\lambda x$	$(D-W)x=\lambda Dx$	$(D-W)x=\lambda x$

$$\min \frac{cut(A,B)}{|A|} + \frac{cut(A,B)}{|B|} \neq \max \frac{assoc(A,A)}{|A|} + \frac{assoc(B,B)}{|B|}$$

Comparison 1





Normalized Cut:

$$(D - W)x = \lambda D x$$

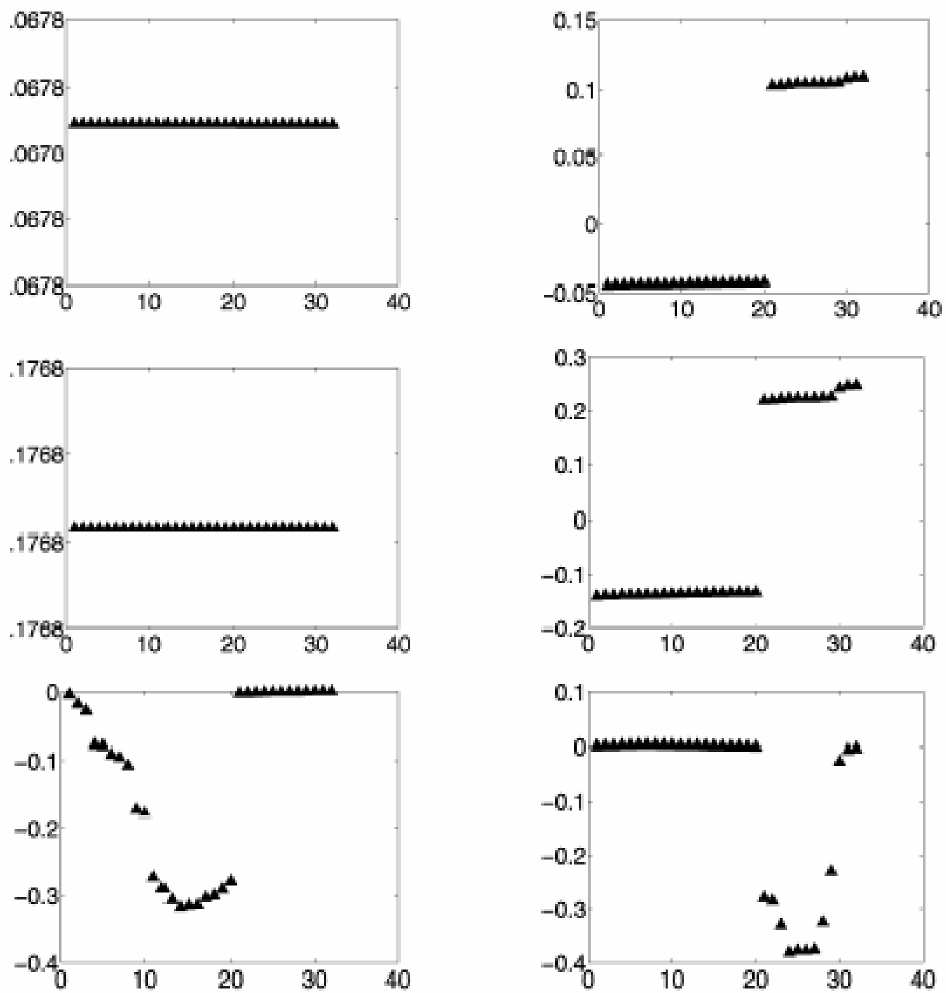
$$W x = (1 - \lambda) D x$$

Average Cut:

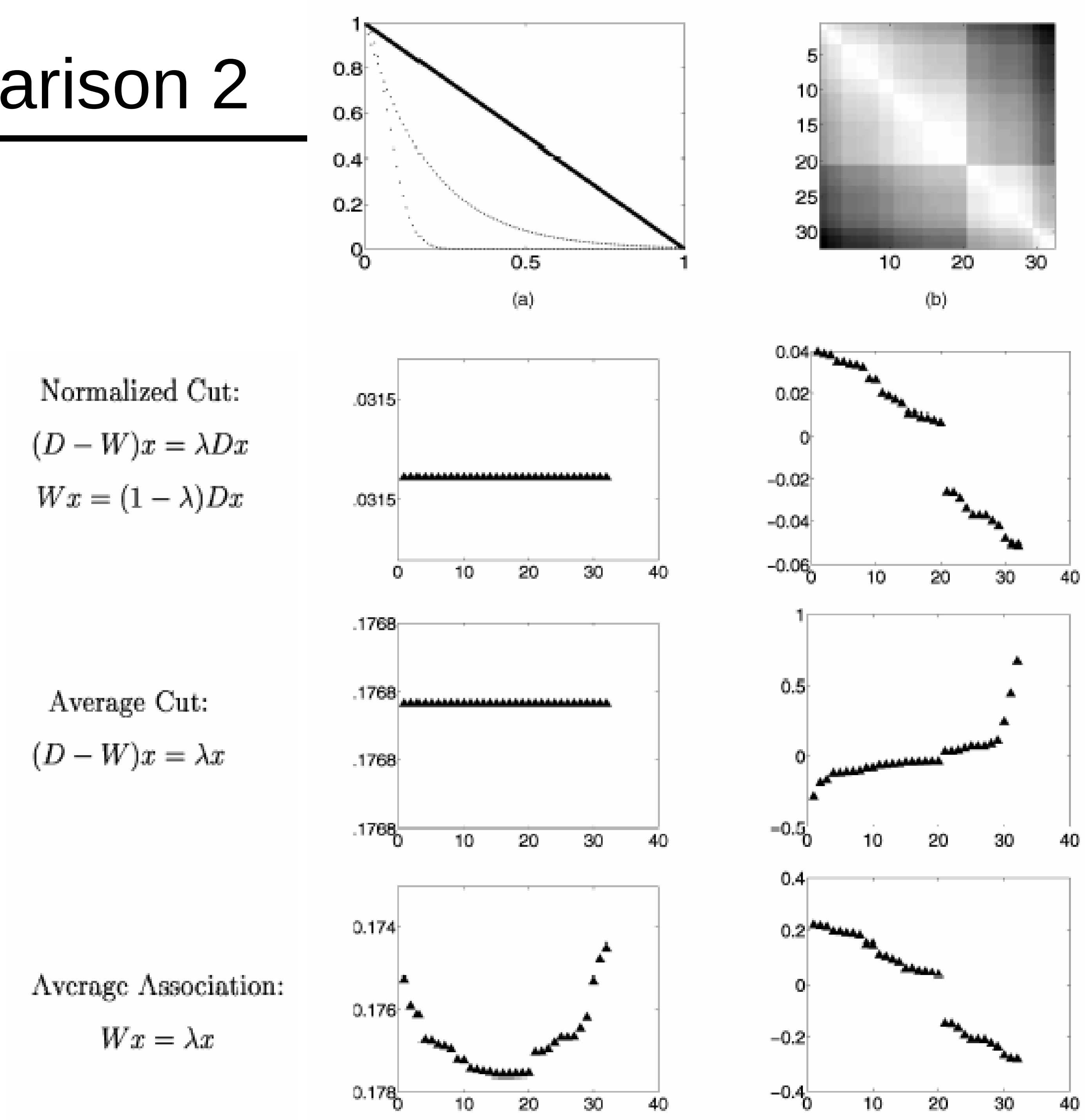
$$(D - W)x = \lambda x$$

Average Association:

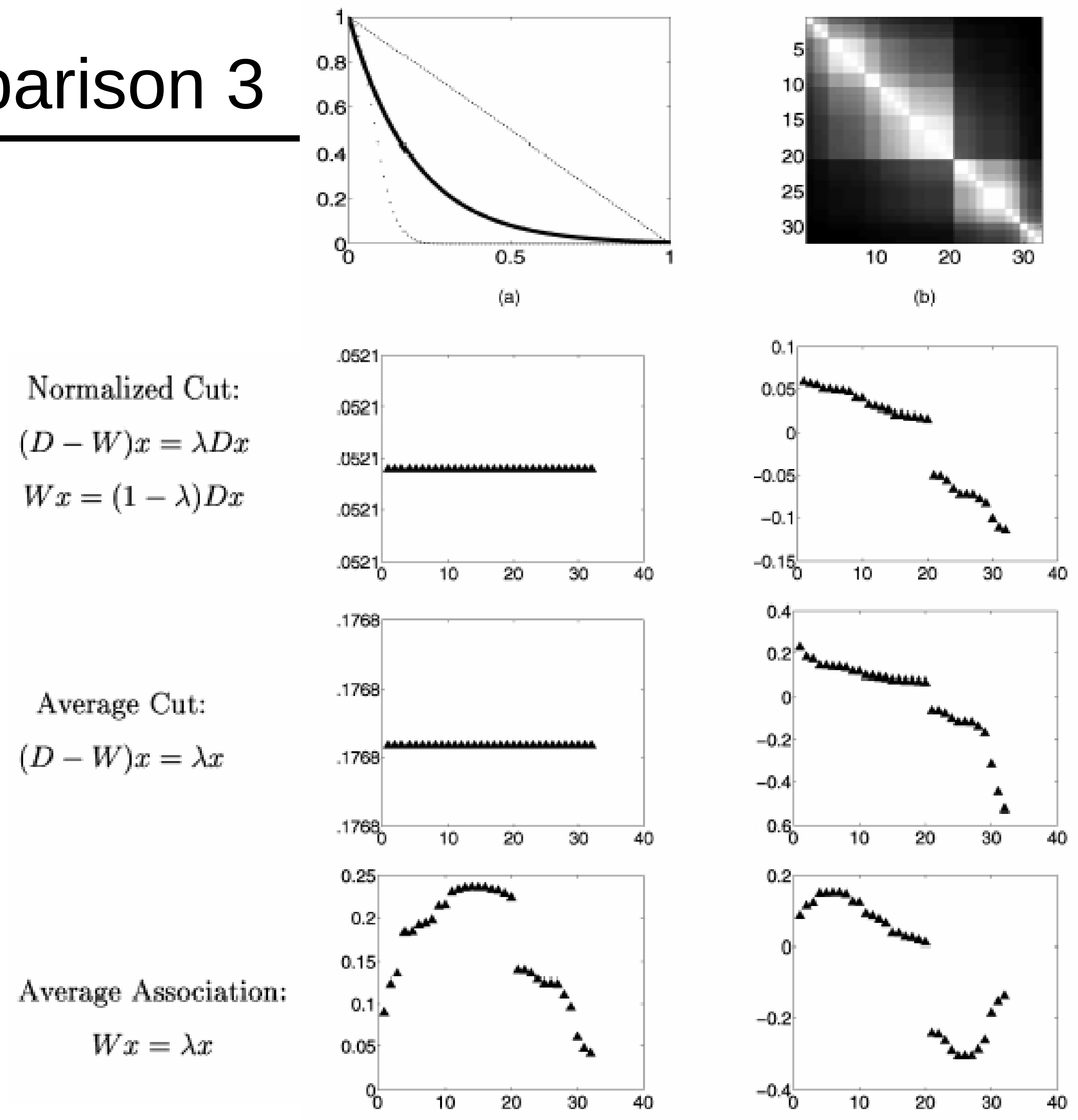
$$W x = \lambda x$$



Comparison 2



Comparison 3



Conclusion

- Image segmentation problem
- Normalized cut presents a new optimality criterion for partitioning a graph into clusters.
- Ncut is normalized measure of disassociation and minimizing it is equivalent to maximizing association.
- The discrete problem corresponding to Min Ncut is NP-Complete.
- We solve an approximate version of the MinNcut problem by converting it into a generalized eigenvector problem.
- Nice results in image segmentation