Introduction to Medical Imaging

MRI Physics

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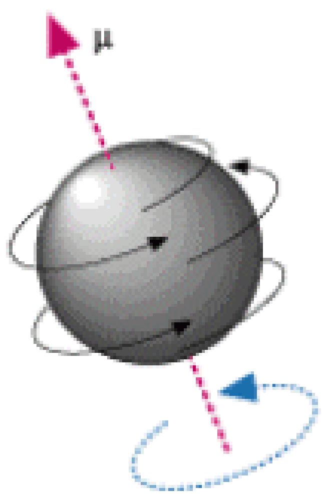
The Essential Element for MRI: Hydrogen

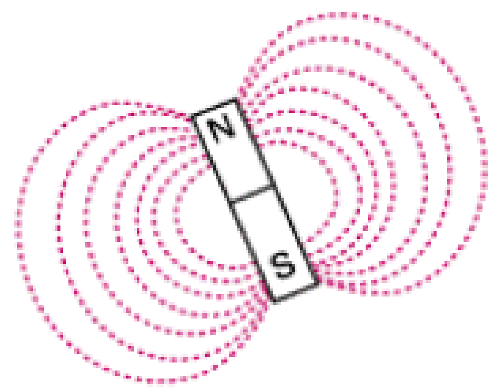
In MRI only hydrogen is used for imaging: ^{1}H

- the hydrogen atom is a component of water: H_20
- the body consists of 2/3 water → a lot of potential signal

The hydrogen atom has only one proton

- this proton has a spin
- it rotates around its own axis which makes it act as a tiny magnet

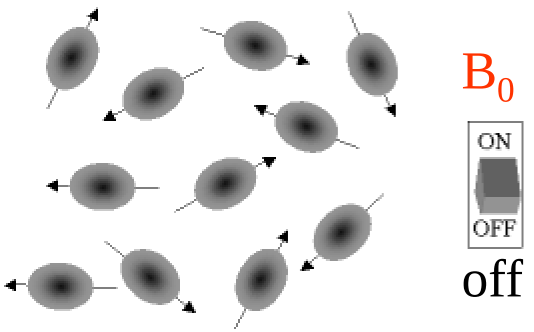




Alignment of Protons

There are millions of protons in human tissue

- they are randomly oriented in the absence of an external magnetic field

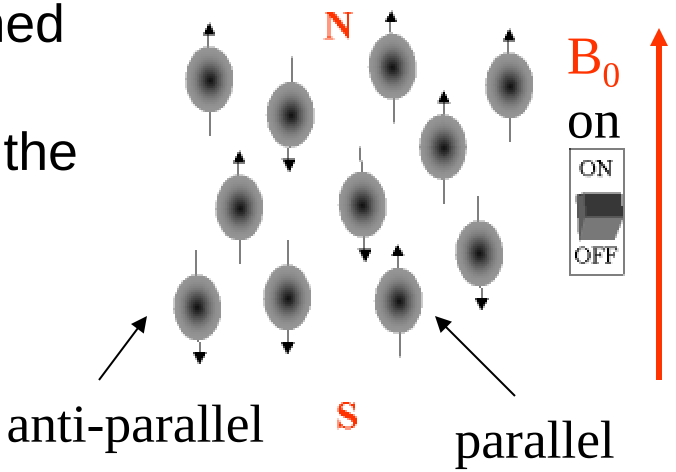


An MRI magnet has a strong magnetic field, B_0 (measured in Tesla)

- it causes the protons to align themselves in the direction of B_0
- some align parallel to B_0, some anti-parallel
- parallel alignment has the higher energy state
- the higher B_0 the more protons will be aligned parallel
- the more protons are in parallel alignment, the higher the *net magnetization* M_{z0}

$$M_{z0} \sim \sum \text{parallel protons}$$

$$- \sum \text{anti-parallel protons}$$



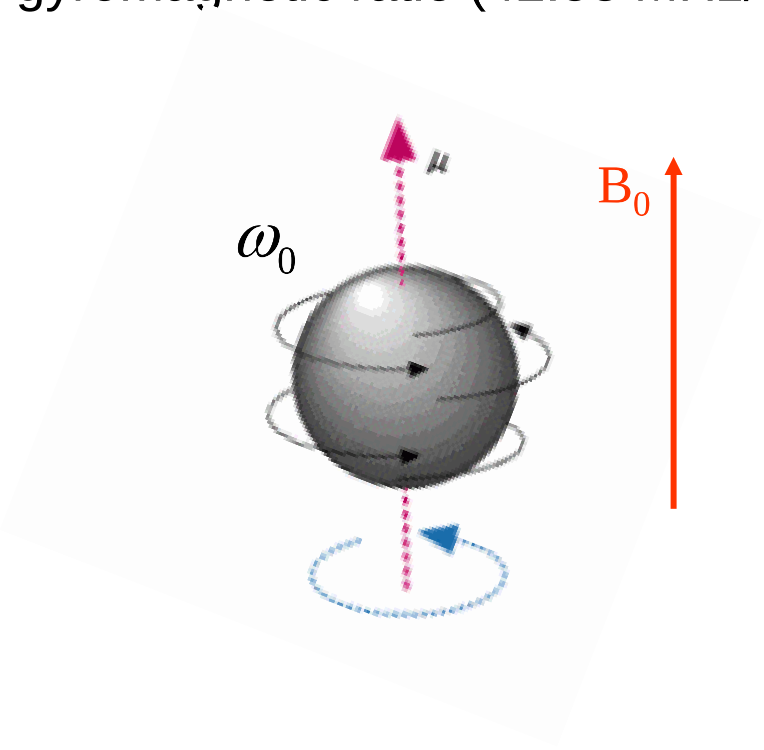
Larmor Frequency

The external magnet field not only aligns the protons

- it also causes the protons to spin at a certain frequency ω_0
- the frequency ω_0 called the *Larmor frequency* and is defined as:

$$\omega_0 = \gamma B_0$$

γ: gyromagnetic ratio (42.58 MHz/T for ^1H)



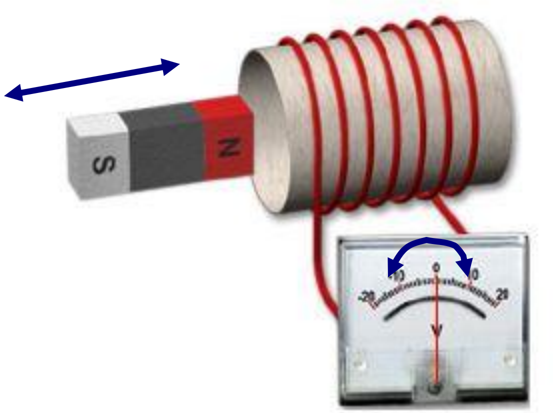
Measuring the Net Magnetization M_{z0}

We suspect that M_{z0} is related to the amount of hydrogen

- but how do we measure M_{z0}?

A way to measure a magnetic field is via electromagnetic induction

- moving the magnet in and out of the coil induces an alternating current which can be measured
- the faster we move the magnet, the more current is induced
- the problem with M_{z0} is that it is not changing and therefore cannot be measured via induction



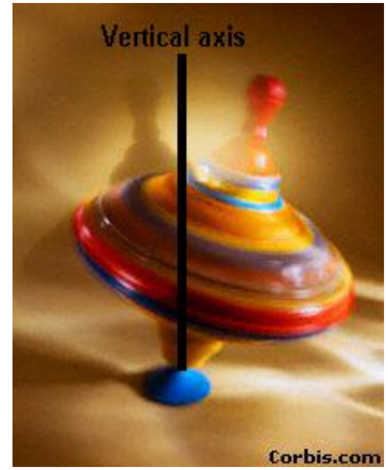
Need a way to turn M_{z0} into an alternating magnet field

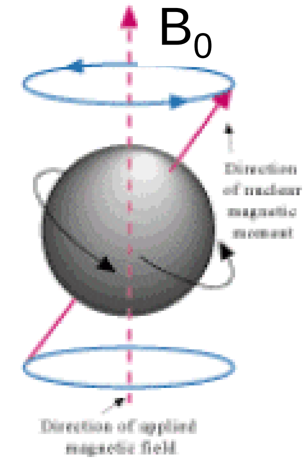
- then the stronger M_{z0}, the more current would be induced
- also need to perform the measurements orthogonal to B_0

Turn M_{z0} into such an orthogonal, alternating magnet field by adding a precession component

Proton Spin Precession: Introduction

Equivalent to a spinning top



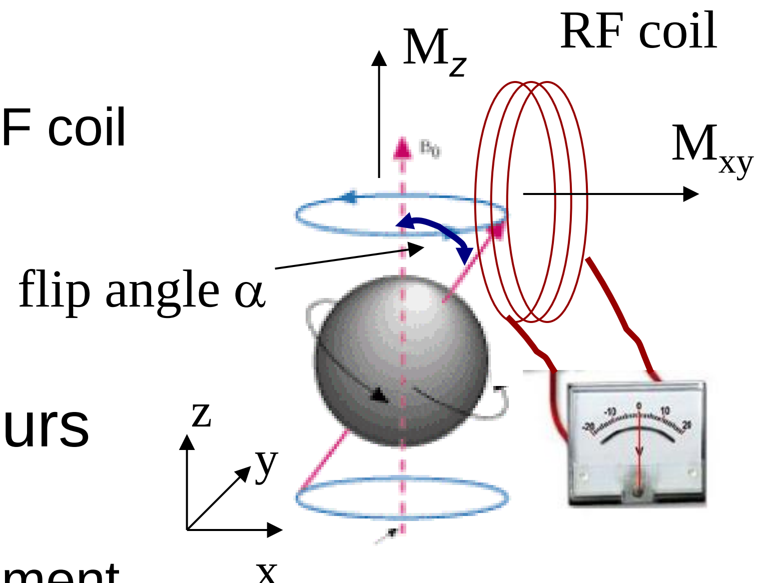


Now the magnetic field has

- a longitudinal (along B_0) component M_z
- a transverse component ($\perp B_0$) M_{xy}

Due to the precession M_{xy} oscillates in a sinusoidal fashion

- can be measured via induction in an RF coil
- will induce a sinusoidal current at frequency ω_0
- the magnitude is $M_{xy} = M_{z0} \sin\alpha$



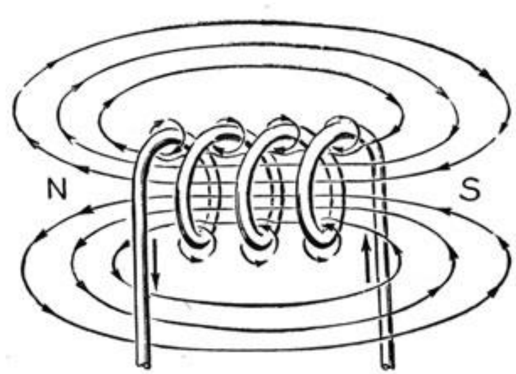
The highest amount of induction occurs when the *flip angle* is 90˚

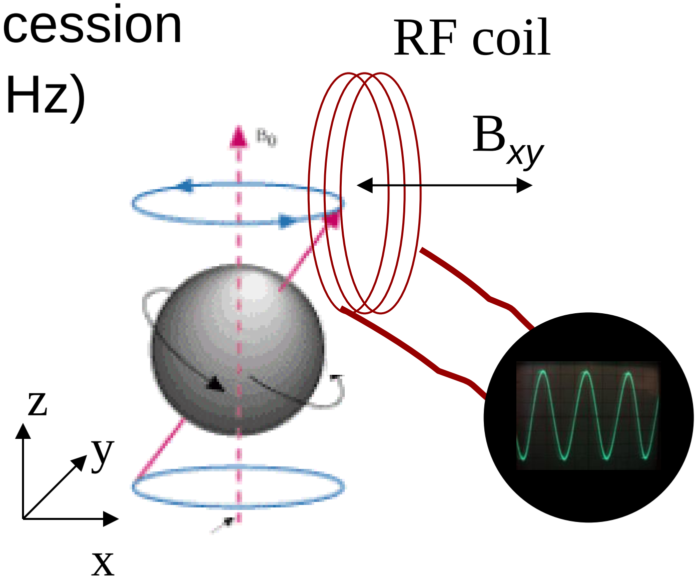
- then $M_{xy} = M_{z0}$ → the desired measurement

How To Create The Precession

We need to add a magnetic field B_{xy} orthogonal to B_0

- this will pull the spinning proton into a precession
- generated by RF pulse (range: 10 - 100MHz)





- note: the same RF coil can also be used for the measurement of the resulting M_{xy}

B_{xy} needs to alternate at Larmor frequency ω_0

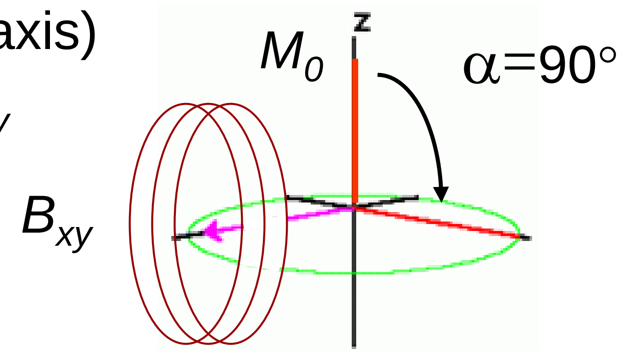
- then we obtain resonance → the magnetic force is applied synchronous to the proton position on the precession circle
- also, the longer the RF signal is left on, the wider the procession
- to get the highest measured signal, one needs to keep B_{xy} on until the flip angle is 90˚

More Formally

The magnet field B_{xy} acts in a similar manner than B_0

- it also causes a spin (around the RF coil axis)
- this spin has also a Larmor frequency, ω_{xy} (orthogonal to ω_0):

$$\omega_{xy} = \gamma B_{xy}$$



- since $B_{xy} \ll B_0 \rightarrow \omega_{xy} \ll \omega_0$

Depending how long B_{xy} is left on (or how large it is), we can rotate M_{z0} into different orientation angles α

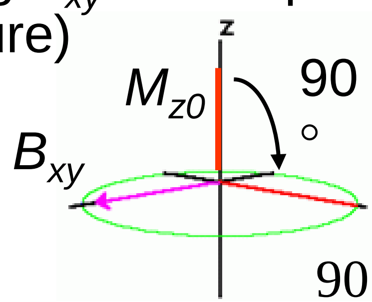
- the angle α is called the *flip angle*

$$\alpha = \int_0^t \gamma B_{xy} d\tau = \gamma B_{xy} t = \omega_{xy} t$$

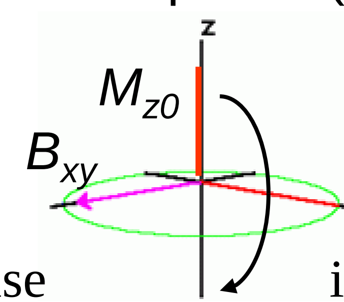
Trade-offs:

- for fast imaging it is desirable to keep *t* short
- this requires doubling B_{xy} which quadruples the power (and the heat and tissue temperature)

Important flip angles:



90° pulse

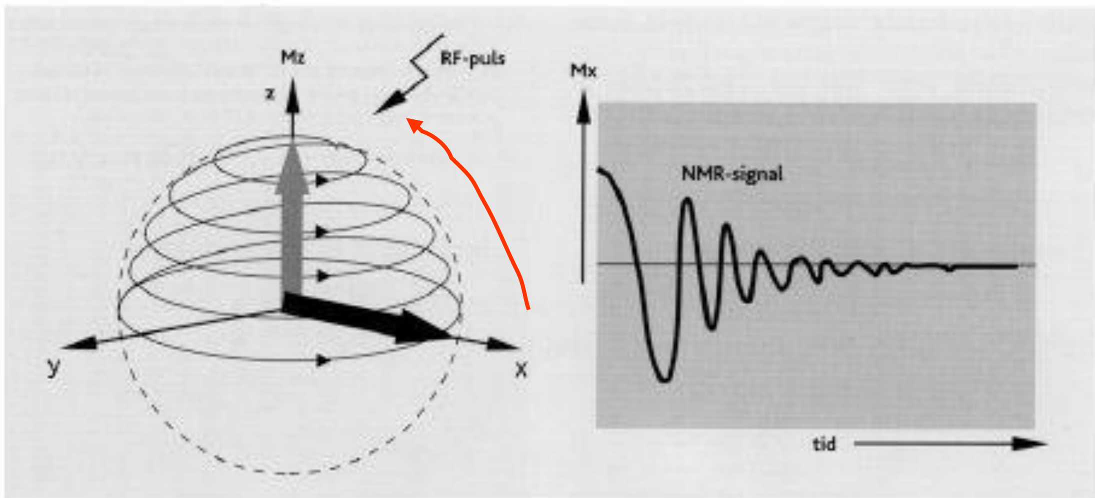


inversion pulse

Relaxation

The tilt (flip) is an unstable situation

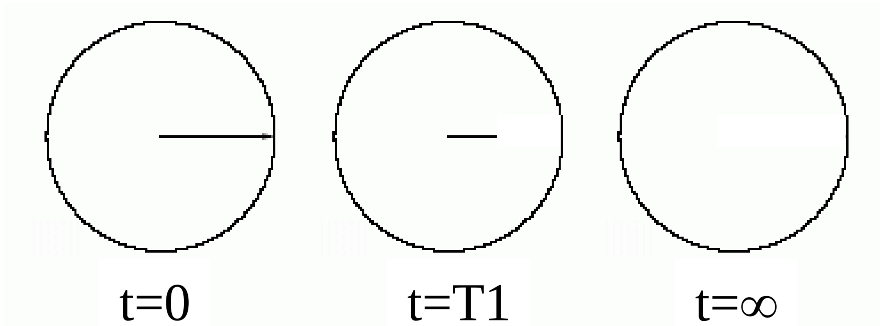
- the proton will rotate back to its original position along the z-axis
- the measured RF signal will decay and eventually go to zero



(also note the sinusoidal form of the induced signal)

- this decay is called *T1-relaxation*

transverse component:

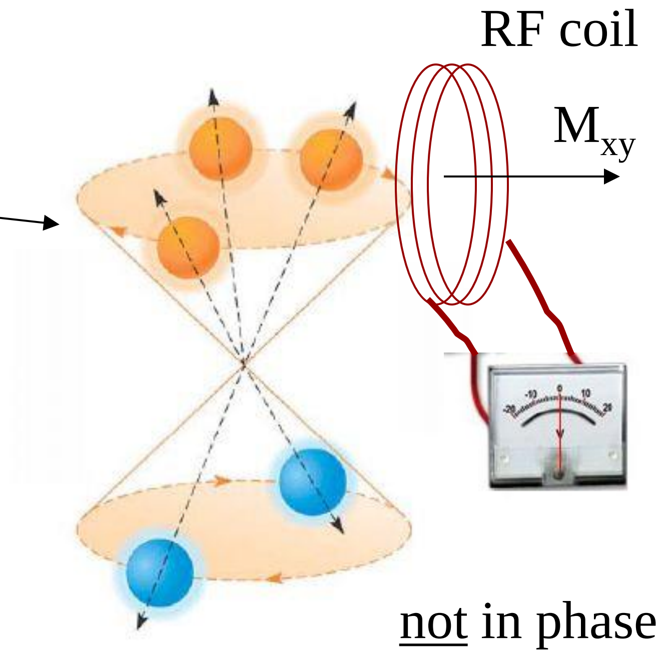


The Net Magnetization M_{xy}

In order to measure a signal of sufficient amplitude, all protons must be precess in phase

- we need to synchronize the spins

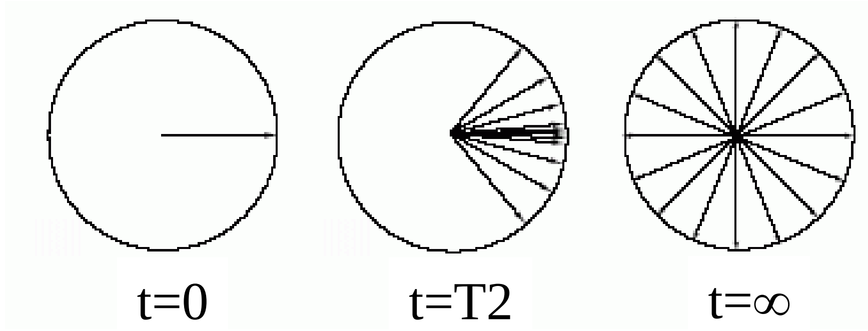
precessing at ω_0



The RF pulse used for flipping also synchronizes the spins

- once the RF pulse is removed the spins go out of phase
- this is called *T2-relaxation*

transverse component:



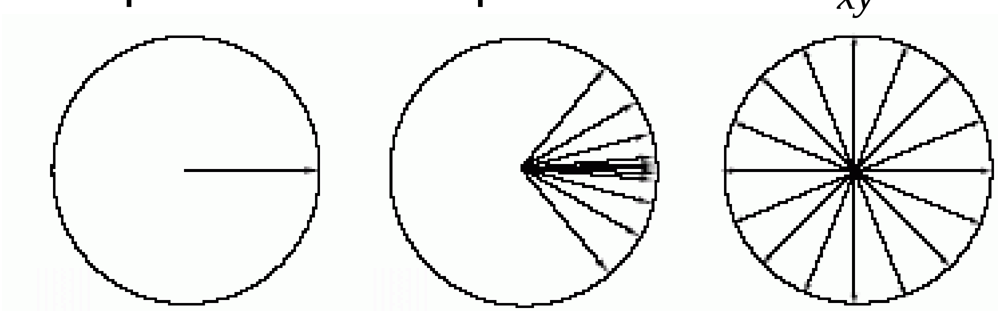
Spin-Spin Relaxation (T_2)

Relaxation due to the gradual disappearance of M_{z0}'s transverse component M_{xy}

- in practice, each spin experiences a slightly different magnet field due to the locally different chemical environments (protons can belong to H_2O, -OH, -CH, …)
- this results the spins to rotate at slightly different angular frequencies
- and as a consequence a loss of phase coherence (*dephasing*) occurs
- the time constant for the exponential decay is called *spin-spin relaxation time T_2*:

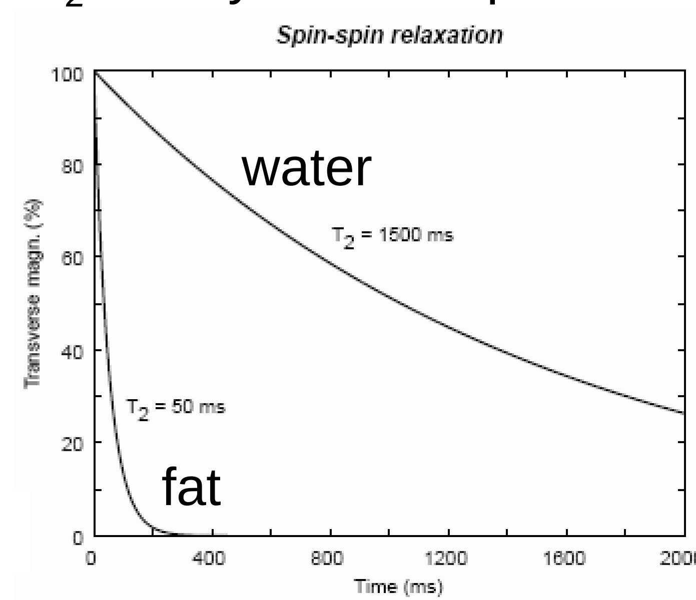
$$M_{xy}(t) = M_{xy}(0)e^{-\frac{t}{T_2}}$$

90° RF pulse 37% dephased no M_{xy} left



t=0 t=T_2 t=∞

T_2 is very tissue-dependent



Spin-Lattice Relaxation (T_1)

In spin-spin relaxation there is no loss of flip angle

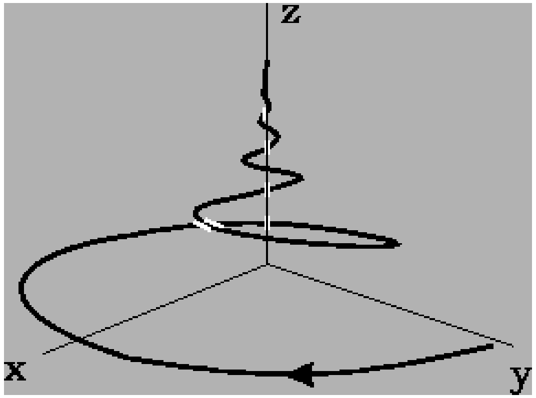
- the system became only disordered and unsynchronized

In spin-lattice relaxation, the flip angle actually changes

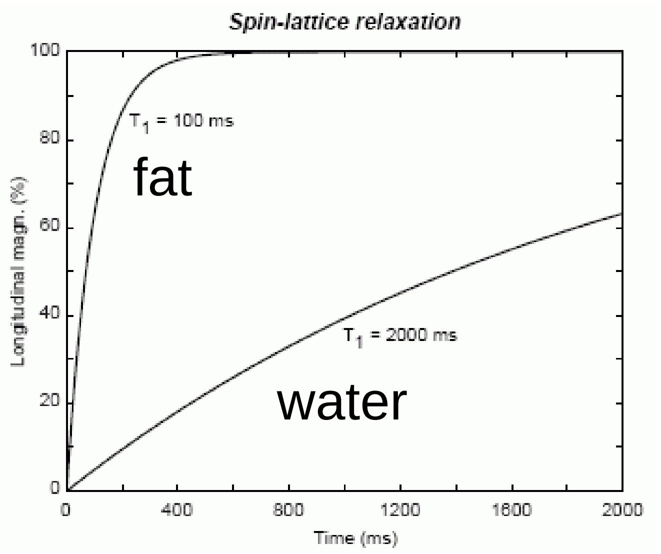
- the longitudinal component M_z will grow from M_{z0} *cos*α *to* M_{z0}
- the energy shift is caused by the (small) heat released through the lattice molecule vibrations
- the time constant for the exponential decay is called *spin-lattice relaxation time* T_1:

$$M_z(t) = M_{z0}\cos\alpha\, e^{-\frac{t}{T_1}} + M_{z0}(1-e^{-\frac{t}{T_1}})$$

will return to the equilibrium value, M_0



Note: T_1 is typically always greater than T_2



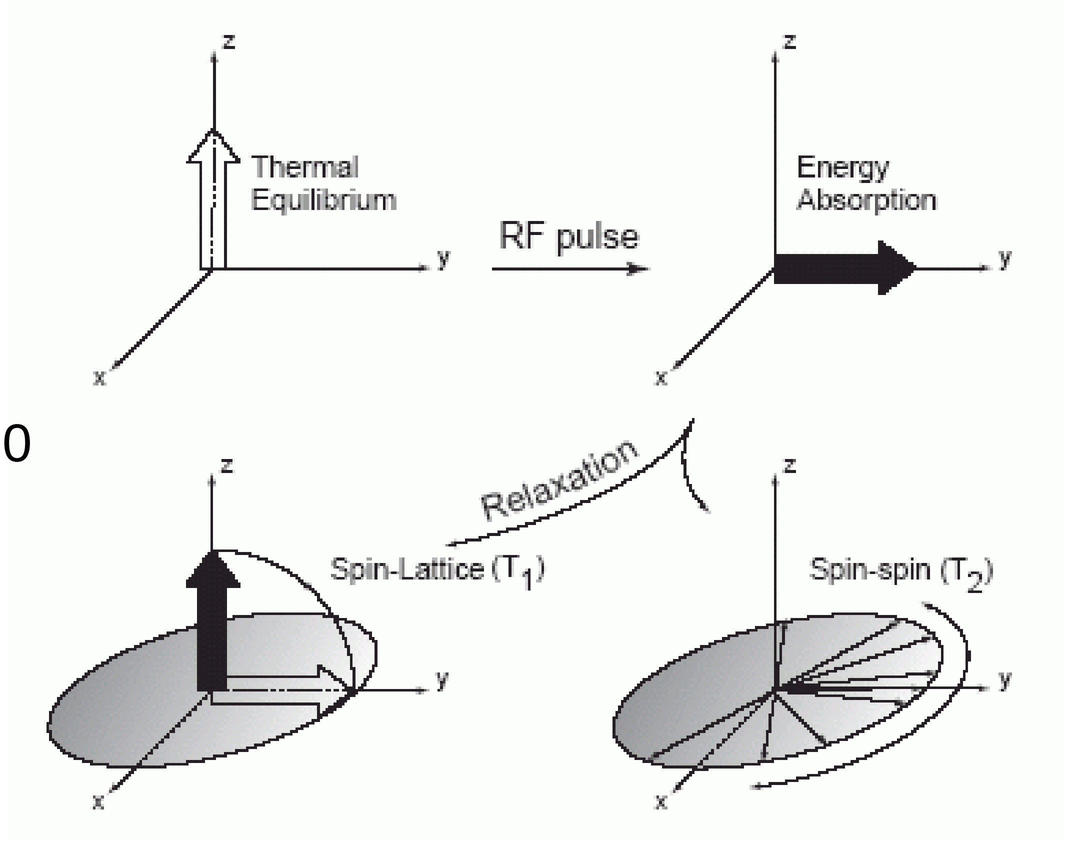
Summary: Energy Absorption and Relaxation

Combining the T1 and T2 effects into a single equation (the Bloch relaxation equation):

$$M_{xy} = M_{xy0}(1\text{-}e^{-\frac{t}{T1}})\, e^{-\frac{t}{T2}}$$

M_{xy} is the measured transverse component at some time t>0

M_{xy0} is the (maximal) transverse component at t=0



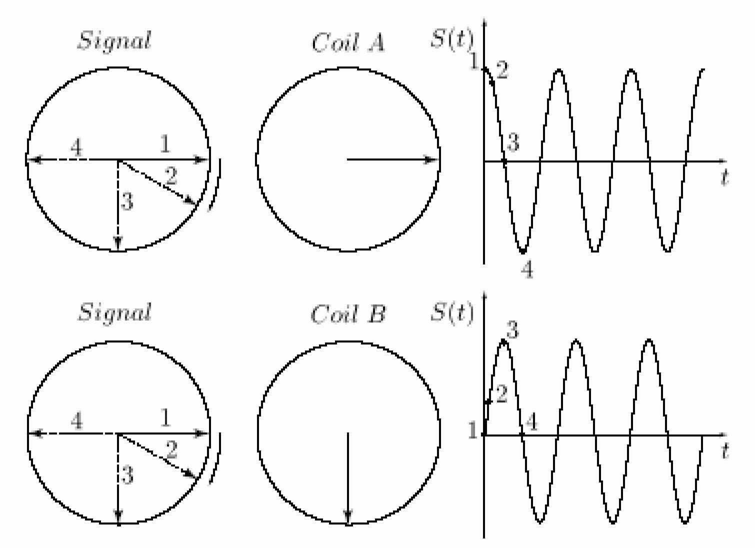
Complex Exponential Representation

To improve SNR, we use two coils, one aligned with the *x*-axis and one aligned with the *y*-axis (*quadrature* scheme)

- the detected signal can then be represented as follows:

$$s_x(t) = Ae^{-\frac{t}{T_2}} \cos(-\omega_0 t)$$

$$s_y(t) = Ae^{-\frac{t}{T_2}} \sin(-\omega_0 t)$$



- thus, coil x gives the real part and coil y the imaginary part of a complex-valued signal:

$$s(t) = Ae^{-\frac{t}{T_2}} e^{-i\omega_0 t}$$

- expressed in a rotating reference frame:

$$s(t) = Ae^{-\frac{t}{T_2}}$$