CSE 564
Introduction to Visualization

System Design and Evaluation

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Lecture	Topic	Projects
1	Intro and logistics	
2	Basic visualizations and tasks, data types, examples, ethical considerations	
3	Data preparation (cleaning, imputation, data set integration)	
4	AI-assisted coding for VIS applications (design, debugging, refactoring)	Project #1 out
5	Big data and data reduction (distance/sim metrics, intro to clustering)	
6	High-D data: concept, subspaces, dimension reduction, PCA	
7	Cluster analysis: hierarchical, density, model, embedding, temporal	
8	Perception and cognition (human visual system, color, contrast)	Project #2(a) out
9	Visual design and aesthetics	
10	Visualization of multivariate and high-D data: linear methods, projections	
11	Vis. of multivariate and high-D data: non-linear methods, embeddings	
12	Visualization and AI: mutual support and capabilities (VIS4AI, AI4VIS)	Project #2(b) out
13	Principles of interaction: drive what is visualized, analyzed & how (HCI4VIS)	
14	Visual analytics, human-centered AI, mixed-initiative & collaborative VA	
15	Midterm #1 (tentative date)	
16	VA system design and evaluation, the nested model	
17	Midterm #1 discussion (tentative date)	Final proj. proposal call out
18	Visualization of hierarchical data	
19	Visualization of maps and data with geo-reference	
20	Visualization of graphs, networks (incl. derivation of causal networks)	Final project proposal due
21	Vis. of time-varying, time-series, streaming data, progressive visualization	
22	Visualization of text, LLMs, and semantic data	
23	Ed Tufte's principles and critiques, responsible visualization, uncertainty	
24	Design of effective infographics	Final proj. prelim report due
25	Foundations scientific and medical visualization, intro to volume rendering	
26	Scientific visualization	Bonus project out (Vol Ren)
27	Story telling with data, data journalism	
28	Midterm #2 (tentative date)	
Final	Final project demo on zoom (public)	All final proj. materials due

Case Study: What Causes Low MPG

The Car Data Set

Consider the salient features of a car (not really big data):

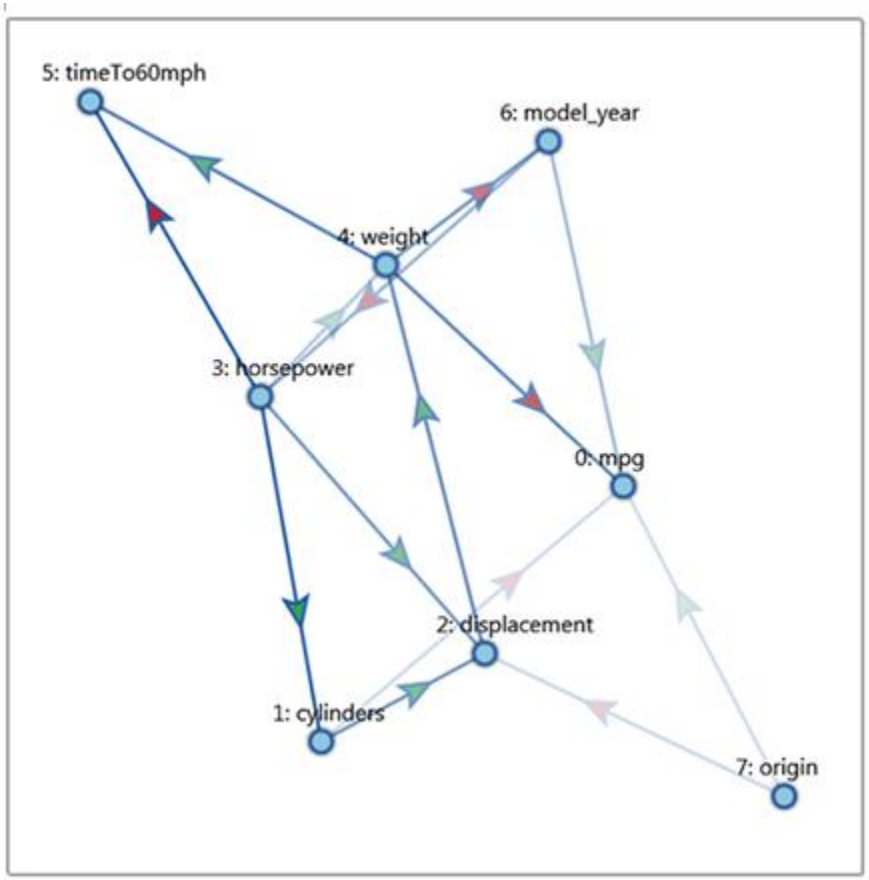
- miles per gallon (MPG)
- top speed
- acceleration (time to 60 mph)
- number of cylinders
- horsepower
- weight
- country origin

400 cars from the 1980s

Shown in a Spreadsheet

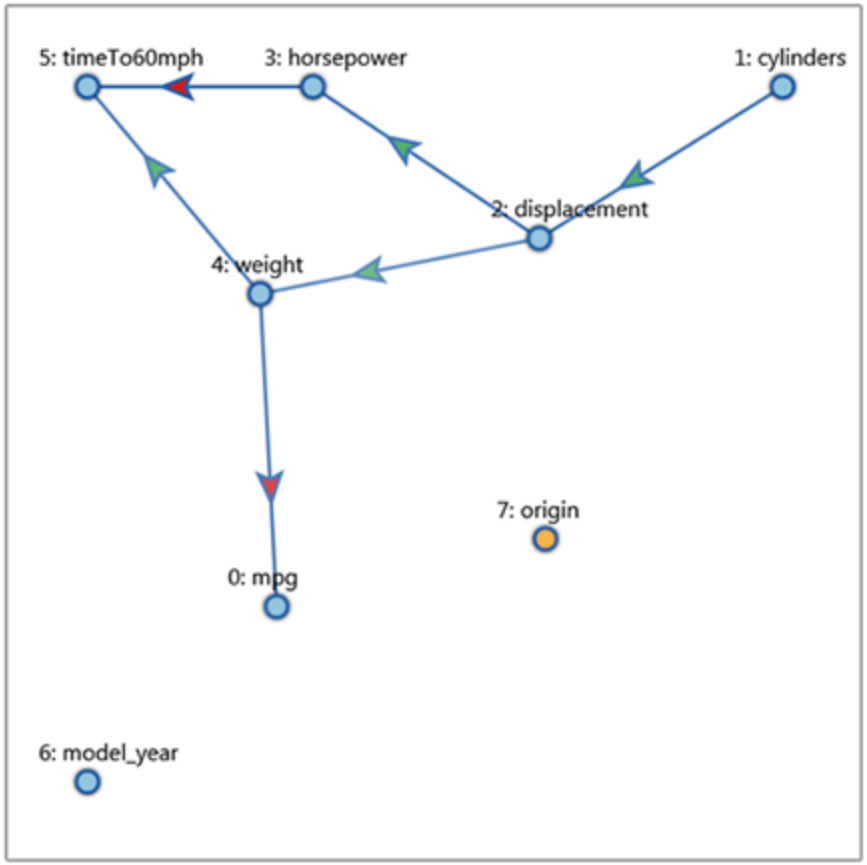
model	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21	6	160	110	3.9	2.62	16.46	0	1	4	4
Mazda RX4 V	21	6	160	110	3.9	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.32	18.61	1	1	4	1
Hornet 4 Dri	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Spor	18.7	8	360	175	3.15	3.44	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.46	20.22	1	0	3	1
Duster 360	14.3	8	360	245	3.21	3.57	15.84	0	0	3	4
Merc 240D	24.4	4	146.7	62	3.69	3.19	20	1	0	4	2
Merc 230	22.8	4	140.8	95	3.92	3.15	22.9	1	0	4	2
Merc 280	19.2	6	167.6	123	3.92	3.44	18.3	1	0	4	4
Merc 280C	17.8	6	167.6	123	3.92	3.44	18.9	1	0	4	4
Merc 450SE	16.4	8	275.8	180	3.07	4.07	17.4	0	0	3	3
Merc 450SL	17.3	8	275.8	180	3.07	3.73	17.6	0	0	3	3
Merc 450SL(	15.2	8	275.8	180	3.07	3.78	18	0	0	3	3
Cadillac Fleet	10.4	8	472	205	2.93	5.25	17.98	0	0	3	4
Lincoln Cont	10.4	8	460	215	3	5.424	17.82	0	0	3	4
Chrysler Imp	14.7	8	440	230	3.23	5.345	17.42	0	0	3	4
Fiat 128	32.4	4	78.7	66	4.08	2.2	19.47	1	1	4	1
Honda Civic	30.4	4	75.7	52	4.93	1.615	18.52	1	1	4	2
Toyota Corol	33.9	4	71.1	65	4.22	1.835	19.9	1	1	4	1
Toyota Coro	21.5	4	120.1	97	3.7	2.465	20.01	1	0	3	1
Dodge Challe	15.5	8	318	150	2.76	3.52	16.87	0	0	3	2
AMC Javelin	15.2	8	304	150	3.15	3.435	17.3	0	0	3	2
Camaro Z28	13.3	8	350	245	3.73	3.84	15.41	0	0	3	4
Pontiac Fireb	19.2	8	400	175	3.08	3.845	17.05	0	0	3	2
Fiat X1-9	27.3	4	79	66	4.08	1.935	18.9	1	1	4	1
Porsche 914	26	4	120.3	91	4.43	2.14	16.7	0	1	5	2
Lotus Europa	30.4	4	95.1	113	3.77	1.513	16.9	1	1	5	2
Ford Pantera	15.8	8	351	264	4.22	3.17	14.5	0	1	5	4
Ferrari Dino	19.7	6	145	175	3.62	2.77	15.5	0	1	5	6
Maserati Bor	15	8	301	335	3.54	3.57	14.6	0	1	5	8
Volvo 142E	21.4	4	121	109	4.11	2.78	18.6	1	1	4	2

Global Layout of The Car Data



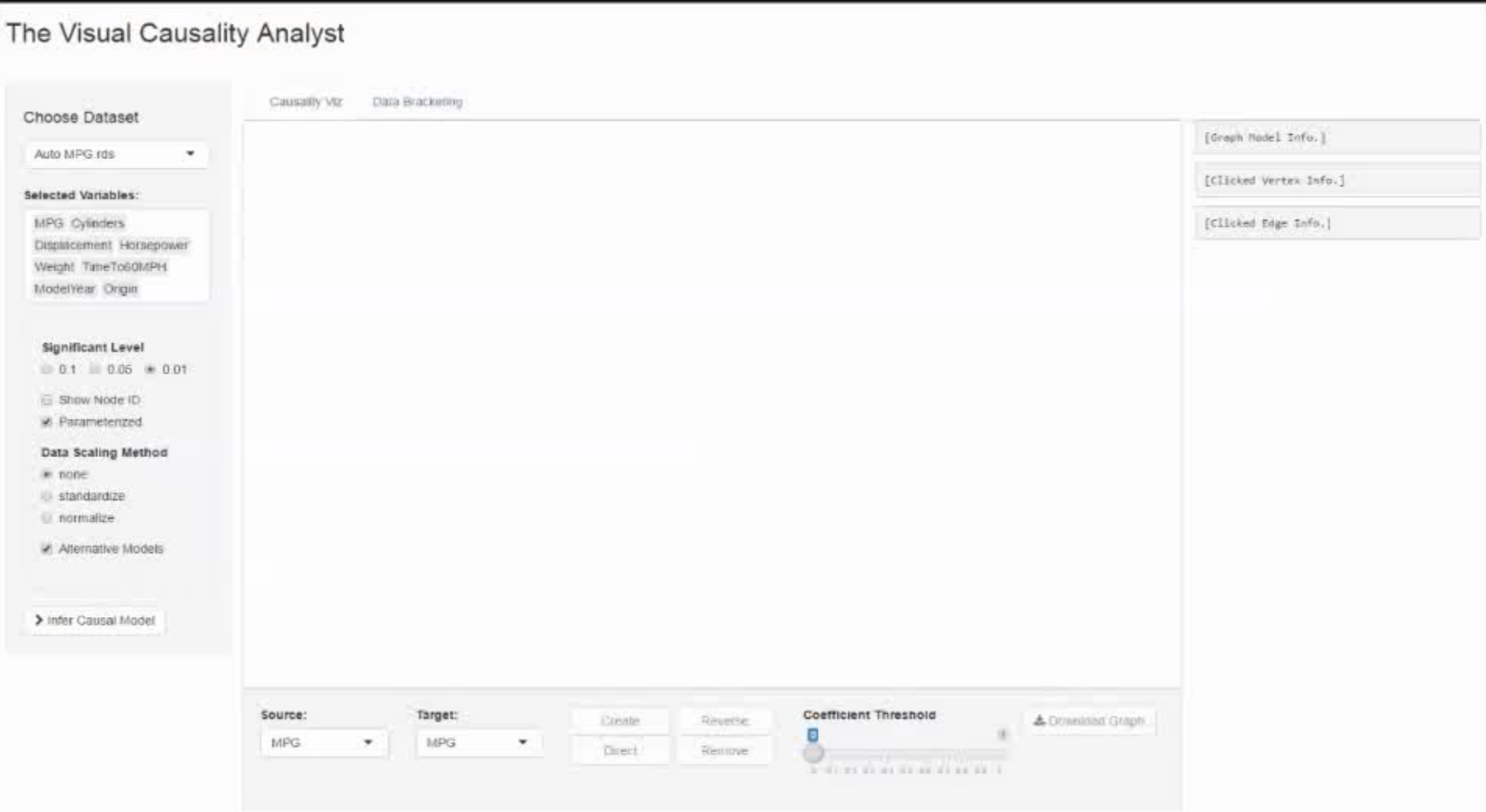
Random

Seeking the Cause of Low MPG



Isolating MPG

The Visual Causality Analyst



How To
Design A Visual Analytics Solution

Use the nested model

- devised by Tamara Munzner (UBC)
- M. Meyer, M. Sedlmair, P. Quinan, T Munzner, "The nested blocks and guidelines model," *Information Visualization*, 2013

Step 1: Characterize the Problem

Define the tasks, data, workflow of target users

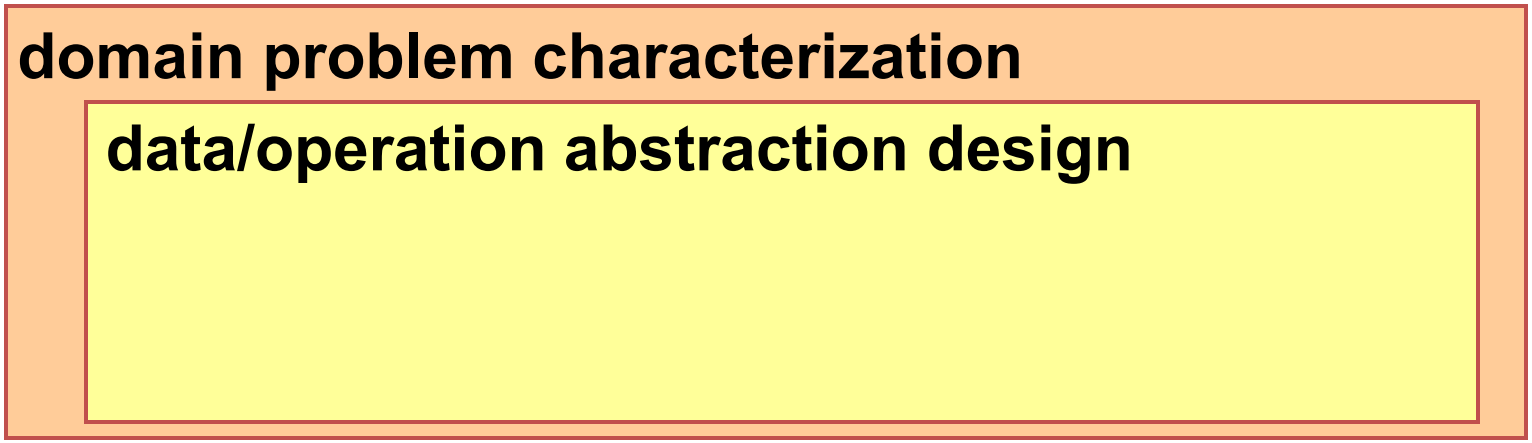
- the tasks are usually described in domain terms
- finding and eliciting the requirements is notoriously hard
- observe how domain users work and perform their tasks
- observe the pains they are having
- what are the limitations?
- what is currently impossible, slow, or tedious?

domain problem characterization

Step 2: Abstract Into a Design

Map from domain vocabulary/concerns to abstraction

- may require some sort of transformation
- data and types are described in abstract terms
- numeric tables, relational/network, spatial, ...
- tasks and operations described in abstract terms
- generic activities: sort, filter, correlate, find trends/outliers...



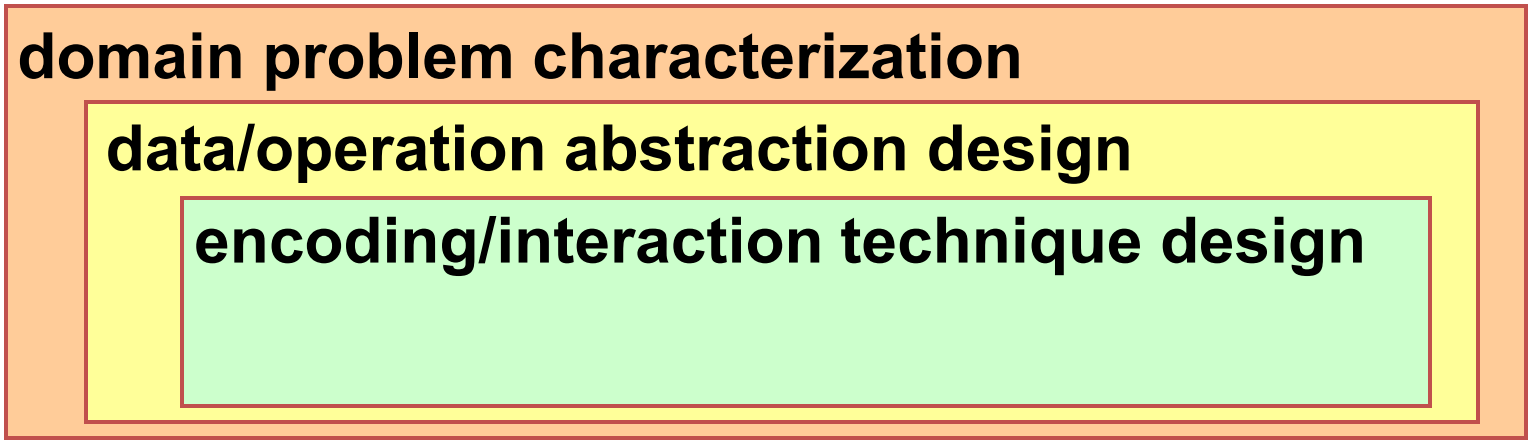
Step 2: Encode Into A Visualization

Visual encoding

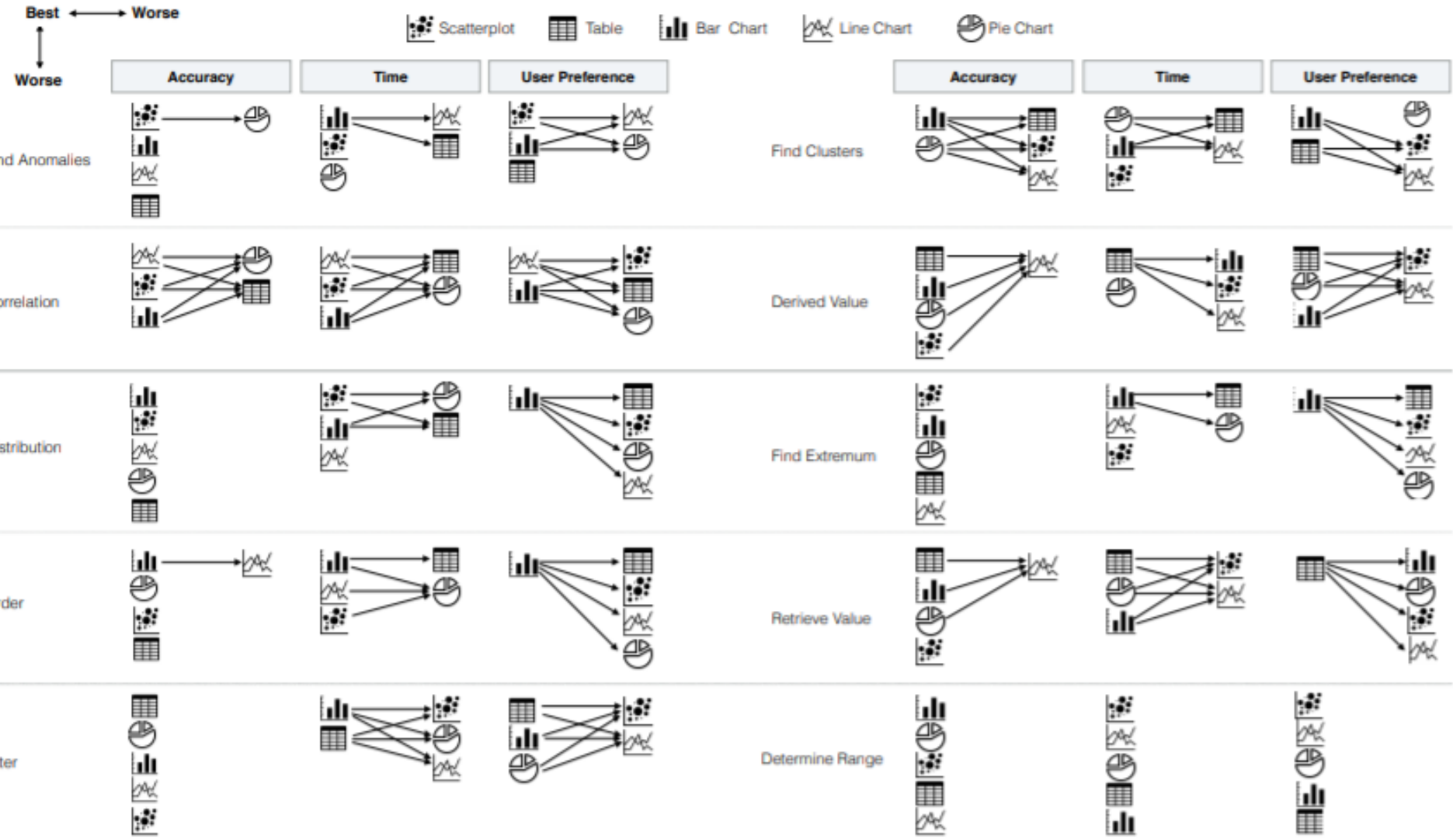
- how to best show the data (also pay tribute to aesthetics)
- bar/pie/line charts, parallel coordinates, MDS plot, scatterplot, tree map, network, etc.

Interaction design

- how to best support the intent a user may have
- select, navigate, order, brush, ...

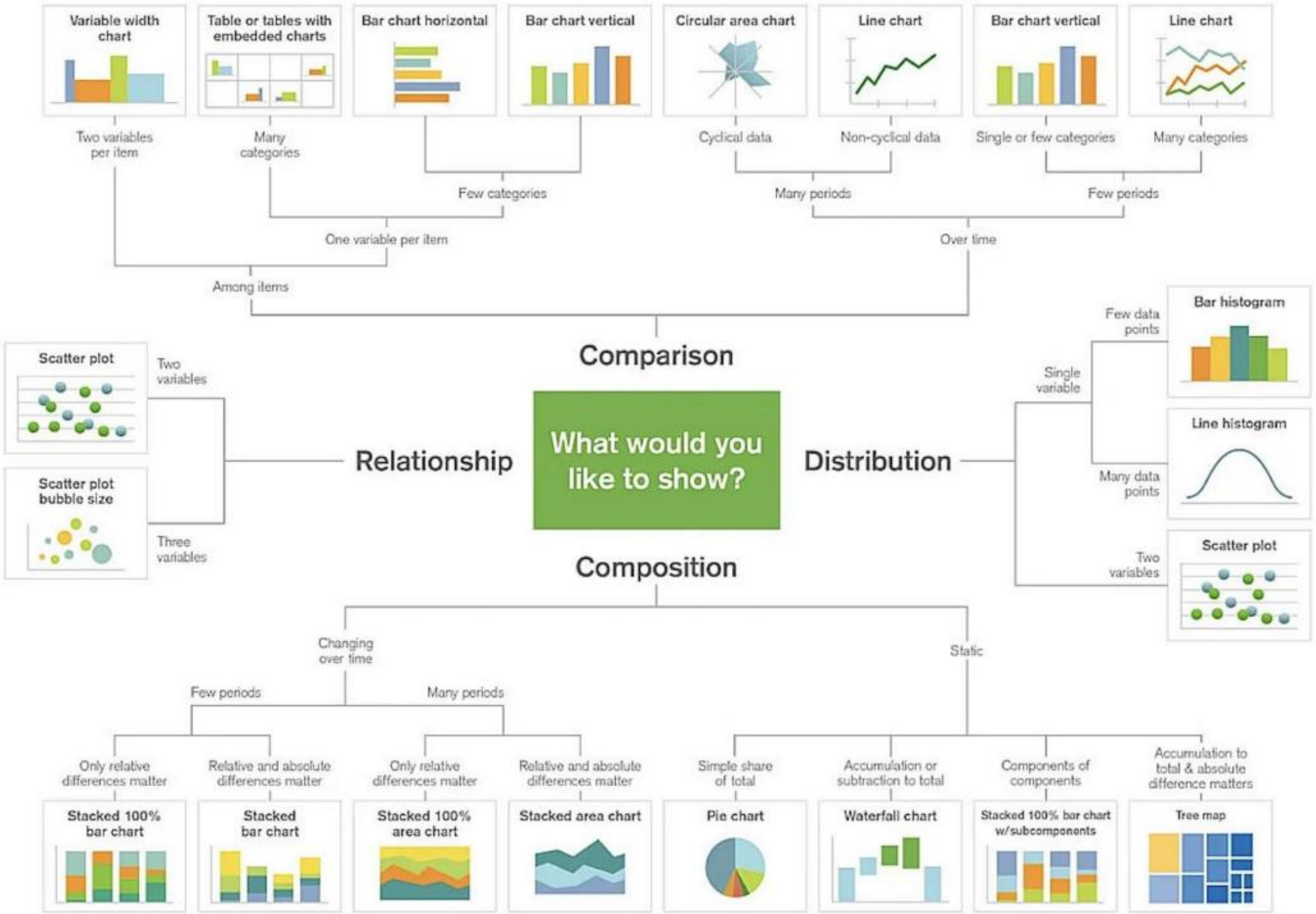


Match Tasks to Visualizations



Saket, Endert, Demiralp, "Task-based effectiveness of basic visualizations." *IEEE TVCG,* 2018

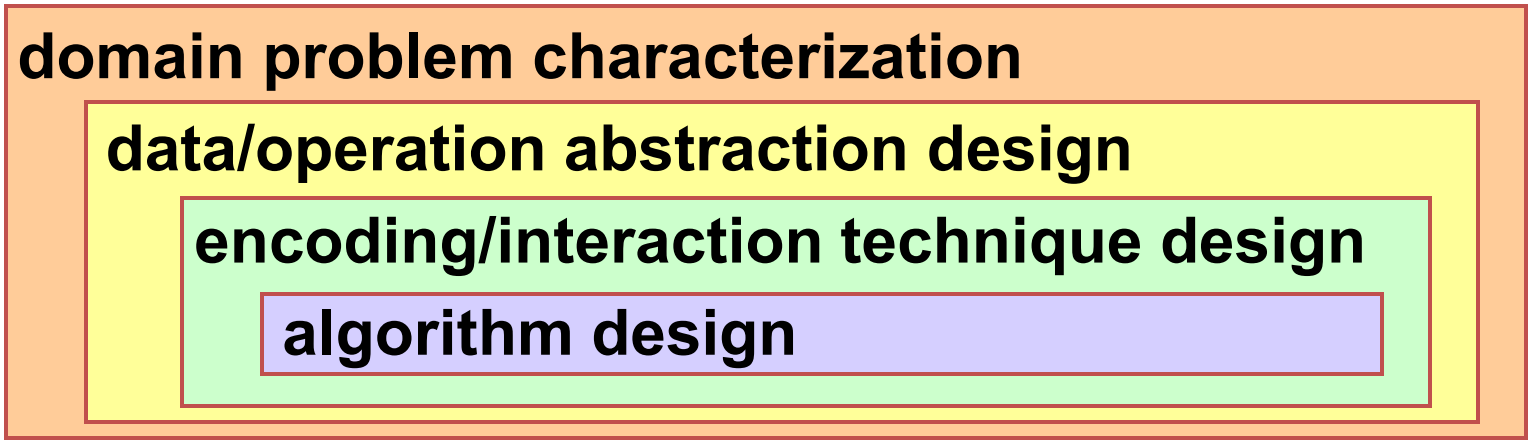
Match Tasks to Visualizations



Step 4: Design an Algorithm

Well-studied computer science problem

- create efficient algorithms
- should support human interaction
- else it would not comply with key principle of visual analytics



Application Example

Let use the causality analyzer framework just presented

- use the car design example

Domain problem characterization

- how to design a faster car without elevating gas consumption

Data/operation abstraction design

- determine how the different car parameters depend on one another (specifically, how do speed/acceleration and mpg relate with respect to design)
- collect data of different car models and compute a causal network

Encoding/interaction technique design

- draw graph where parameters are nodes and causal links are edges
- provide interactions that allows users to test causal links and compute a score

Algorithm design

- Partial correlation followed by causal inferencing/conditioning
- Bayesian Information Criterion (BIC) to model Occam's Razor

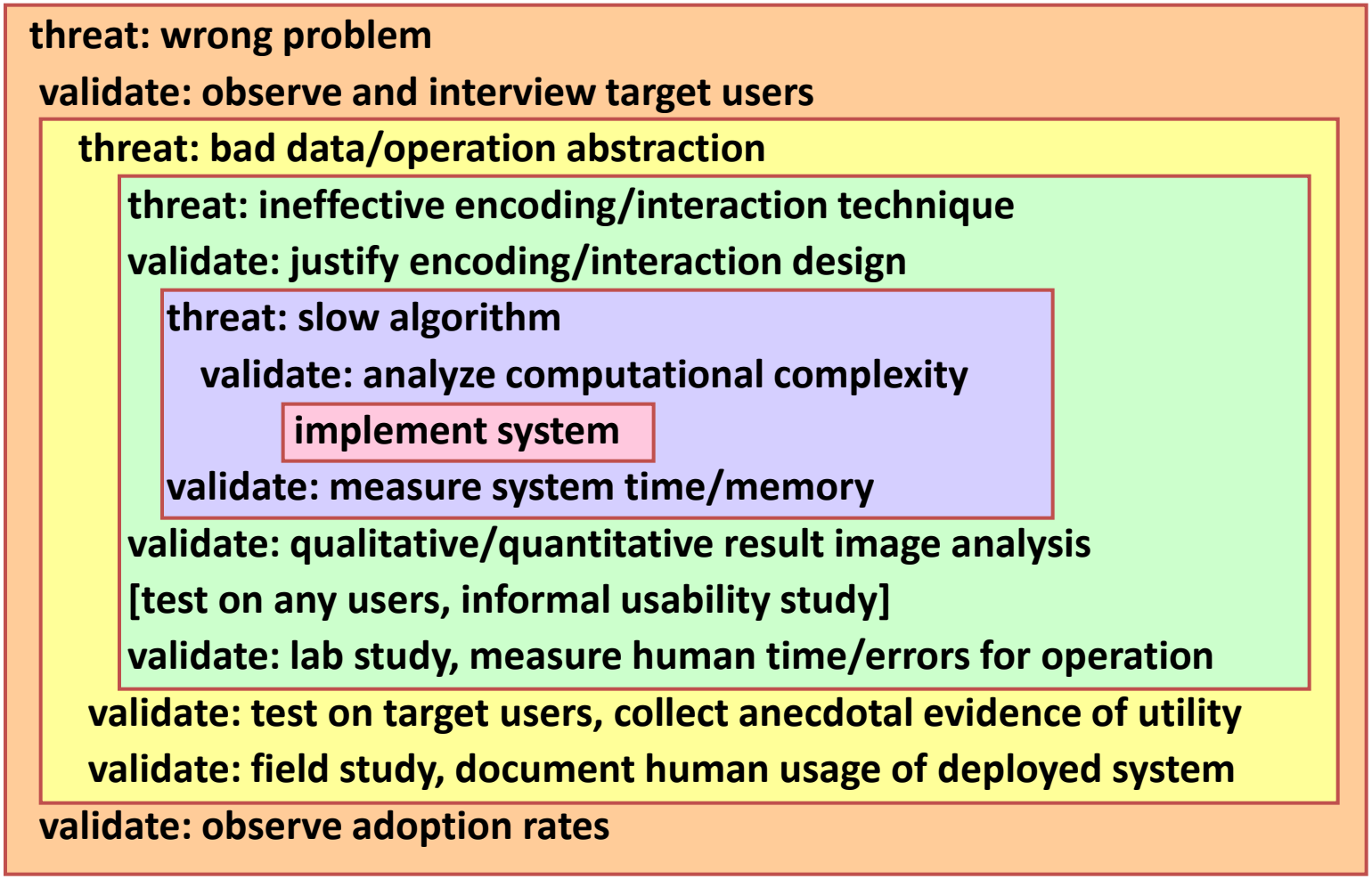
Another Application Example

How the iPhone came about

- domain problem characterization (define need)
- data/operation abstraction design
- encoding/interaction technique design
- algorithm design

June 29, 2007

Gauge Success



Gauge Success

Validate along the way and refine

- formative user study

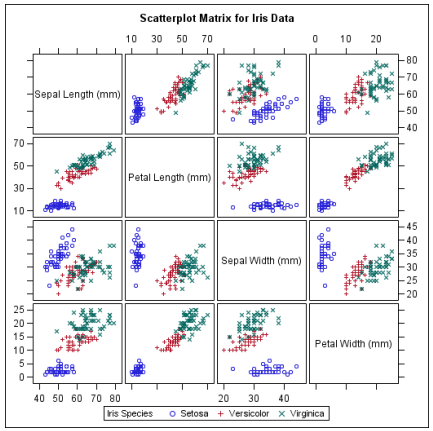
Extend to general user studies of the final design

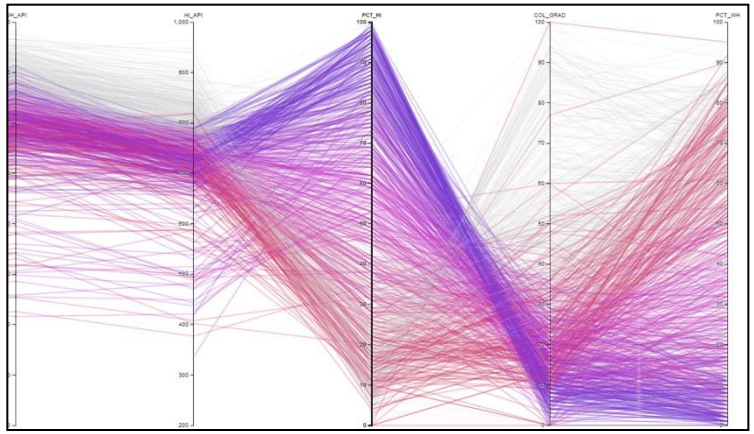
- possibly with mock-ups first (Wizard of Oz designs)
- summative user study
- laboratory study
- smaller number of subjects but can use speak aloud protocol
- crowd-sourced via internet
- potentially greater number of subjects to yield better statistics but can be superficial

Let's discuss evaluation studies next

Suppose…

- You boss asks you to come up with a visualization that can show 4 variables
- This reminds you of the great times at CSE 564
- You also remember these three visualizations





Which One Will You Implement?

Let's Ask

- Your best friend
  - but will he/she be an unbiased judge?
- Ask more people

Testing with Users

- You will need
  - implementations
  - some users
  - a few tasks they can solve

- Ask each user to
  - find a certain relationship in the data
  - find certain data elements
  - and so on

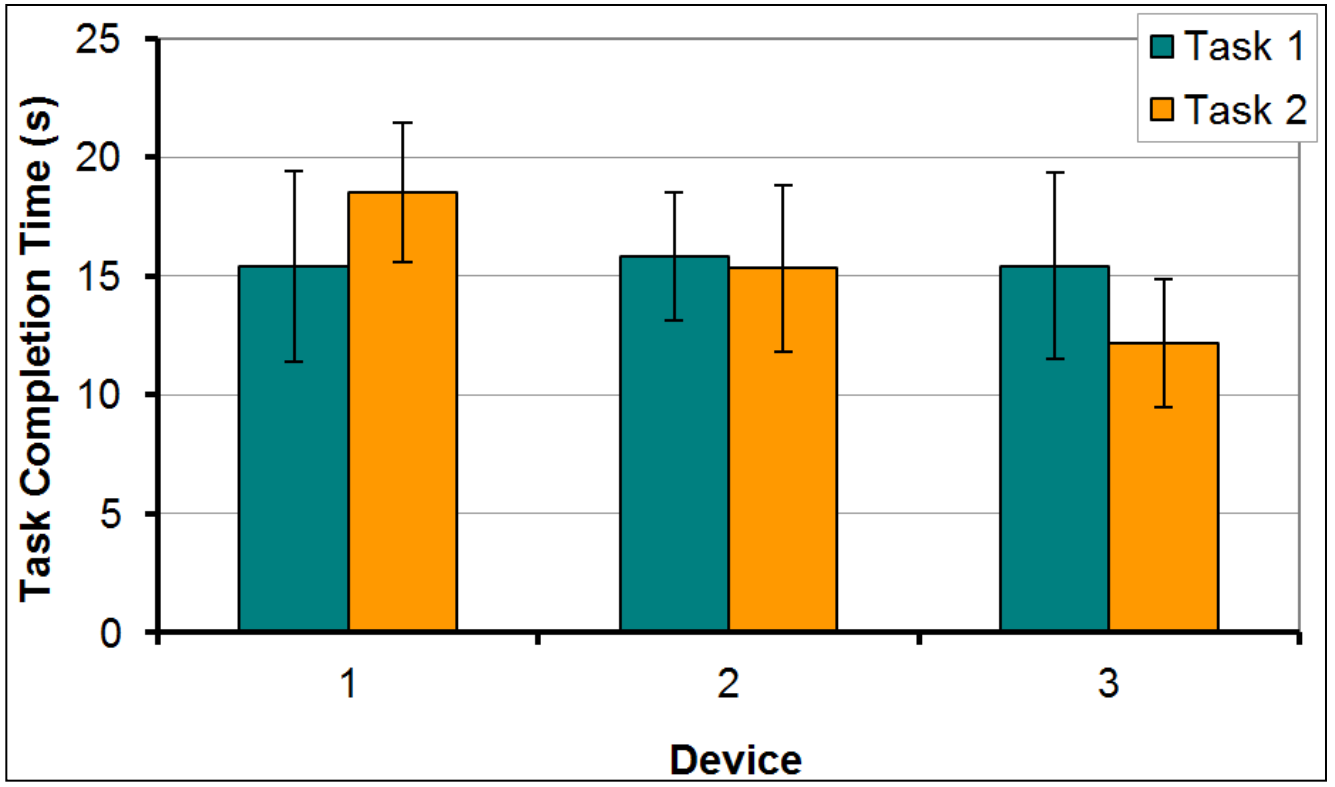
- Measure time and accuracy
- Do this for each of the three visualizations

You Get a Result Like This

Participant	Device 1		Device 2		Device 3	
	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	11	18	15	13	20	14
2	10	14	17	15	11	13
3	10	23	13	20	20	16
4	18	18	11	12	11	10
5	20	21	19	14	19	8
6	14	21	20	11	17	13
7	14	16	15	20	16	12
8	20	21	18	20	14	12
9	14	15	13	17	16	14
10	20	15	18	10	11	16
11	14	20	15	16	10	9
12	20	20	16	16	20	9
Mean	15.4	18.5	15.8	15.3	15.4	12.2
SD	4.01	2.94	2.69	3.50	3.92	2.69

You Get a Result Like This

- Which visualization is best (1, 2, or 3)?



Next Some Basics

Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

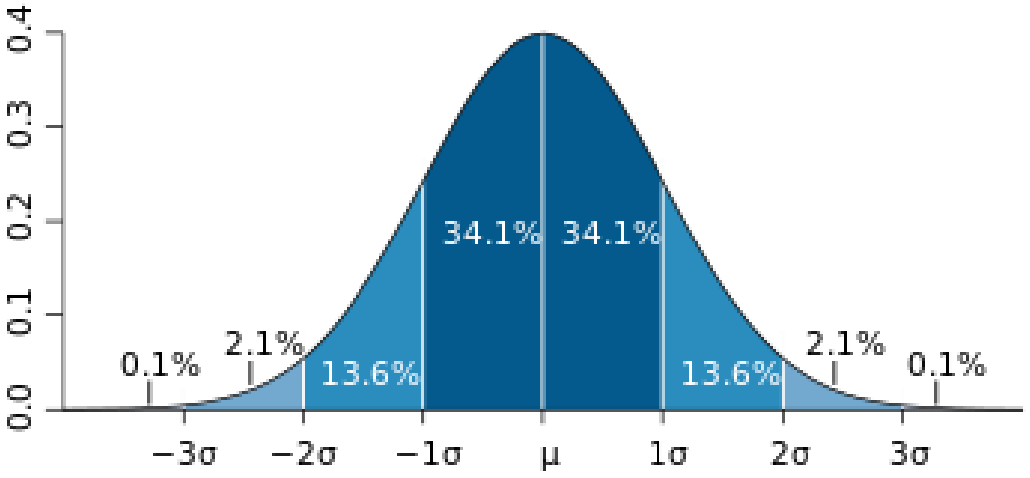
σ = standard deviation

$\sum$ = sum of

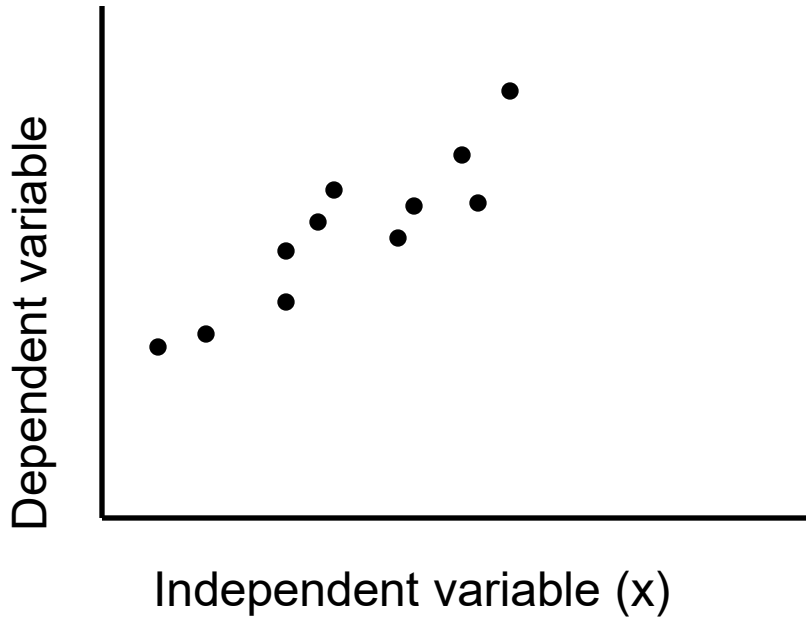
x = each value in the data set

$\bar{x}$ = mean of all values in the data set

n = number of value in the data set



Regression

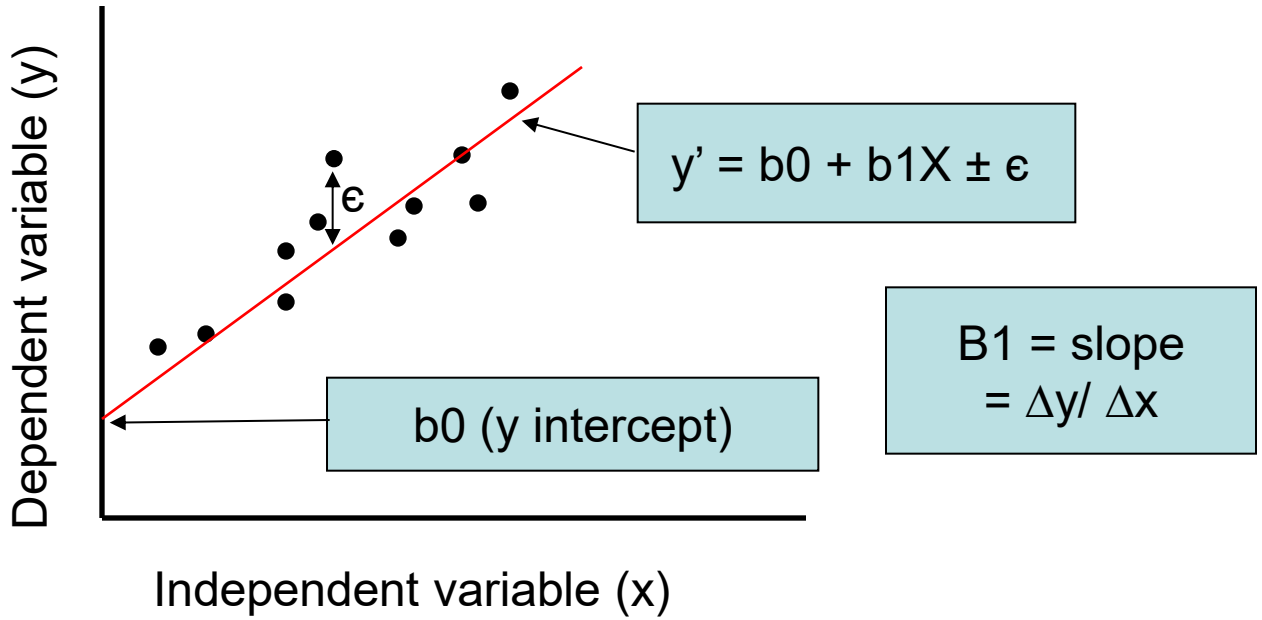


Regression is the attempt to explain the variation in a dependent variable using the variation in independent variables.

Regression is thus an explanation of causation.

If the independent variable(s) sufficiently explain the variation in the dependent variable, the model can be used for prediction.

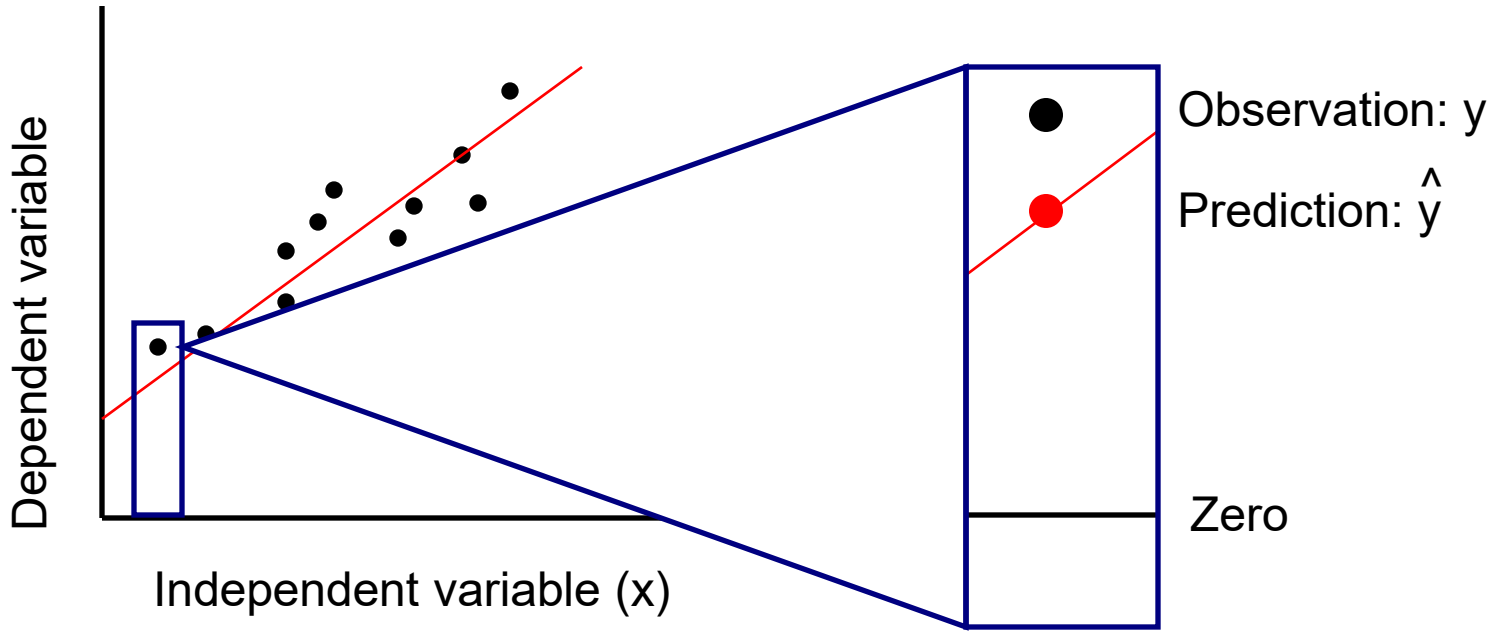
Simple Linear Regression



The output of a regression is a function that predicts the dependent variable based upon values of the independent variables.

Simple regression fits a straight line to the data.

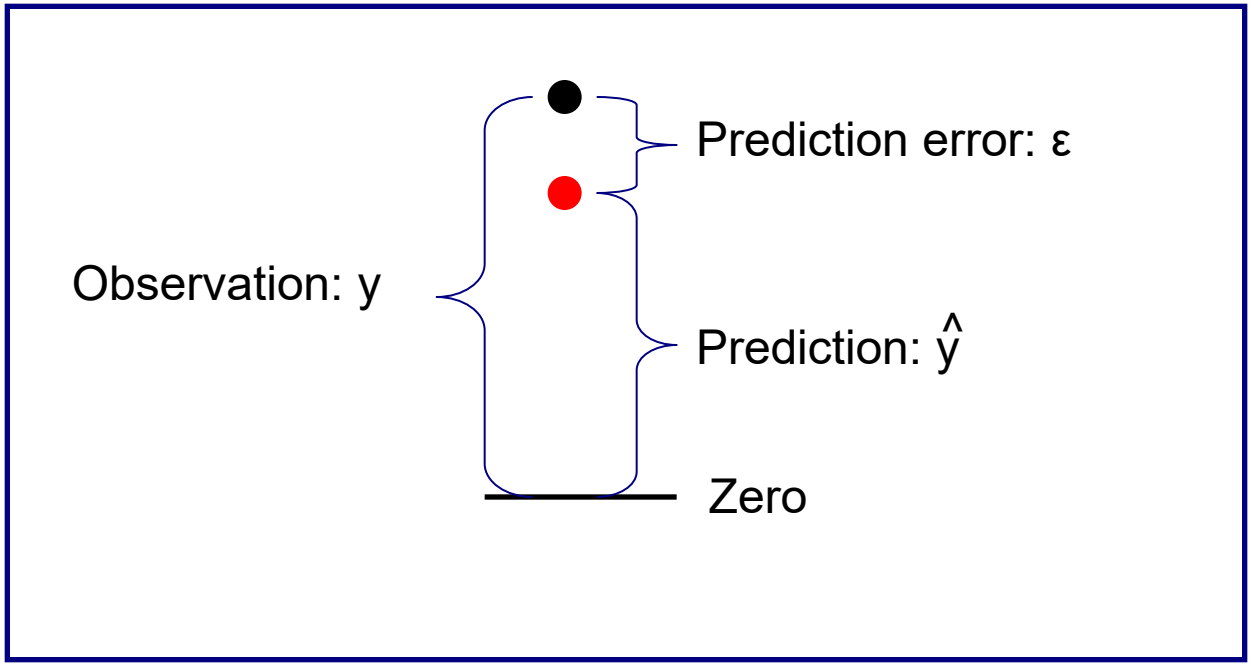
Simple Linear Regression



The function will make a prediction for each observed data point.

The observation is denoted by y and the prediction is denoted by $\hat{y}$.

Simple Linear Regression

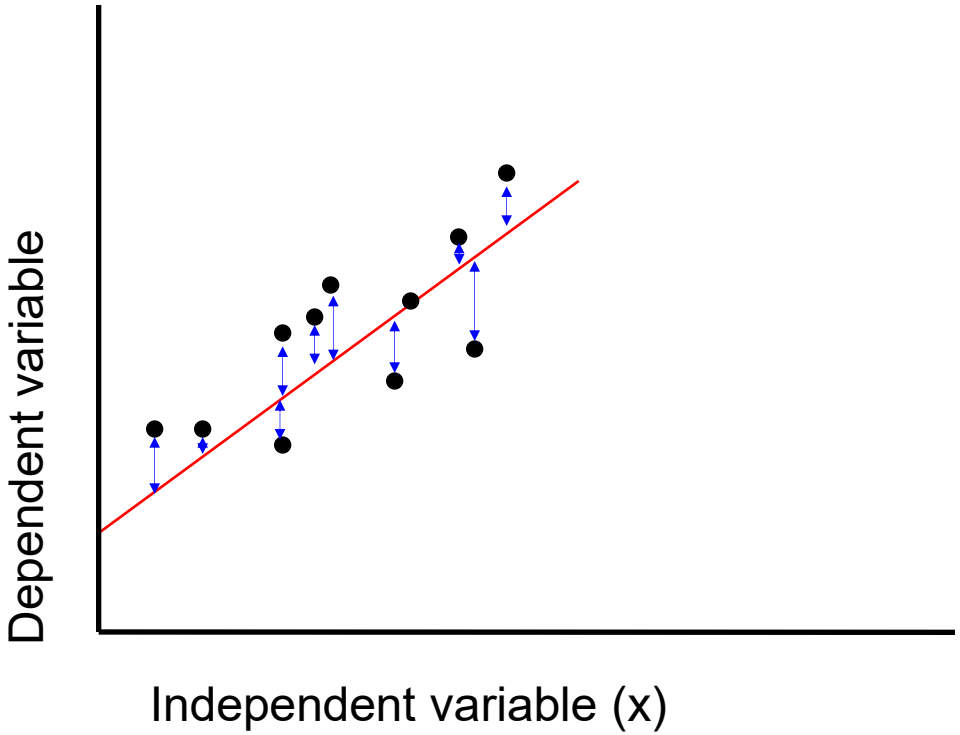


For each observation, the variation can be described as:

$$y = \hat{y} + \varepsilon$$

Actual = Explained + Error

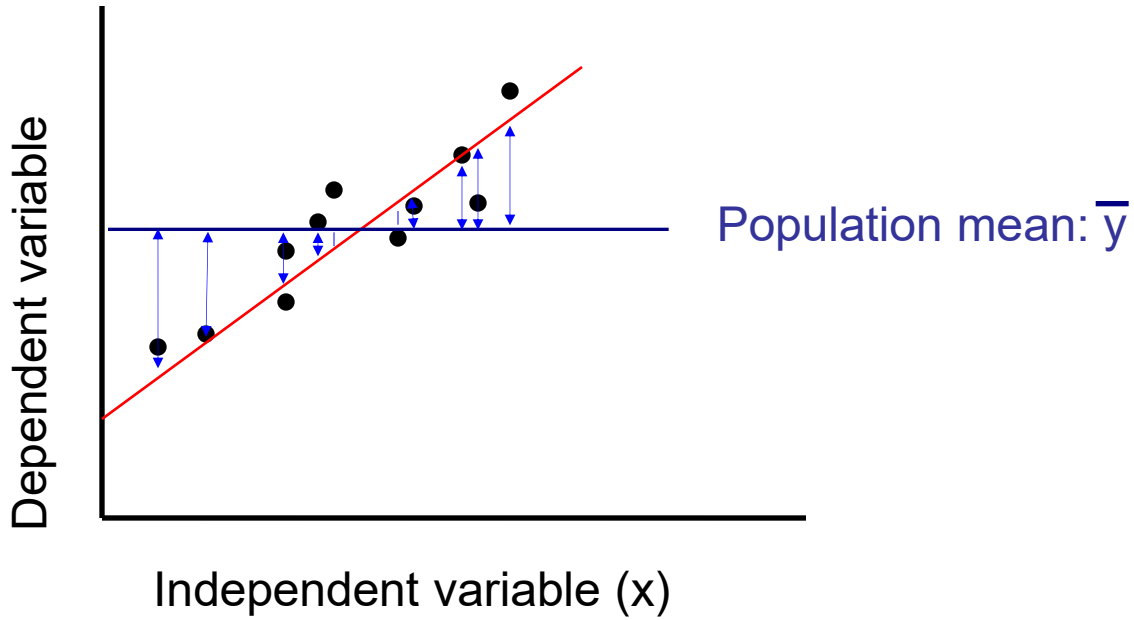
Regression



A least squares regression selects the line with the lowest total sum of squared prediction errors.

This value is called the Sum of Squares of Error, or SSE.

Calculating SSR



The Sum of Squares Regression (SSR) is the sum of the squared differences between the prediction for each observation and the population mean.

Regression Formulas

The Total Sum of Squares (SST) is equal to SSR + SSE.

Mathematically,

$$SSR = \sum (\hat{y} - \bar{y})^2$$ **(measure of explained variation)**

$$SSE = \sum (y - \hat{y})^2$$ **(measure of unexplained variation)**

$$SST = SSR + SSE = \sum (y - \bar{y})^2$$ **(measure of total variation in y)**

remaining slides courtesy of Scott MacKenzie (York University)
“Human-Computer Interaction: An Empirical Research Perspective”

What is Hypothesis Testing?

- … the use of statistical procedures to answer research questions
- Typical research question (generic):

> Is the time to complete a task less using Method A than using Method B?

- For hypothesis testing, research questions are statements:

> There is no difference in the mean time to complete a task using Method A vs. Method B.

- This is the *null hypothesis* (assumption of "no difference")
- Statistical procedures seek to reject or accept the null hypothesis (details to follow)

Analysis of Variance

- The *analysis of variance* (ANOVA) is the most widely used statistical test for hypothesis testing in factorial experiments
- Goal → determine if an independent variable has a significant effect on a dependent variable
- Remember, an independent variable has at least two levels (test conditions)
- Goal (put another way) → determine if the test conditions yield different outcomes on the dependent variable (e.g., one of the test conditions is faster/slower than the other)

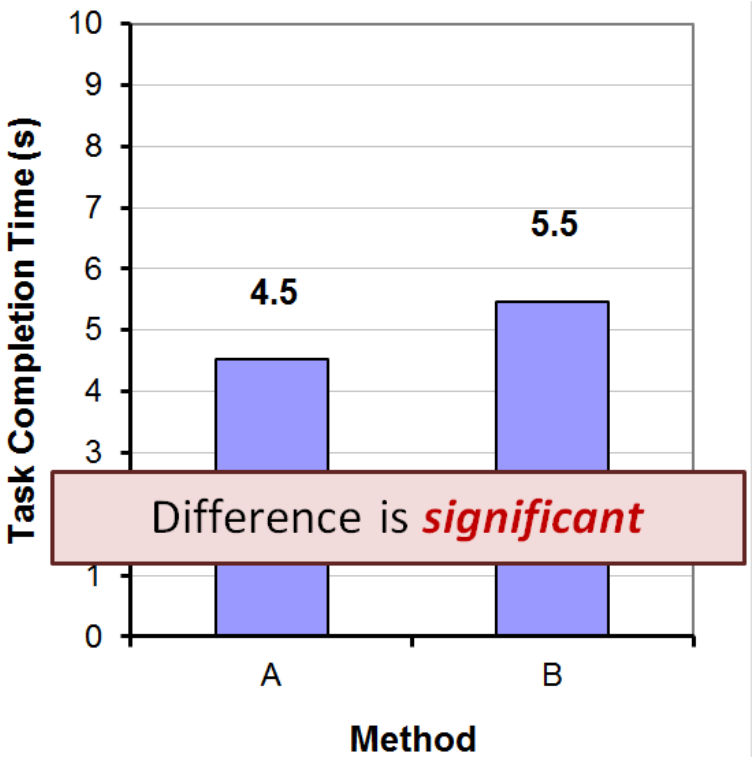
Why Analyze the Variance?

- Seems odd that we analyse the variance when the research question is concerned with the overall means:

> Is the time to complete a task less using Method A than using Method B?

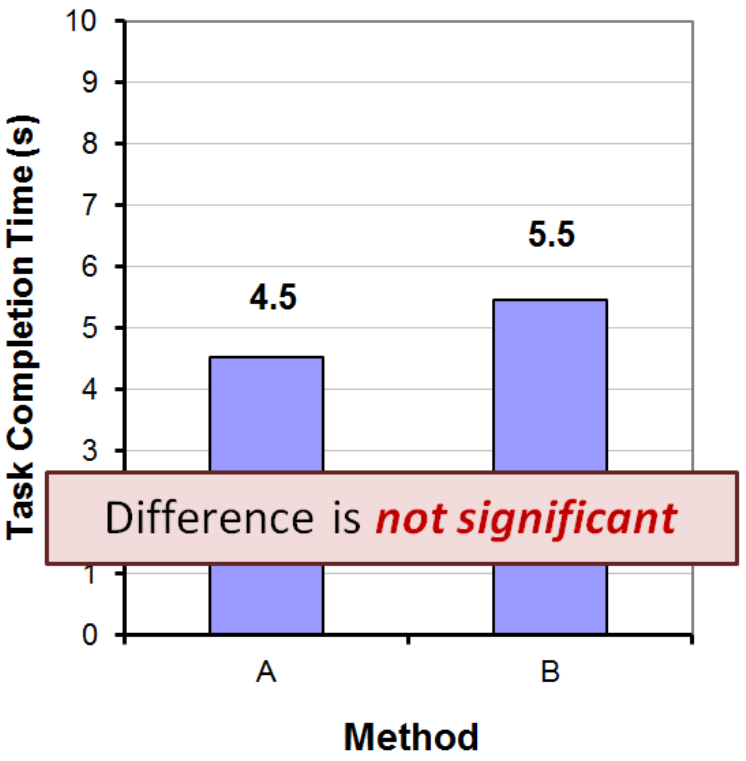
- Let's explain through two simple examples (next slide)

Example #1



"Significant" implies that in all likelihood the difference observed is due to the test conditions (Method A vs. Method B).

Example #2

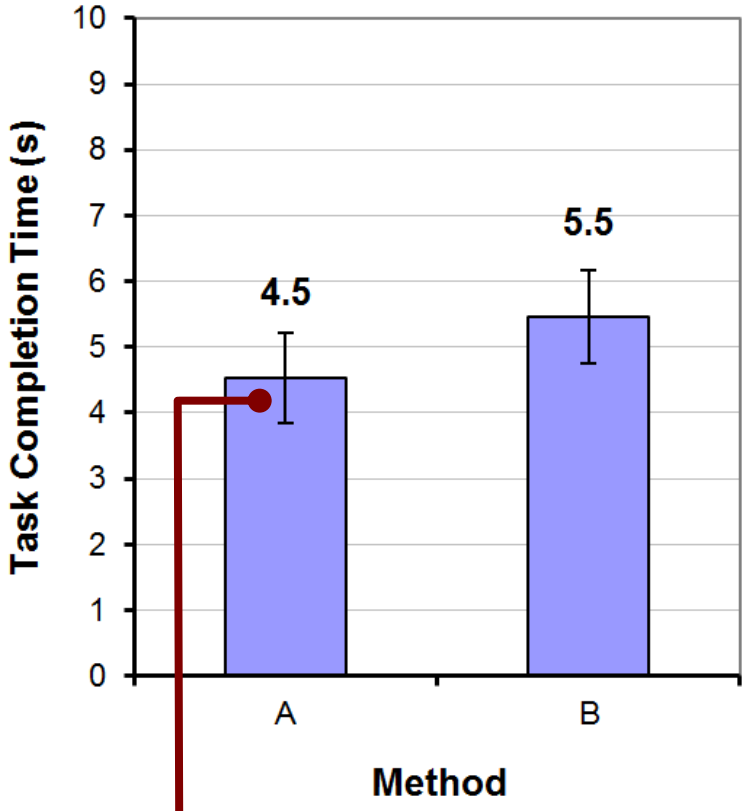


"Not significant" implies that the difference observed is likely due to chance.

File: `06-AnovaDemo.xlsx`

Example #1 - Details

Note: Within-subjects design



Participant	Method	
	A	B
1	5.3	5.7
2	3.6	4.8
3	5.2	5.1
4	3.6	4.5
5	4.6	6.0
6	4.1	6.8
7	4.0	6.0
8	4.8	4.6
9	5.2	5.5
10	5.1	5.6
Mean	4.5	5.5
SD	0.68	0.72

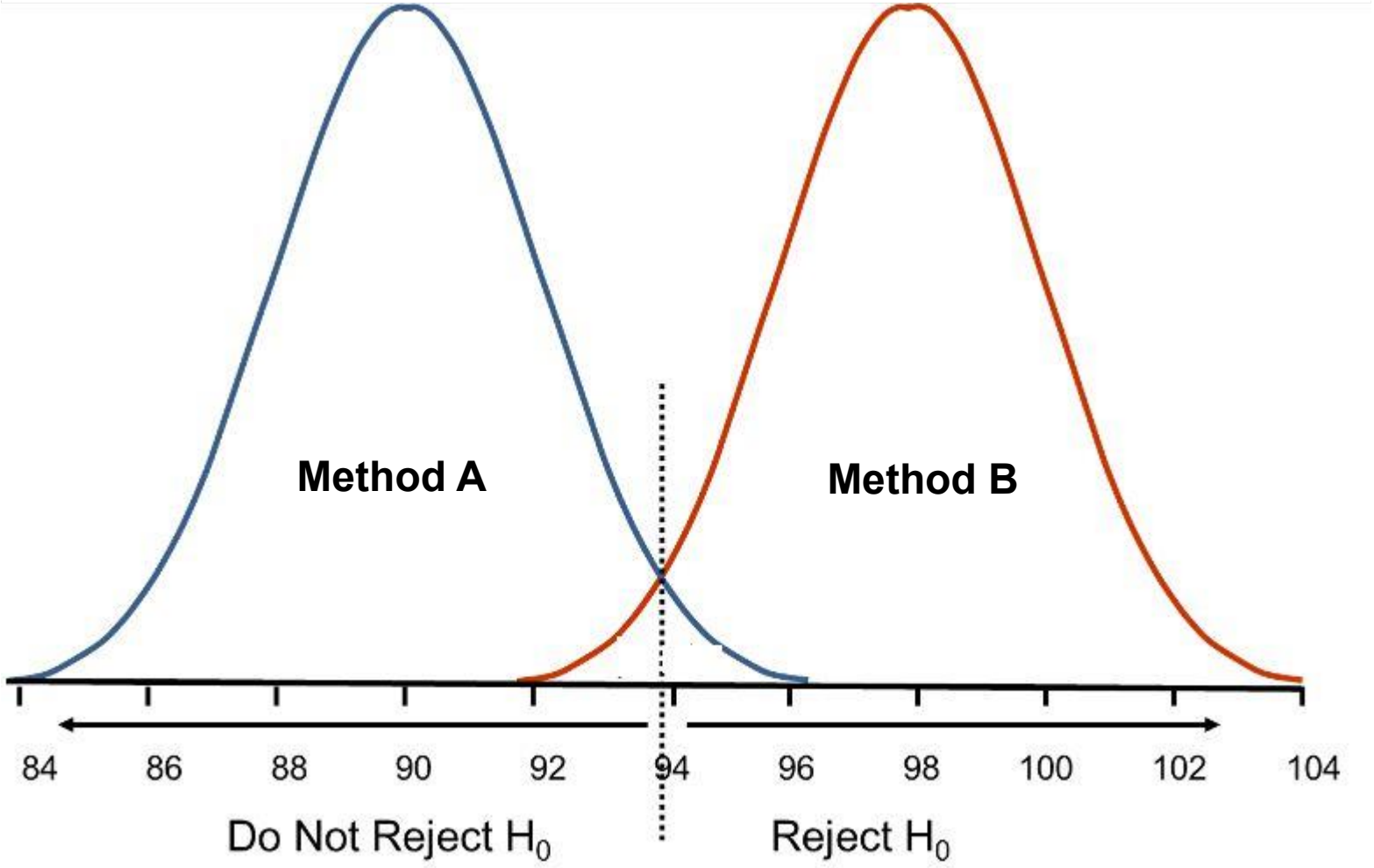
Error bars show
±1 standard deviation

Note: *SD* is the square root of the variance

Make Sure to Randomize

- Eliminate any effect than the one you're after
- Randomize the order in which the subjects run method A and B
  - else may get learning effects of the overall problem
  - method B may turn out better just because users learnt about the problem with method A
- Randomize the data sets or tasks they are asked to use when running method A and B
  - one dataset may be easier than the other
  - method B may turn out better just because the data or tasks was easier

Reject or Not Reject – That's the Question



Example #1 – ANOVA[1]

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	9	5.080	.564				
Method	1	4.232	4.232	9.796	.0121	9.796	.804
Method * Subject	9	3.888	.432				

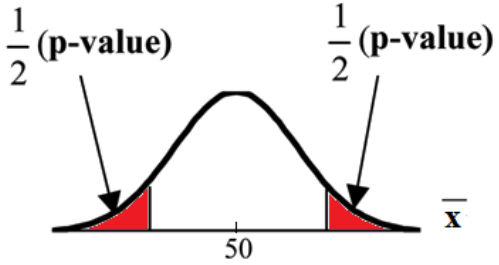
Probability of obtaining the observed data if the null hypothesis is true

Reported as…

$F_{1,9} = 9.80, p < .05$

Thresholds for "p"

- .05
- .01
- .005
- .001
- .0005
- .0001



[1] ANOVA table created by *StatView* (now marketed as *JMP*, a product of SAS; www.sas.com)

Example #1 – ANOVA[1]

MS=SS/df

MS between/within
here: 4.232/0.432 = 9.796

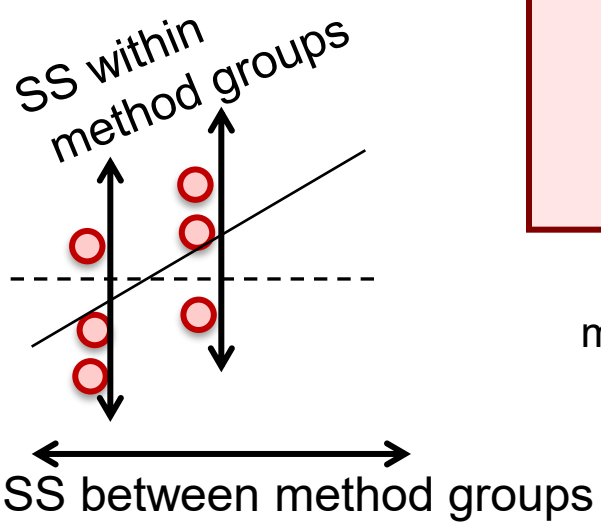
ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	9	5.080	.564				
Method	1	4.232	4.232	9.796	.0121	9.796	.804
Method * Subject	9	3.888	.432				

SS *between* method groups (difference of average treatment effect across groups)

SS *within* method groups (variation of subjects w/r to each treatment mean)

Probability of obtaining the observed data if the null hypothesis is true



Reported as…

$F_{1,9} = 9.80, p < .05$

Thresholds for "p"
- .05
- .01
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- .001
- .0005
- .0001

more explanation, see here

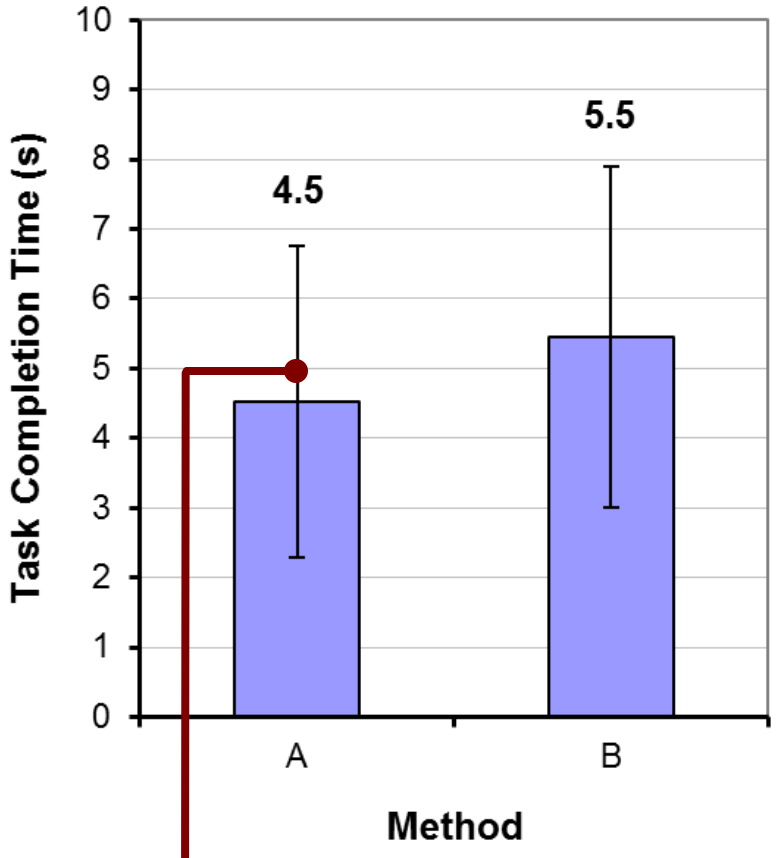
[1] ANOVA table created by *StatView* (now marketed as *JMP*, a product of SAS; www.sas.com)

How to Report an *F*-statistic

> The mean task completion time for Method A was 4.5 s. This was 20.1% less than the mean of 5.5 s observed for Method B. The difference was statistically significant ($F_{1,9} = 9.80$, $p < .05$).

- Notice in the parentheses
  - Uppercase for *F*
  - Lowercase for *p*
  - Italics for *F* and *p*
  - Space both sides of equal sign
  - Space after comma
  - Space on both sides of less-than sign
  - Degrees of freedom are subscript, plain, smaller font
  - Three significant figures for *F* statistic
  - No zero before the decimal point in the *p* statistic (except in Europe)

Example #2 - Details



Error bars show
±1 standard deviation

Participant	Method	
	A	B
1	2.4	6.9
2	2.7	7.2
3	3.4	2.6
4	6.1	1.8
5	6.4	7.8
6	5.4	9.2
7	7.9	4.4
8	1.2	6.6
9	3.0	4.8
10	6.6	3.1
Mean	4.5	5.5
SD	2.23	2.45

Example #2 – ANOVA

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	9	37.372	4.152				
Method	1	4.324	4.324	.626	.4491	.626	.107
Method * Subject	9	62.140	6.904				

Probability of obtaining the observed data if the null hypothesis is true

Reported as…

$F_{1,9} = 0.626$, ns

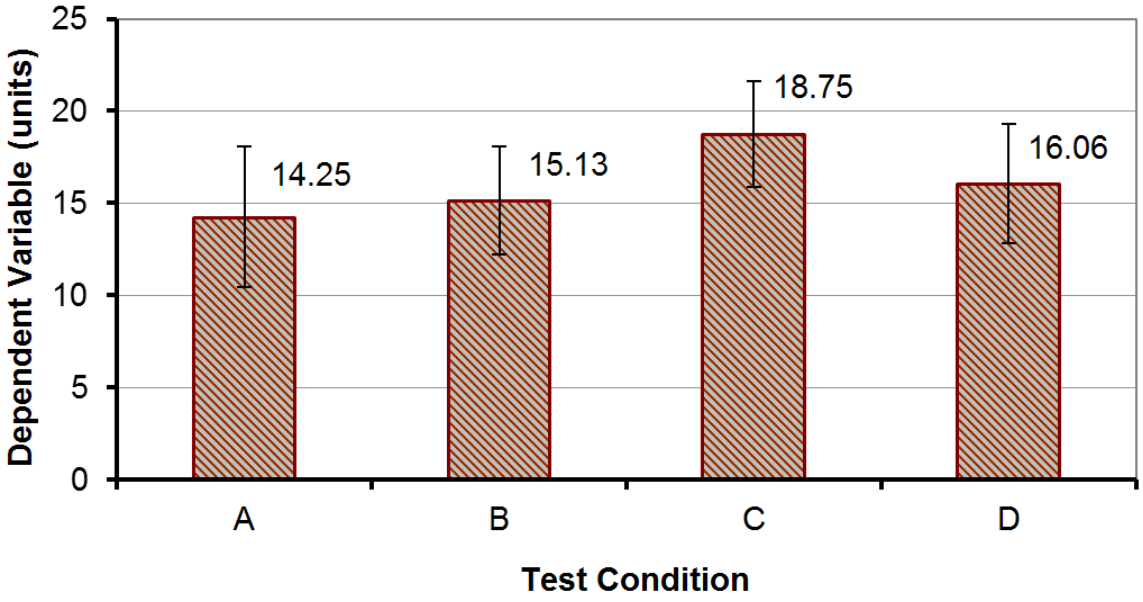
Note: For non-significant effects, use "ns" if $F < 1.0$, or "$p > .05$" if $F > 1.0$.

Example #2 - Reporting

The mean task completion times were 4.5 s for Method A and 5.5 s for Method B. As there was substantial variation in the observations across participants, the difference was not statistically significant as revealed in an analysis of variance ($F_{1,9}$ = 0.626, ns).

More Than Two Test Conditions

Participant	Test Condition			
	A	B	C	D
1	11	11	21	16
2	18	11	22	15
3	17	10	18	13
4	19	15	21	20
5	13	17	23	10
6	10	15	15	20
7	14	14	15	13
8	13	14	19	18
9	19	18	16	12
10	10	17	21	18
11	10	19	22	13
12	16	14	18	20
13	10	20	17	19
14	10	13	21	18
15	20	17	14	18
16	18	17	17	14
Mean	14.25	15.13	18.75	16.06
SD	3.84	2.94	2.89	3.23



ANOVA

ANOVA Table for Dependent Variable (units)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	15	81.109	5.407				
Test Condition	3	182.172	60.724	4.954	.0047	14.862	.896
Test Condition * Subject	45	551.578	12.257				

- There was a significant effect of Test Condition on the dependent variable ($F_{3,45} = 4.95$, $p < .005$)
- Degrees of freedom
  - If n is the number of test conditions and m is the number of participants, the degrees of freedom are…
  - Effect → $(n - 1)$
  - Residual → $(n - 1)(m - 1)$
  - Note: single-factor, within-subjects design

Post Hoc Comparisons Tests

- A significant *F*-test means that at least one of the test conditions differed significantly from one other test condition
- Does not indicate which test conditions differed significantly from one another
- To determine which pairs differ significantly, a post hoc comparisons tests is used
- Examples:
  - Fisher PLSD, Bonferroni/Dunn, Dunnett, Tukey/Kramer, Games/Howell, Student-Newman-Keuls, orthogonal contrasts, Scheffé
- Scheffé test on next slide

Scheffé Post Hoc Comparisons

Scheffe for Dependent Variable (units)
Effect: Test Condition
Significance Level: 5 %

	Mean Diff.	Crit. Diff.	P-Value	
A, B	-.875	3.302	.9003	
A, C	-4.500	3.302	.0032	S
A, D	-1.813	3.302	.4822	
B, C	-3.625	3.302	.0256	S
B, D	-.938	3.302	.8806	
C, D	2.688	3.302	.1520	

- Test conditions A:C and B:C differ significantly (see chart three slides back)

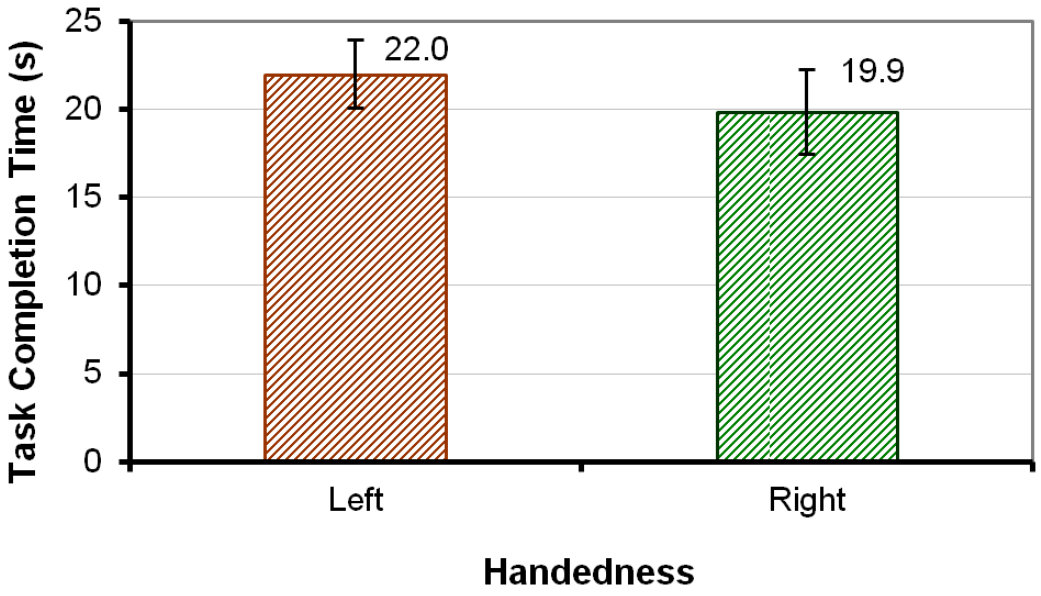
Between-subjects Designs

- Research question:
  - *Do left-handed users and right-handed users differ in the time to complete an interaction task?*
- The independent variable (handedness) must be assigned between-subjects
- Example data set →

Participant	Task Completion Time (s)	Handedness
1	23	L
2	19	L
3	22	L
4	21	L
5	23	L
6	20	L
7	25	L
8	23	L
9	17	R
10	19	R
11	16	R
12	21	R
13	23	R
14	20	R
15	22	R
16	21	R
Mean	20.9	
SD	2.38	

Summary Data and Chart

Handedness	Task Completion Time (s)	
	Mean	*SD*
Left	22.0	1.93
Right	19.9	2.42



ANOVA

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Handedness	1	18.063	18.063	3.781	.0722	3.781	.429
Residual	14	66.875	4.777				

- The difference was not statistically significant ($F_{1,14} = 3.78$, $p > .05$)
- Degrees of freedom:
  - Effect → $(n - 1)$
  - Residual → $(m - n)$
  - Note: single-factor, between-subjects design

Two-way ANOVA

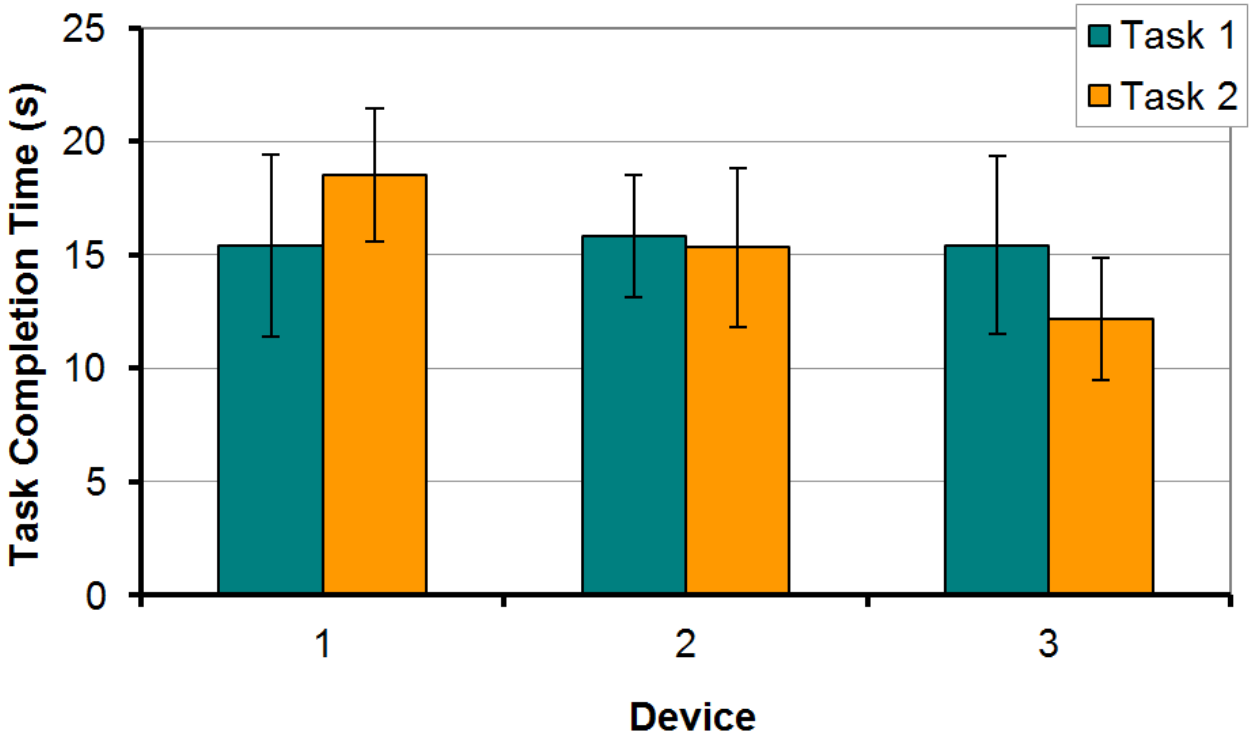
- An experiment with two independent variables is a *two-way design*
- ANOVA tests for
  - Two main effects + one interaction effect
- Example
  - Independent variables
    - Device → D1, D2, D3 (e.g., mouse, stylus, touchpad)
    - Task → T1, T2 (e.g., point-select, drag-select)
  - Dependent variable
    - Task completion time (or something, this isn't important here)
  - Both IVs assigned within-subjects
  - Participants: 12
  - Data set (next slide)

Data Set

Participant	Device 1		Device 2		Device 3	
	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	11	18	15	13	20	14
2	10	14	17	15	11	13
3	10	23	13	20	20	16
4	18	18	11	12	11	10
5	20	21	19	14	19	8
6	14	21	20	11	17	13
7	14	16	15	20	16	12
8	20	21	18	20	14	12
9	14	15	13	17	16	14
10	20	15	18	10	11	16
11	14	20	15	16	10	9
12	20	20	16	16	20	9
Mean	15.4	18.5	15.8	15.3	15.4	12.2
SD	4.01	2.94	2.69	3.50	3.92	2.69

Summary Data and Chart

	Task 1	Task 2	*Mean*
Device 1	15.4	18.5	17.0
Device 2	15.8	15.3	15.6
Device 3	15.4	12.2	13.8
Mean	15.6	15.3	15.4



ANOVA

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	11	134.778	12.253				
Device	2	121.028	60.514	5.865	.0091	11.731	.831
Device * Subject	22	226.972	10.317				
Task	1	.889	.889	.076	.7875	.076	.057
Task * Subject	11	128.111	11.646				
Device * Task	2	121.028	60.514	5.435	.0121	10.869	.798
Device * Task * Subject	22	244.972	11.135				

Can you pull the relevant statistics from this chart and craft statements indicating the outcome of the ANOVA?

ANOVA - Reporting

The grand mean for task completion time was 15.4 seconds. Device 3 was the fastest at 13.8 seconds, while device 1 was the slowest at 17.0 seconds. The main effect of device on task completion time was statistically significant ($F_{2,22} = 5.865$, $p < .01$). The task effect was modest, however. Task completion time was 15.6 seconds for task 1. Task 2 was slightly faster at 15.3 seconds; however, the difference was not statistically significant ($F_{1,11} = 0.076$, ns). The results by device and task are shown in Figure x. There was a significant Device $\times$ Task interaction effect ($F_{2,22} = 5.435$, $p < .05$), which was due solely to the difference between device 1 task 2 and device 3 task 2, as determined by a Scheffé post hoc analysis.

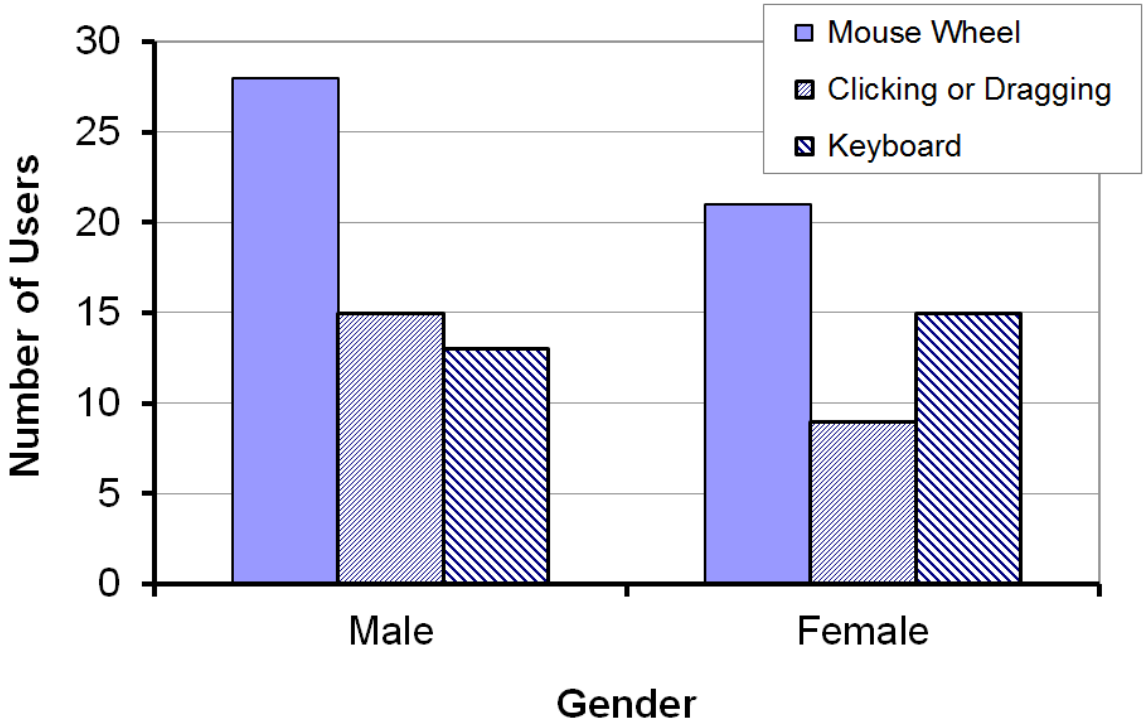
Chi-square Test (Nominal Data)

- A *chi-square test* is used to investigate relationships
- Relationships between categorical, or nominal-scale, variables representing attributes of people, interaction techniques, systems, etc.
- Data organized in a *contingency table* – cross tabulation containing counts (frequency data) for number of observations in each category
- A chi-square test compares the *observed values* against *expected values*
- Expected values assume "no difference"
- Research question:
  - *Do males and females differ in their method of scrolling on desktop systems?* (next slide)

Chi-square – Example #1

Observed Number of Users				
Gender	Scrolling Method			Total
	MW	CD	KB	
Male	28	15	13	56
Female	21	9	15	45
Total	49	24	28	101

MW = mouse wheel
CD = clicking, dragging
KB = keyboard



Chi-square – Example #1

56.0·49.0/101=27.2

Expected Number of Users				
Gender	Scrolling Method			Total
	MW	CD	KB	
Male	27.2	13.3	15.5	56.0
Female	21.8	10.7	12.5	45.0
Total	49.0	24.0	28.0	101

(Expected-Observed)^2/Expected=(28-27.2)^2/27.2

Chi Squares				
Gender	Scrolling Method			Total
	MW	CD	KB	
Male	0.025	0.215	0.411	0.651
Female	0.032	0.268	0.511	0.811
Total	0.057	0.483	0.922	**1.462**

Significant if it exceeds critical value (next slide)

$\chi^2 = 1.462$

(See **HCI:ERP** for calculations)

Chi-square Critical Values

- Decide in advance on *alpha* (typically .05)
- Degrees of freedom
  - $df = (r - 1)(c - 1) = (2 - 1)(3 - 1) = 2$
  - r = number of rows, c = number of columns

Significance Threshold (α)	Degrees of Freedom							
	1	2	3	4	5	6	7	8
.1	2.71	4.61	6.25	7.78	9.24	10.65	12.02	13.36
.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
.01	6.64	9.21	11.35	13.28	15.09	16.81	18.48	20.09
.001	10.83	13.82	16.27	18.47	20.52	22.46	24.32	26.13

$\chi^2 = 1.462$ (< 5.99 ∴ not significant)