This homework is also meant as a conversation starter for the last class of the semester, where you can ask questions about all class material. Please, do not look in homeworks from previous years for solutions. This quick fix may help you now, but not in the exam, where good performance is much more relevant to your final grade. The homework is meant to make you think about the material you heard about in class, to help you make sure you really understand it.

1. (MRI) Magnetization:
   Assume a high quality MR scanner with field strength $B_0 = 5$ Tesla and with no magnetic gradient applied.
   a) What is the frequency of the RF pulse required to flip the protons (Hydrogen atoms) 90 degrees? (The gyromagnetic ratio for Hydrogen is: $\gamma = 42.58 \text{ MHz/T}$).
   b) 200 ms after the correct 90 degree pulse is stopped, what percentage of the transverse component of the net magnetization vector remains in the fat molecules? Hint: what is the time constant relevant for this component?
   c) 200 ms after the correct 90 degree pulse is stopped, what percentage of the longitudinal component of the net magnetization vector of the fat molecules has been restored? Hint: what is the time constant relevant for this component?
   d) Now solve b) and c) for water molecules.

2. (MRI) Gradients:
   A patient is positioned in a 2.5 Tesla magnet; a gradient in the $z$ (axial) direction is applied with strength $G_z = 15 \text{ mT/m}$ ($15 \times 10^{-3} \text{ Tesla/meter}$). The zero crossing of the gradient occurs at the patient’s chin ($B_z(\text{chin}) = 0$ Tesla), and it becomes positive upwards, towards the eyes.
   a) What is both the magnet field due to the gradient and that due to the overall magnet field at the patient’s nose (which is 11 cm from the chin)?
   b) What frequency should the RF pulse be centered on to flip the protons in the slice including the nose?
   c) If a RF pulse centered at your favorite radio station frequency of 106.3 MHz is applied, name a potential organ or feature of the body that will be imaged? Assume the patient’s head is the straight up (not looking up or down). Justify your answer.
   d) Imagine you just invented the concept of k-space acquisition. An important conference where all major medical physicists of the world will be gathering is about to come up, and all you have time and resources for is to communicate your great discovery in a short scientific essay, which is bound to make you eternally famous. For this, you must explain how you were able to spatially encode the slice in such a way that the signals acquired with RF coils can be translated to the Fourier spectrum of the selected slice. Of course, once this is known, obtaining the actual slice in the spatial domain is relatively easy, but explain this as well, to form the punch line of your article. Since this is a scientific conference, you need to be specific, also with the mathematics. But knowing your celebrity science audience, you can rest assured that they know Fourier transforms and MRI physics.
3. (MRI) Weighting:
The patient, whose brain slice is shown on the right, shows some strange behavior, which leads the neurologist to suspect one or two types of (rather aggressive) brain tumors. He prescribes a few MRI scans to determine a diagnosis. Assume the following (fictional) numbers for T1/T2 constants (in ms):
brain (100/10), brain stem (200/50), tumor A (30/20), tumor B (190/180)
a) What weighting do you suggest to image tumor A with good contrast?
b) What weighting do you suggest to image tumor B with good contrast?
For both answers, specify an approximate TE and TR

4. (MRI) Signal acquisition:
In the EPI pulse sequence to the right, plot the resulting k-space traversal.

5. (Ultrasound) Doppler:
Assume that a certain individual has a slight aortic regurgitation, that is, the aortic valve does not close properly after blood ejection and due to this, the aortic pressure pushes some blood back into the left ventricle. Let us assume that the maximum blood velocity $v_{out \text{ max}}$ out of the left ventricle into the aorta is 3 m/s, and further assume that the blood velocity in regurgitation $v_{reg \text{ max}}$ is 1 m/s. Using a Doppler pulse frequency of 2.5 MHz, what is the frequency of the Doppler sound:
a) at maximum ejection speed
b) at maximum regurgitation speed
Assume that you are pointing the ultrasound beam right orthogonal to the valve, in the direction of the aorta.