MIC-GPU: High-Performance Computing for Medical Imaging on Programmable Graphics Hardware (GPUs)

SPIE Medical Imaging

Parallelism in CT Reconstruction

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CT Reconstruction Pipeline

A CT reconstruction pipeline is typically composed of a number of serial components

Example 1: Filtered Backprojection

- projection filtering
- backprojection
- post-weighting

Example 2: Iterative 3D reconstruction in blocks

- forward projection of volume into set's views
- correction factor computation
- backprojection of correction factors
- post-weighting (normalization)

Kernel-Centric Decomposition

By locating parallelism, we can consider each of these steps to be a SIMD kernel, as follows:

Example 1: Filtered Backprojection

- projection filtering → *filtering kernel*
- backprojection → *backprojection kernel*
- post-weighting → *post-weighting kernel*

Example 2: Iterative 3D reconstruction in blocks

- backprojection of volume into set's views → *projection kernel*
- correction factor computation → *correction factor kernel*
- backprojection of correction factors → *backprojection kernel*
- normalization → *normalization kernel*

vector operations — projector with interpolation

Kernel Scheduling

SIMD can only execute one kernel at a time

- this prohibits kernel overlap, even if mathematically correct
- we may merge kernels if targets are identical → this favors load balancing and the reduction of passes

Therefore a decomposition of a reconstruction pipeline into components is advisable

- develop an optimized kernel for each component
- overlap (=hide) the loading of data (if needed) with execution of a prior kernel (or within kernel)
- also optimize what platform to run the computations (CPU, GPU), but then consider transfer of data

Popular CT Reconstruction Pipelines

We will discuss:

- analytical schemes (Feldkamp)
- iterative schemes (SART, SIRT, EM)
- in terms of anatomical and metabolic (functional) CT

The projection/backprojection is typically the most expensive operation

- it is part of every algorithm and application
- with variations in
  - beam geometry
  - modeling of tissue (attenuation, scattering) and detector effect
  - each is implemented with a dedicated kernel
  - each such kernel is loaded into the GPU on demand

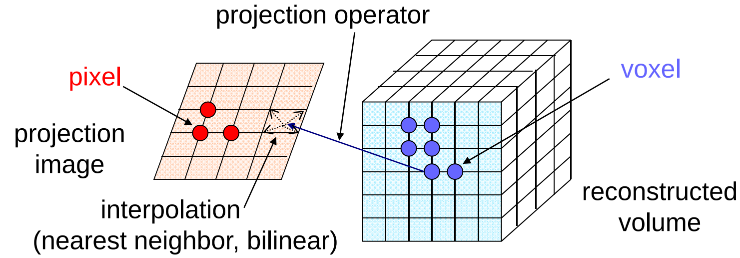
Terminology

We shall discuss all material in terms of 3D reconstruction

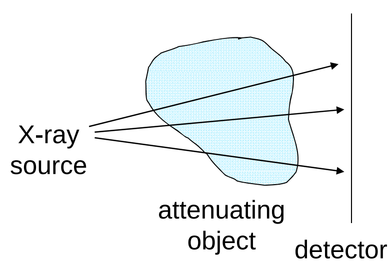
- the reduction to 2D slice reconstruction is straightforward

Pixels: the basis elements (point samples) of the projection image (the photon measurements)

Voxels: the basis elements (point samples) of the reconstruction volume (the attenuation densities or the tracer photon emissions)

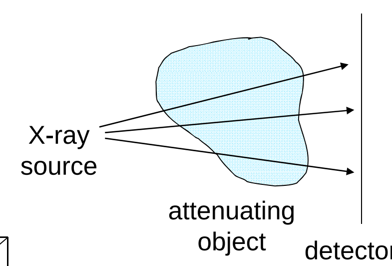


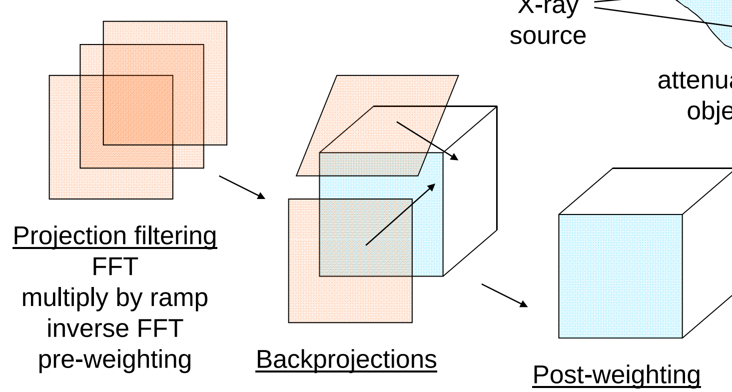
Transmission CT



Transmission CT

A typical reconstruction algorithm is Filtered Backprojection



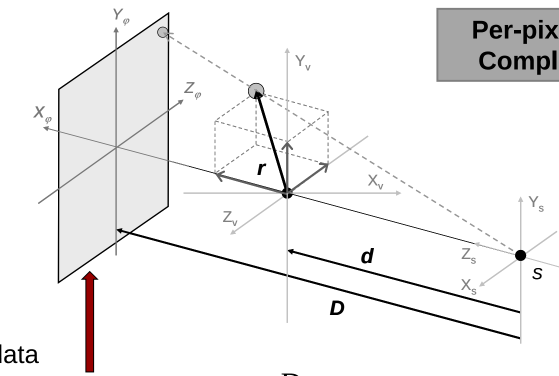


Example

Feldkamp-Davis-Kress (FDK) Cone-beam reconstruction

FDK: Filtering

Per-pixel operation
Complexity: O(N^2)



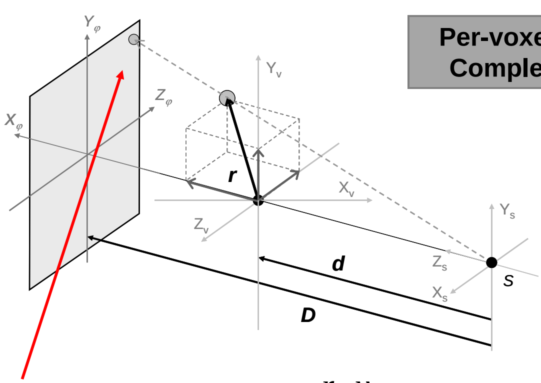
filtered projection data

$$\hat{P}_\phi(Y,Z) = \frac{D}{\sqrt{D^2+Y^2+Z^2}} P_\phi(Y,Z) ** g(Y)$$

circular pre-weighting — projection data — ramp filter

FDK: Backprojection

Per-voxel operation
Complexity: O(N^3)

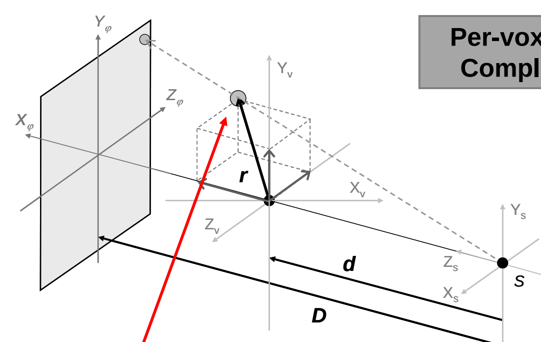


$$\hat{P}_\varphi(r) = P_\varphi(Y(r), Z(r)) \qquad Y(r) = \frac{r \cdot y_\phi}{d + r \cdot x_\phi} D, Z(r) = \frac{r \cdot z_\phi}{d + r \cdot x_\phi} D$$

voxel→projection mapping — projection coordinates of mapped voxel

FDK: Accumulation, Depth-Weighting

Per-voxel operation
Complexity: O(N^3)



reconstructed voxel

$$f(\mathbf{r}) = \frac{1}{4\pi^2} \int_0^{2\pi} \frac{d^2}{(d + \mathbf{r} \cdot x_\phi)^2} \hat{P}_\phi(\mathbf{r}) d\phi$$

accumulation for all projections — depth-weighting

Other Analytical Algorithms

Similar concepts apply for other analytical CT algorithms

- modified FDK, multi-orbit cone-beam CT
- helical CT with exact and non-exact algorithms

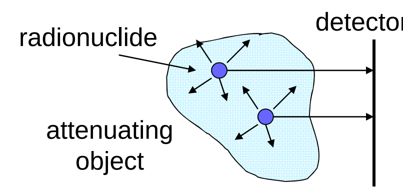
Always a sequence of serial steps

1. projection filtering, possibly rebinning
2. backprojection
3. accumulation and weighting

Only backprojection (and rebinning) requires interpolation

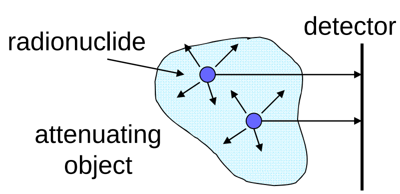
The remaining operations are straight vector arithmetic

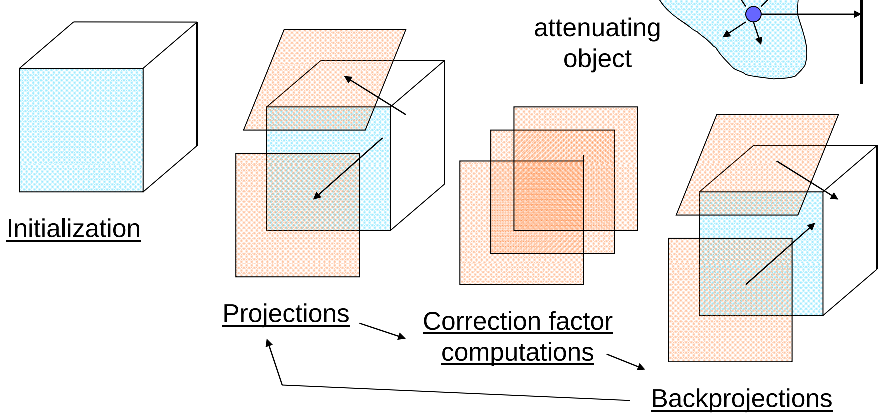
Emission CT



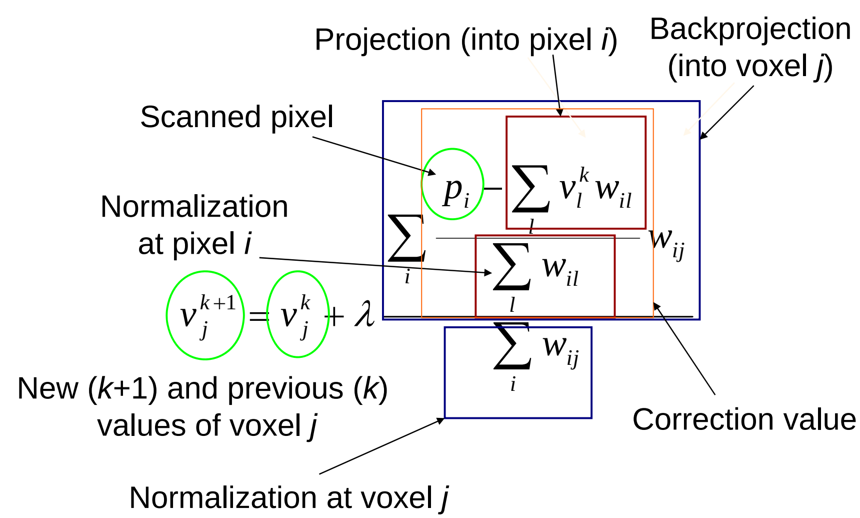
Emission CT

Typically reconstructions are done in an iterative manner





Example: SART/SIRT



$$v_j^{k+1} = v_j^k + \lambda \frac{\sum_i \frac{p_i - \sum_l v_l^k w_{il}}{\sum_l w_{il}} w_{ij}}{\sum_i w_{ij}}$$

Example: EM (OS-EM)

Maximizes the likelihod of the values of voxels *j*, given values at pixels *i*

New (*k*+1) and previous (*k*) values of voxel *j*

Backprojection (into voxel *j*)

$$v_j^{k+1} = \frac{v_j^k}{\sum_i w_{ij}} \sum_i \frac{p_i}{\sum_j v_j^k w_{ij}}$$

Correction value

weight factor

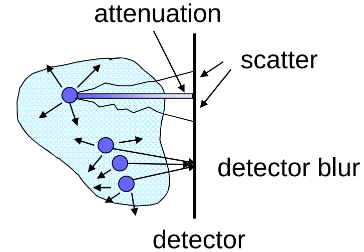
Normalization at voxel *j*

Projection (into pixel *i*)

The Weight Factor

The weight factor w_{ij} can model various effects:

- interpolation filter factors (nearest neighbor, bilinear, Gaussian, Bessel, etc)
- detector geometric response (blurring due to off-angle photon contributions)
- photon attenuation (requires an attenuation map μ obtained via transmission CT)
- photon scattering (requires a gradient map, typically the transmission CT reconstruction)
- …



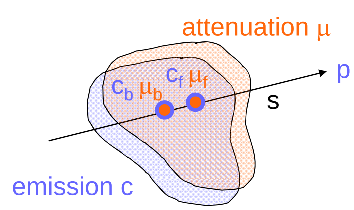
We will focus on

- attenuation modeling
- scatter effect

Attenuation Modeling: Theory

Forward projection:

- the energy arriving at a detector pixel is: $p = \int_{s=0}^{l} c(s)\, e^{-\int_{t=0}^{s} \mu(t)dt}\, ds$

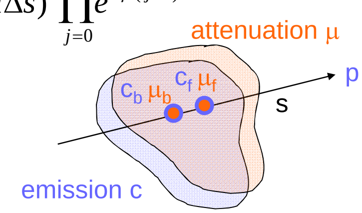


Attenuation Modeling: Theory

Forward projection:

- the energy arriving at a detector pixel is: $p = \int_{s=0}^{l} c(s)\, e^{-\int_{t=0}^{s} \mu(t)dt}\, ds$
- in discrete terms:

$$p \approx \sum_{i=0}^{L/\Delta s} c(i\Delta s)\, e^{-\sum_{j=0}^{i-1} \mu(j\Delta s)} = \sum_{i=0}^{L/\Delta s} c(i\Delta s) \prod_{j=0}^{i-1} e^{-\mu(j\Delta s)}$$



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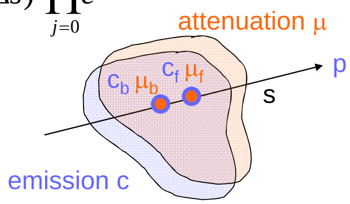
- using a Taylor series approximation:

$$p \approx \sum_{i=0}^{L/\Delta s} c(i\Delta s) \prod_{j=0}^{i-1} (1-\mu(j\Delta s))$$



note: all values are normalized to [0,1]

Attenuation Modeling: Theory

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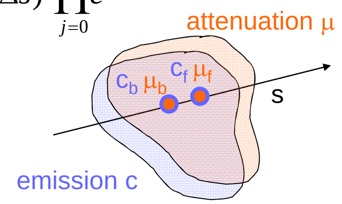


note: all values are normalized to [0,1]

- formulated as a recursive back-to-front compositing equation:

$$c_b = c_b(1-\mu_f) + c_f = c_b t_f + c_f$$

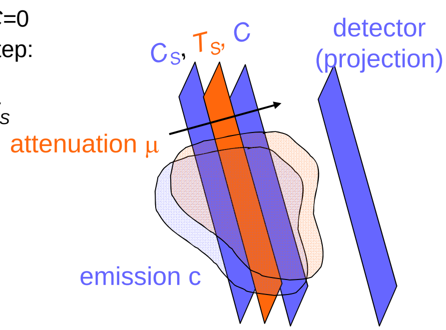
Attenuation Modeling: Practice

Notice that "recursive back-to-front compositing" operation is performed for EVERY pixel on the projection → parallelism

Assuming volume consists of slices parallel to the detector

Forward projection (back-to-front traversal):

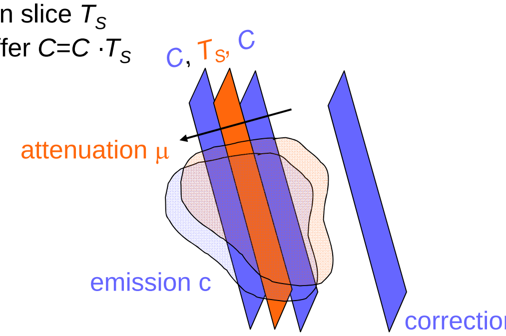
- for every pixel on the projection C=0
- step from back to front, at each step:
  - interpolate emission slice C_S
  - Interpolate attenuation slice T_S
  - composite $C = C \cdot T_S + C_S$



Attenuation Modeling: Practice

Backprojection (front-to-back traversal):

- initialize correction buffer *C*
- step from front to back, at each step:
  - spread (and add) C into emission volume affected by slice
  - interpolate attenuation slice T_S
  - update correction buffer $C = C \cdot T_S$



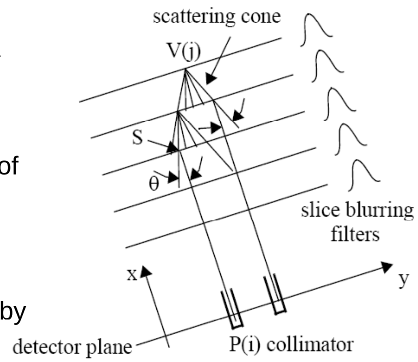
Scatter Modeling: Theory

Idea:

- scattering can be modeled by a phase function resembling a Gaussian
- the anatomical density map determines the parameters (σ) of this Gaussian

Approach:

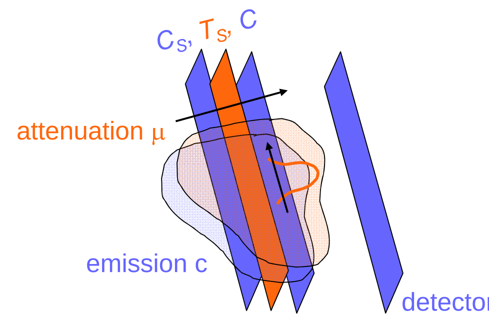
- similar to attenuation modeling except the "weight" is provided by the phase function based on attenuation values



Scatter Modeling: Practice

Forward projection (back-to-front traversal):

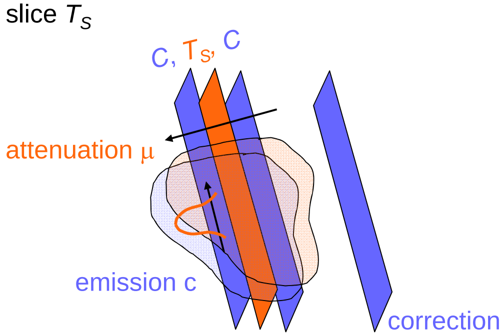
- emission buffer $C = 0$
- step from back to front, at each step:
  - interpolate emission slice C_S and attenuation slice T_S
  - blur C using T_S
  - $C = C + C_S$



Scatter Modeling: Practice

Backprojection (front-to-back traversal):

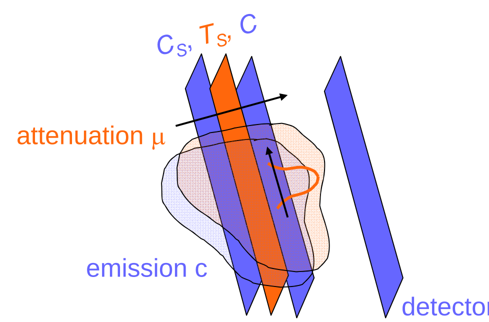
- initialize correction buffer C
- step from front to back, at each step:
  - spread (and add) C into emission volume
  - interpolate attenuation slice T_S
  - blur C using T_S



Combining Both Effects

Forward projection (back-to-front traversal):

- emission buffer C=0
- step from back to front, at each step:
  - interpolate emission slice C_S and attenuation slice T_S
  - blur C using T_S
  - $C = C \cdot T_S + C_S$



Combining Both Effects

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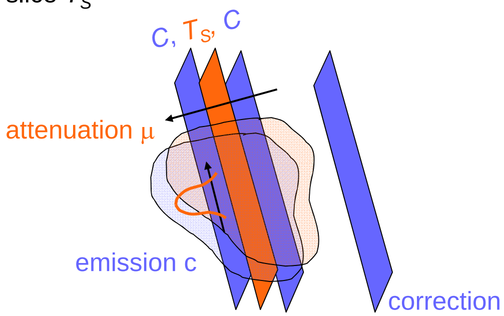
Backprojection (front-to-back traversal):

- initialize correction buffer C
- step from front to back, at each step:
  - spread (and add) C into the emission volume
  - interpolate attenuation slice T_S
  - blur C using T_S
  - update $C = C \cdot T_S$



Conclusion

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Reconstruction pipeline can be parallelized (analytical or iterative)

Different stages/effects are represented by different kernels:

- forward / backward projection
- attenuation modeling / scattering

Implementation needs more hardware (GPU) details

- data representation
- memory model and constraints
- …