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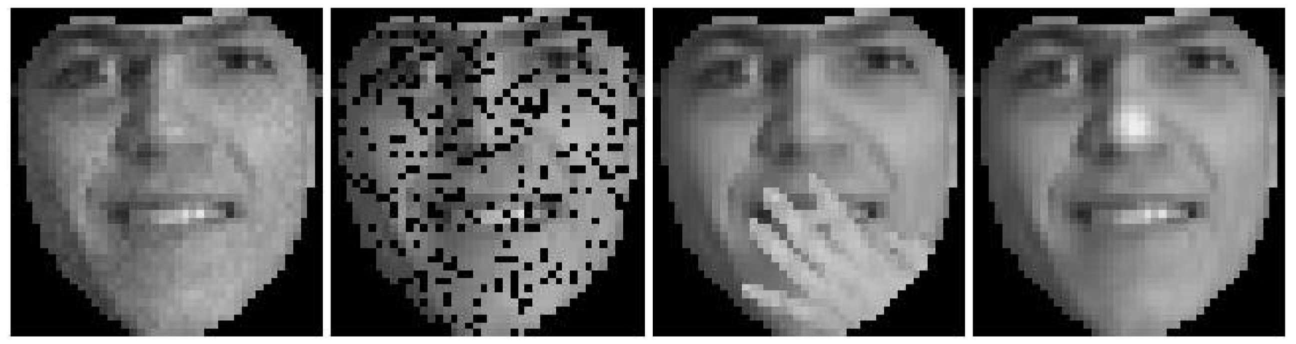
The Problem

Data corruption

- Noise *e.g.* from capturing devices
- Missing data *e.g.* due to transmission
- Outliers *e.g.* occlusion, specular reflection



noise | missing data | outliers

Goal

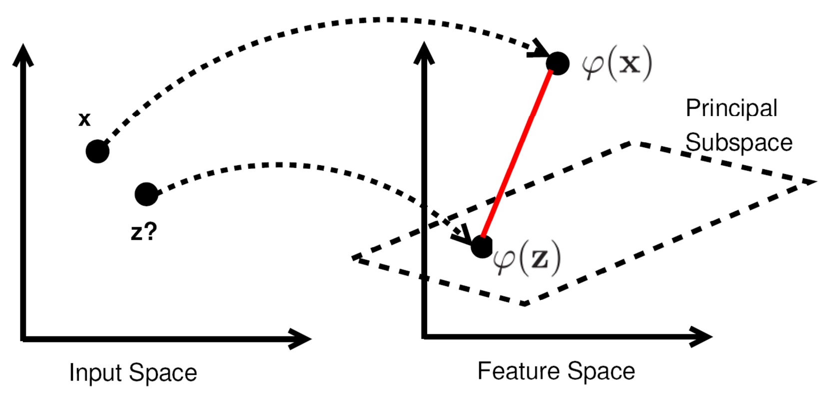
?

Data modeling using subspace

- Principal Component Analysis (PCA)
  - Feature extraction, dimensionality reduction
  - Linear structure
- Kernel PCA
  - Extension of PCA for non-linear data structure
  - Robustness?

Kernel PCA & Pre-image

- Mapping to feature space is implicit. , non-reversible.
- Feature mapping is non-reversible. How to find pre-images?



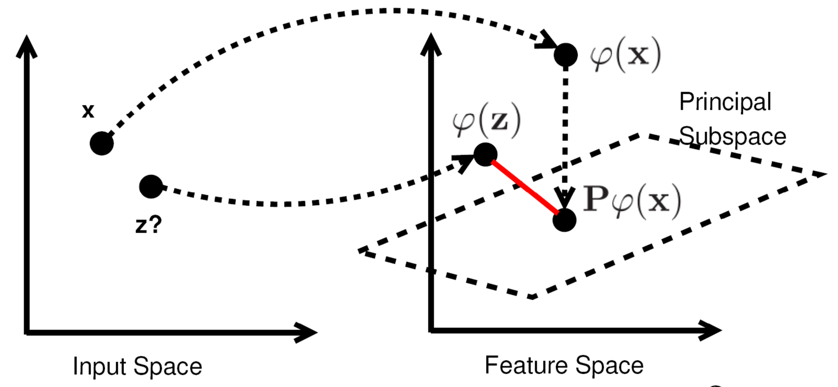
- The pre-image problem:

$$\arg\min_{\mathbf{z}} ||\varphi(\mathbf{z}) - \varphi(\mathbf{x})||^2 \text{ s.t. } \varphi(\mathbf{z}) \in \mathcal{PS}$$

- Problem: There might be no feasible solution!

KPCA reconstruction revisited

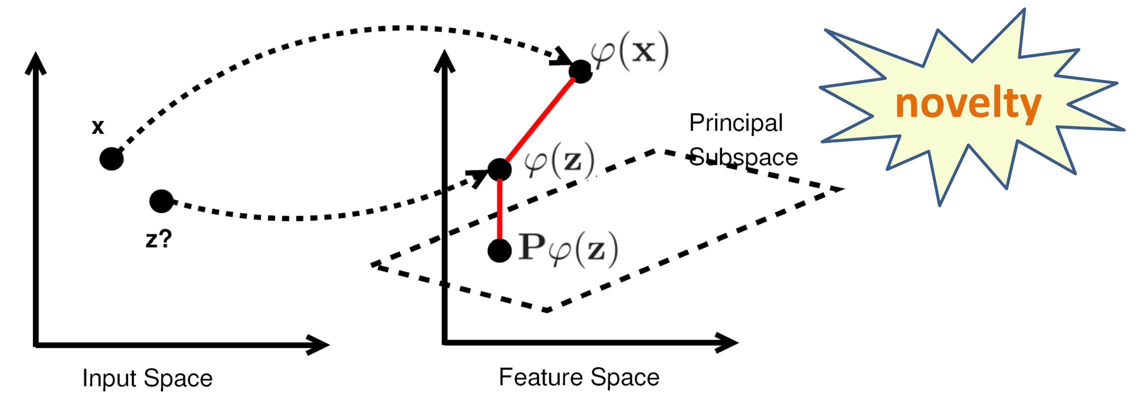
- **Traditional approach:**



$$\arg\min_{\mathbf{z}} ||\varphi(\mathbf{z}) - \mathbf{P}\varphi(\mathbf{x})||_2^2$$

- Disadvantage: Incorporation of robustness???

- **Our proposed method:**



$$\arg\min_{\mathbf{z}} ||\varphi(\mathbf{x}) - \varphi(\mathbf{z})||_2^2 + C||\varphi(\mathbf{z}) - \mathbf{P}\varphi(\mathbf{z})||_2^2$$

- What we gain: easy incorporation of robustness

$$\arg\min_{\mathbf{z}} E_0(\mathbf{x}, \mathbf{z}) + C||\varphi(\mathbf{z}) - \mathbf{P}\varphi(\mathbf{z})||_2^2$$

Robust measure

Robustness

Dealing with missing data

$$E_0(\mathbf{x}, \mathbf{z}) = -\exp\left(-\gamma_2 \sum (x_i - z_i)^2 . \delta(x_i \text{ not missing})\right)$$

Dealing with intra-sample outliers

$$E_0(\mathbf{x}, \mathbf{z}) = -\exp\left(-\gamma_2 \sum \frac{(x_i - z_i)^2}{(x_i - z_i)^2 + \sigma^2}\right)$$

Dealing with missing data and intra-sample outliers in training data

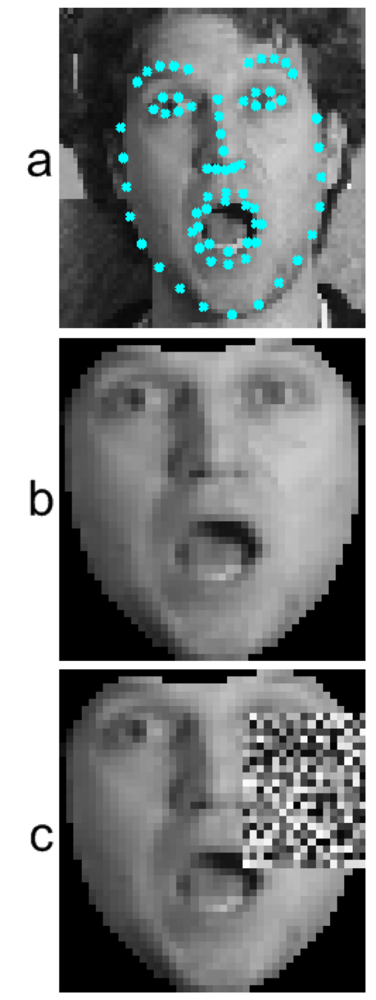
- Iterative procedure
- Alternate b/t modeling and robust fitting
- Divide training data into several partitions
- Modeling with leave-one-out partitions

Optimization

- Fix point update

Experiments

RKPCA for intra-sample outliers



	Occ.Sz	Region Type	Base Line	Mika et al	Kwok&Tsang	Robust PCA	Ours
Energy 80%	20	Whole face	14.0±5.5	13.5±3.3	14.1±3.4	10.8±2.4	**8.1±2.3**
		Occ. Reg.	71.5±5.5	22.6±7.9	17.3±6.6	**13.3±5.5**	16.1±6.1
		Non-occ Reg.	0.0±0.0	11.3±2.3	13.2±2.9	10.1±2.2	**6.0±1.7**
	30	Whole face	27.7±10.2	17.5±4.8	16.6±4.6	12.2±3.2	**10.9±4.2**
		Occ. Reg.	70.4±3.9	24.2±7.1	19.3±6.6	**15.4±5.1**	18.4±5.8
		Non-occ Reg.	0.0±0.0	13.3±3.0	14.7±3.8	9.6±2.3	**5.7±4.3**
	40	Whole face	40.2±12.7	20.9±5.9	18.8±5.8	16.4±7.1	**14.3±6.3**
		Occ. Reg.	70.6±3.6	25.7±7.2	21.1±7.1	20.1±8.0	**19.8±6.3**
		Non-occ Reg.	0.0±0.0	15.2±4.2	16.1±5.3	9.4±2.3	**8.8±8.1**
Energy 95%	20	Whole face	14.2±5.3	12.6±3.1	13.8±3.2	9.1±2.3	**7.0±2.1**
		Occ. Reg.	71.2±5.4	29.2±8.4	**17.3±6.4**	18.6±7.1	18.1±6.1
		Non-occ Reg.	0.0±0.0	8.6±1.6	12.9±2.9	6.5±1.4	**4.1±1.6**
	30	Whole face	26.8±9.5	17.4±4.4	16.2±4.1	13.4±5.0	**10.2±3.7**
		Occ. Reg.	70.9±4.4	30.0±7.6	**19.5±6.5**	23.8±7.8	21.0±6.3
		Non-occ Reg.	0.0±0.0	10.1±1.9	14.1±3.2	6.3±1.4	**3.1±1.7**
	40	Whole face	40.0±11.9	22.0±5.9	18.9±6.0	22.7±11.7	**14.3±5.8**
		Occ. Reg.	70.7±3.6	30.1±7.2	**21.4±7.4**	32.4±11.9	22.4±7.0
		Non-occ Reg.	0.0±0.0	12.1±3.3	15.9±5.2	7.0±2.5	**5.0±6.7**

- Figure: a) 68 landmarks, b) shape-normalized , c) synthetic occlusion
- Table: mean + std of the absolute differences b/t reconstructed images and the ground-truth on Multi-PIE database.

RKPCA for denoising

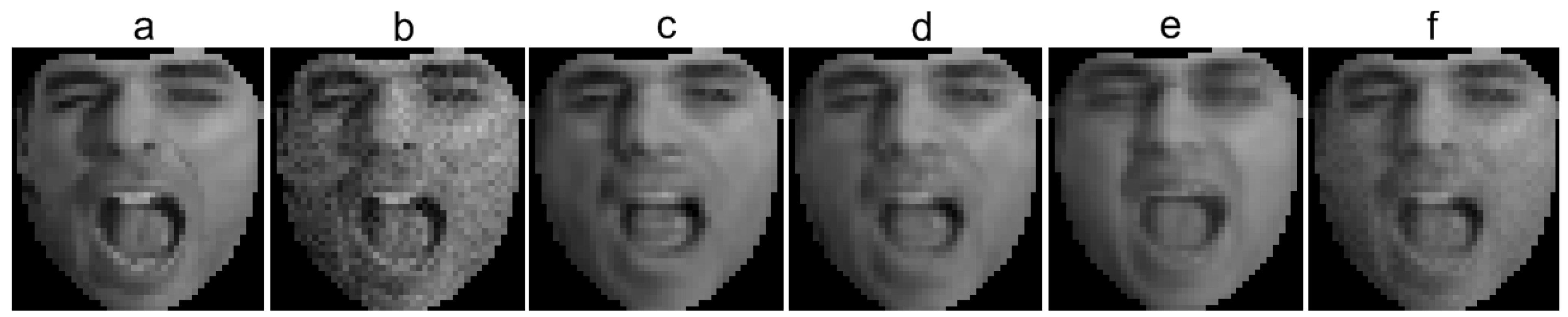


Figure 6: Example of denoised images. a) original image, b) corrupted by Gaussian noise, c) de-noised using PCA, d) using Mika *et al*, e) using Kwok & Tsang method, f) result of our method.

Energy	Base Line	Mika	Kwok& Tsang	PCA	Ours
80%	8.14±0.16	9.07±1.86	11.79±2.56	10.04±1.99	**7.01±1.27**
95%	8.14±0.16	6.37±1.30	11.55±2.52	6.70±1.20	**5.70±0.96**
100%	8.14±0.16	5.55±0.97	11.52±2.52	6.44±0.39	**5.43±0.78**

- Image denoising on Multi-PIE database. BaseLine: amount of noise
- Our methods better than the others.

RKPCA for incomplete training data

p(del)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
mean	13 ± 4	28 ± 4	43 ± 7	53 ± 8	70 ± 9	81 ± 9	97 ± 9	109 ± 8	124 ± 7	139 ± 7
1-NN	5 ± 3	14 ± 5	30 ± 10	60 ± 20	90 ± 20	NA	NA	NA	NA	NA
PPCA	3.7 ± .6	9 ± 2	17 ± 5	25 ± 9	50 ± 10	90 ± 30	110 ± 30	110 ± 20	120 ± 30	140 ± 30
PKPCA	5 ± 1	12 ± 3	19 ± 5	24 ± 6	32 ± 6	40 ± 7	45 ± 4	61 ± 8	70 ± 10	100 ± 20
Ours	**3.2 ± 1.9**	**8 ± 4**	**12 ± 4**	**19 ± 6**	**27 ± 8**	**34 ± 10**	**44 ± 9**	**53 ± 12**	**69 ± 13**	**83 ± 15**

- Reconstruction errors for 5 different methods and 10 probabilities of missing values for the Oil Flow dataset.
- Our method outperforms others for all levels of missing data.