CSE509 Computer System Security



2023-03-02 Public Key Cryptography

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Public Key Cryptography

Many algorithms with different purposes

One common property: pair of keys, one public and one secret

Session key establishment

Exchange messages to create a shared secret key

Encryption

Anyone can encrypt a message using a recipient's public key Only the recipient can decrypt a message using their private key *No shared secret!* Private key (secret) is stored only at one side

Digital signatures

Sign a message with a private key

Diffie–Hellman Key Exchange

Allows two parties to jointly *establish a shared secret key* over an insecure communication channel

The established key can then be used to encrypt subsequent communication using a symmetric key cipher

"New Directions in Cryptography" by Whitfield Diffie and Martin Hellman, 1976

Based on the discrete logarithm problem

$$3^{29} \mod 17 \xrightarrow{easy} ??$$

$$3^{??} \mod 17 \xrightarrow{hard} 12$$

Diffie-Hellman Key Exchange

Alice and Bob agree on a large (at least 1024 bit) prime number p and a base g - both public

p is usually of the form 2q+1 where q is also prime

g is a generator of the multiplicative group of integers modulo p (for every x coprime to p there is a k such that $g^k \equiv x \mod p$)

Alice picks a secret large random number *a* and sends to Bob $g^a \mod p$ Bob picks a secret large random number *b* and sends to Alice $g^b \mod p$ Alice calculates $s = (g^b \mod p)^a = g^{ba} \mod p$ Bob calculates $s = (g^a \mod p)^b = g^{ab} \mod p$



Man-in-the-Middle Attack

Alice and Bob share no secrets



Mallory actively decrypts and re-encrypts all messages No authentication: Alice and Bob assume that they communicate directly General problem: *need for a root of trust (future lecture)*

Symmetric Key Cryptography



Public Key Cryptography



Advantages

No shared secrets

Only private keys need to be kept secret, but they are never shared

Easier key management

No need to transmit any secret key beforehand

For *n* parties, *n* key pairs are needed (instead of n(n-1)/2 shared symmetric keys)

Provides both secrecy and authenticity

Disadvantages

More computationally intensive

Encryption/decryption is 2–3 orders of magnitude slower than symmetric key primitives

About one order of magnitude larger keys

Key generation is more difficult

RSA Asymmetric Encryption

Named after its inventors: Rivest, Shamir, Adleman

Based on the assumption that factoring large numbers is hard

Relatively easy to find two large prime numbers p and qNo efficient methods are known to factor their product N

Variable key length

- Largest (publicly known) factored RSA modulus is 768 795 829 bits long
- February 2020: took roughly 2700 core-years

It is believed that 1024-bit keys may already (or in the near future) be breakable by a sufficiently powerful attacker

2048-bit keys should be the absolute minimum

👿 RSA Factoring Challenge - Wiki 🗙 🕂

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| https://en.wikip | edia.org/wiki/RSA | _Factoring_Cha | llenge | | ▣ … ☆ △ II\ 🗉 👪 ≫ |
|------------------------|-------------------|----------------|--------------------------|------------------------------------|---|
| RSA-150 | 150 | 496 | | April 16, 2004 | Kazumaro Aoki et al. |
| RSA-155 | 155 | 512 | US\$9,383 ^[8] | August 22, 1999 | Herman te Riele et al. |
| RSA-160 | 160 | 530 | | April 1, 2003 | Jens Franke et al., University of Bonn |
| RSA-170 ^[b] | 170 | 563 | | December 29, 2009 | D. Bonenberger and M. Krone ^[c] |
| RSA-576 | 174 | 576 | US\$10,000 | December 3, 2003 | Jens Franke et al., University of Bonn |
| RSA-180 ^[b] | 180 | 596 | | May 8, 2010 | S. A. Danilov and I. A. Popovyan, Moscow State University ^[11] |
| RSA-190 ^[b] | 190 | 629 | | November 8, 2010 | A. Timofeev and I. A. Popovyan |
| RSA-640 | 193 | 640 | US\$20,000 | November 2, 2005 | Jens Franke et al., University of Bonn |
| RSA-200 [b] ? | 200 | 663 | | May 9, 2005 | Jens Franke et al., University of Bonn |
| RSA-210 ^[b] | 210 | 696 | | September 26, 2013 ^[12] | Ryan Propper |
| RSA-704 ^[b] | 212 | 704 | US\$30,000 | July 2, 2012 | Shi Bai, Emmanuel Thomé and Paul Zimmermann |
| RSA-220 ^[b] | 220 | 729 | | May 13, 2016 | S. Bai, P. Gaudry, A. Kruppa, E. Thomé and P. Zimmermann |
| RSA-230 [b] | 230 | 762 | | August 15, 2018 | Samuel S. Gross, Noblis, Inc. @ |
| RSA-232 ^[b] | 232 | 768 | | February 17, 2020 ^[13] | N. L. Zamarashkin, D. A. Zheltkov and S. A. Matveev. |
| RSA-768 ^[b] | 232 | 768 | US\$50,000 | December 12, 2009 | Thorsten Kleinjung et al. ^[14] |
| RSA-240 [b] | 240 | 795 | | Dec 2, 2019 ^[15] | F. Boudot, P. Gaudry, A. Guillevic, N. Heninger, E. Thomé and P. Zimmermann |
| RSA-250 ^[b] | 250 | 829 | | Feb 28, 2020 ^[16] | F. Boudot, P. Gaudry, A. Guillevic, N. Heninger, E. Thomé and P. Zimmermann |
| RSA-260 | 260 | 862 | | | |
| RSA-270 | 270 | 895 | | | |

RSA

Choose two distinct large prime numbers p and qLet n = pq (modulus)

Select *e* as a relative prime to (p - 1)(q - 1)Calculate *d* such that $de \equiv 1 \mod (p - 1)(q - 1)$ Public key = (e, n)Private key = *d*

To encrypt *m*, calculate $c \equiv m^e \mod n$

Plaintext block must be smaller than the key length

To decrypt *c*, calculate $m \equiv c^d \mod n$

Ciphertext block will be as long as the key

RSA in Practice

RSA calculations are computationally expensive

Two to three orders of magnitude slower than symmetric key primitives → RSA is used in combination with symmetric key encryption

Sending an encrypted message:

Encrypt message with a random symmetric key Encrypt the symmetric key with recipient's public key

Transmit both the encrypted message and the encrypted key

Setting up an encrypted communication channel:

Negotiate a symmetric key using RSA

Use the symmetric key for subsequent communication

PKCS: Public-Key Cryptography Standards (#1–#15)

Make different implementations interoperable Avoid various known pitfalls in commonly used schemes

Forward Secrecy

Threat: capture encrypted traffic now, use it in the future

Private keys may be compromised later on (e.g., infiltrate system)

A cryptanalytic breakthrough may be achieved

FS: Even if current keys are leaked, past encrypted traffic cannot be decrypted

Generate random secret keys without using a deterministic algorithm

Cannot read old messages

Cannot forge a message and claim that it was sent in the past

Support

IPsec, SSH, Off-the-Record messaging (OTR), TLS (Diffie-Hellman instead of RSA key exchange)

Not a panacea

Ephemeral keys may be kept in memory for hours

Server could be forced to record all session keys

TLS session resumption needs careful treatment

Elliptic Curve Cryptography

Proposed in 1985, but not used until 15 years later

Relies on the intractability of a different mathematical problem: *elliptic curve discrete logarithm*

Main benefit over RSA: shorter key length

Example: a 256-bit elliptic curve public key is believed to provide comparable security to a 3072-bit RSA public key

Endorsed by NIST

Key exchange: elliptic curve Diffie–Hellman (ECDH)

Digital signing: elliptic curve digital signature algorithm (ECDSA)



Commercial National Security Algorithm Suite and Quantum Computing FAQ



Q: What is the Commercial National Security Algorithm Suite?

A: The Commercial National Security Algorithm Suite is the suite of algorithms identified in CNSS Advisory Memorandum 02-15 for protecting NSS up to and including TOP SECRET classification. This suite of algorithms will be incorporated in a new version of the National Information Assurance Policy on the Use of Public Standards for the Secure Sharing of Information Among National Security Systems (CNSSP-15 dated October 2012). The Advisory

| Algorithm | Usage |
|--|--------------------------------------|
| RSA 3072-bit or larger | Key Establishment, Digital Signature |
| Diffie-Hellman (DH) 3072-bit or larger | Key Establishment |
| ECDH with NIST P-384 | Key Establishment |
| ECDSA with NIST P-384 | Digital Signature |
| SHA-384 | Integrity |
| AES-256 | Confidentiality |

Table I: CNSA 2.0 algorithms for software and firmware updates

September 2022: CNSA v2.0



Public-key CRYSTALS-Dilithium CRYSTALS-Kyber

Symmetric-key Advanced Encryption Standard (AES) Secure Hash Algorithm (SHA)

Software and Firmware Updates

Xtended Merkle Signature Scheme (XMSS) Leighton-Micali Signature (LMS)

| Algorithm | Function | Specification | Parameters |
|--|--|------------------------|--|
| Leighton-Micali Signature (LMS) | Asymmetric algorithm for digitally signing firmware and software | <u>NIST SP 800-208</u> | All parameters approved for all classification levels. SHA-256/192 recommended. |
| Xtended Merkle Signature Scheme (XMSS) | Asymmetric algorithm for digitally signing firmware and software | NIST SP 800-208 | All parameters approved for all classification levels. |

Table II: CNSA 2.0 symmetric-key algorithms

| Algorithm | Function | Specification | Parameters |
|---------------------------------------|---|----------------|--|
| Advanced Encryption Standard (AES) | Symmetric block cipher for information protection | FIPS PUB 197 | Use 256-bit keys for all classification levels. |
| Secure Hash Algorithm (SHA) | Algorithm for computing a condensed representation of information | FIPS PUB 180-4 | Use SHA-384 or SHA- 512 for all classification levels. |

Table III: CNSA 2.0 quantum-resistant public-key algorithms

| Algorithm | Function | Specification | Parameters |
|--------------------|---|---------------|---|
| CRYSTALS-Kyber | Asymmetric algorithm for key establishment | TBD | Use Level V parameters for all classification levels. |
| CRYSTALS-Dilithium | Asymmetric algorithm for digital signatures | TBD | Use Level V parameters for all classification levels. |

Cryptographic Hash Functions

Hash functions that are considered practically impossible to invert



Properties of an ideal cryptographic hash function

Easy to compute the hash value for any given message Infeasible to generate a message that has a given hash Infeasible to modify a message without changing the hash Infeasible to find two different messages with the same hash

Many-to-one function: collisions can happen

Cryptographic Hash Function Properties

Pre-image resistance

Given a hash value **h**, it should be computationally infeasible to find any input **m** such that **h** = **hash**(**m**)

Example: break a hashed password

Second pre-image resistance

Given an input m_1 , it should be computationally infeasible to find another input m_2 such that $m_1 \neq m_2$ and $hash(m_1) = hash(m_2)$

Example: forge an existing certificate

Collision Resistance

It should be computationally infeasible to find two different inputs m_1 and m_2 such that $hash(m_1) = hash(m_2)$ (collision)

Example: prepare two contradicting versions of a contract

Birthday Paradox

How many people does it take before the odds are 50% or better of having

... another person with the same birthday as you? 253 Second pre-image resistance

... two people with the same birthday? 23

Collision resistance



Uses of Cryptographic Hash Functions

- Data integrity
- **Digital signatures**
- Message authentication
- User authentication
- Timestamping
- Certificate revocation management

Common Hash Functions

MD5: 128-bit output

1993: Boer and Bosselaers, "pseudo-collision" in which 2 different IVs produce an identical digest

1996: Dobbertin, collision of the MD5 compression function

2004: Wang, Feng, Lai, and Yu, collisions for the full MD5

2005: Lenstra, Wang, and de Weger, construction of X.509 certs with different public keys but same hash

2008: Sotirov, Stevens, Appelbaum, Lenstra, Molnar, Osvik, de Wege, creation of rogue CA certificates

Use it? NO, it's unsafe

SHA-1: 160-bit output

2005: Rijmen and Oswald, attack on a reduced version of SHA1 (53 out of 80 rounds)

2005: Wang, Yao, and Yao, lowered the complexity for finding a collision to 2⁶³

2006: Rechberger, attack with 2³⁵ compression function evaluations

2015: Stevens, Karpman, and Thomas, freestart collision attack

2017: SHAttered attack, generated two different PDF files with the same SHA-1 hash

2020: Leurent and Peyrin, chosen-prefix collision attack with a complexity of 2^{63.4} (~45K USD per collision)

Use it? NO, use SHA-256 or better instead

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Read more

For more details on our schedule, please see Windows Enforcement of Authenticode Code Signing and Timestamping on Technet, or

Message Authentication Codes (MACs) (AKA authentication "tag")

Verify both message *integrity* and *authenticity*



Why Are MACs Needed?

Tampering

Any modification to the message will result in a different MAC value Recipient will detect the tampering because the authentication check will fail

Forgery/Impersonation

MACs are generated using a secret key: only authorized parties in possession of the key can generate a valid MAC

Replay

An attacker could capture a message from Alice to Bob and replay it later pretending to be Alice

Protocols that use MACs include a timestamp or sequence number in each message, which makes it impossible for an attacker to replay an old message

MAC = H(key || message)

|| denotes concatenation

Problem: easy to append data to the message without knowing the key and obtain another valid MAC

Length-extension attack: calculate $H(m_1 \parallel m_2)$ for an attacker-controlled m_2 given only $H(m_1)$ and the length of m_1

Keyed-hash message authentication code (HMAC)

$\mathsf{HMAC}(K, m) = \mathsf{H}((K \bigoplus opad) || \mathsf{H}(K \bigoplus ipad || m))$

opad/ipad: outer/inner padding

Impossible to generate the HMAC of a message without knowing the secret key

Double nesting prevents various forms of length-extension attacks

Order of Encryption and MACing

Encrypted data usually must be protected with a MAC

Encryption alone protects only against passive adversaries

Different options:

MAC-and-Encrypt $E(P) \parallel M(P)$

No integrity of the ciphertext

MAC-then-Encrypt $E(P \parallel M(P))$

No integrity of the ciphertext (have to decrypt it first)

Encrypt-then-MAC $E(P) \parallel M(E(P))$

Provides integrity of the ciphertext

Preferable option – *always MAC the ciphertext*

Digital Signatures

Use RSA backwards:

Sign (encrypt) with the private key *Verify* (decrypt) with the public key

Ownership of a private key turns it into a *digital signature*

Anyone can verify that a message was signed by its owner *> Non-repudiation*

Again, too expensive to sign the whole message

Calculate a cryptographic hash of the message and then sign the hash

What if a private key was stolen or deliberately leaked?

All signatures (past and future) of that signer become suspect

The signer might know which signatures were issued legitimately, but there is no way for the verifier to distinguish between them

Digital Signatures



Hashes vs. MACs vs. Digital Signatures

| | Hash | MAC | Signature |
|-----------------|--------------|--------------|--------------|
| Integrity | \checkmark | \checkmark | \checkmark |
| Authentication | | \checkmark | \checkmark |
| Non-repudiation | | | \checkmark |
| Keys | None | Symmetric | Asymmetric |

Public Key Authenticity

Authentication without confidence in the keys used is pointless

Need to obtain evidence that a given public key is authentic

It is correct and belongs to the person or entity claimed Has not been tampered with or replaced by an attacker

Different ways to establish trust (future lecture)

- **TOFU:** trust on first use (e.g., SSH)
- Web of trust: decentralized trust model (e.g., PGP)
- **PKI:** public key infrastructure (e.g., TLS)

Adi Shamir: Crypto is typically bypassed, not penetrated

