# Public Key Cryptography 

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## Public Key Cryptography

## Many algorithms with different purposes

One common property: pair of keys, one public and one secret

## Session key establishment

Exchange messages to create a shared secret key

## Encryption

Anyone can encrypt a message using a recipient's public key Only the recipient can decrypt a message using their private key

No shared secret! Private key (secret) is stored only at one side

## Digital signatures

Sign a message with a private key

## Diffie-Hellman Key Exchange

Allows two parties to jointly establish a shared secret key over an insecure communication channel

The established key can then be used to encrypt subsequent communication using a symmetric key cipher
"New Directions in Cryptography" by Whitfield Diffie and Martin Hellman, 1976 Based on the discrete logarithm problem


## Diffie-Hellman Key Exchange

Alice and Bob agree on a large (at least 1024 bit) prime number $\boldsymbol{p}$ and a base $g$ - both public
$\boldsymbol{p}$ is usually of the form $\mathbf{2 q + 1}$ where $\boldsymbol{q}$ is also prime
$g$ is a generator of the multiplicative group of integers modulo $p$
(for every $\boldsymbol{x}$ coprime to $\boldsymbol{p}$ there is a $\boldsymbol{k}$ such that $\boldsymbol{g}^{\boldsymbol{k}} \equiv \boldsymbol{x} \boldsymbol{\operatorname { m o d }} \boldsymbol{p}$ )
Alice picks a secret large random number $a$ and sends to $\operatorname{Bob} \boldsymbol{g}^{a} \bmod \boldsymbol{p}$
Bob picks a secret large random number $b$ and sends to Alice $\boldsymbol{g}^{b} \bmod \boldsymbol{p}$
Alice calculates $\boldsymbol{s}=\left(\boldsymbol{g}^{b} \bmod \boldsymbol{p}\right)^{a}=\boldsymbol{g}^{b a} \bmod \boldsymbol{p}$
Bob calculates $\boldsymbol{s}=\left(\boldsymbol{g}^{a} \bmod \boldsymbol{p}\right)^{b}=\boldsymbol{g}^{a b} \bmod \boldsymbol{p}$


## Man-in-the-Middle Attack

Alice and Bob share no secrets


Mallory actively decrypts and re-encrypts all traffic
No authentication: Alice and Bob assume that they communicate directly
General problem: need for a root of trust

## Symmetric Key Cryptography



## Public Key Cryptography



## Advantages

No shared secrets
Only private keys need to be kept secret, but they are never shared

## Easier key management

No need to transmit any secret key beforehand
For $\boldsymbol{n}$ parties, $\boldsymbol{n}$ key pairs are needed (instead of $n(n-1) / 2$ shared keys)
Provides both secrecy and authenticity

## Disadvantages

More computationally intensive
Encryption/decryption is 2-3 orders of magnitude slower than symmetric key primitives
About one order of magnitude larger keys
Key generation is more difficult

## RSA Asymmetric Encryption

Named after its inventors: Rivest, Shamir, Adleman
Based on the assumption that factoring large numbers is hard
Relatively easy to find two large prime numbers $p$ and $q$
No efficient methods are known to factor their product $N$
Variable key length
Largest (publicly known) factored RSA modulus is 768795829 bits long
February 2020: took roughly 2700 core-years
It is believed that 1024-bit keys may already (or in the near future) be breakable by a sufficiently powerful attacker

2048-bit keys should be the absolute minimum

W／RSA Factoring Challenge－Wiki $\times$
https：／／en．wikipedia．org／wiki／RSA＿Factoring＿Challenge
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| RSA－150 | 150 | 496 |  | April 16， 2004 | Kazumaro Aoki et al． |
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| RSA－155 | 155 | 512 | US\＄9，383 ${ }^{[8]}$ | August 22， 1999 | Herman te Riele et al． |
| RSA－160 | 160 | 530 |  | April 1， 2003 | Jens Franke et al．，University of Bonn |
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| RSA－704 ${ }^{[b]}$ | 212 | 704 | US\＄30，000 | July 2， 2012 | Shi Bai，Emmanuel Thomé and Paul Zimmermann |
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| RSA－260 | 260 | 862 |  |  |  |
| RSA－270 | 270 | 895 |  |  |  |

## RSA

Choose two distinct large prime numbers $\boldsymbol{p}$ and $\boldsymbol{q}$
Let $\boldsymbol{n}=\boldsymbol{p q}$ (modulus)
Select $\boldsymbol{e}$ as a relative prime to $(\boldsymbol{p - 1})(\boldsymbol{q}-\mathbf{1})$
Calculate $\boldsymbol{d}$ such that $\boldsymbol{d e} \equiv \mathbf{1} \bmod (p-1)(q-1)$
Public key $=(\boldsymbol{e}, \boldsymbol{n})$
Private key $=d$
To encrypt $\boldsymbol{m}$, calculate $\boldsymbol{c} \equiv \boldsymbol{m}^{e} \boldsymbol{\operatorname { m o d }} \boldsymbol{n}$
Plaintext block must be smaller than the key length
To decrypt $\boldsymbol{c}$, calculate $\boldsymbol{m} \equiv \boldsymbol{c}^{d} \bmod \boldsymbol{n}$
Ciphertext block will be as long as the key

## RSA in Practice

## RSA calculations are computationally expensive

Two to three orders of magnitude slower than symmetric key primitives $\rightarrow$ RSA is used in combination with symmetric key encryption

## Sending an encrypted message:

Encrypt message with a random symmetric key
Encrypt the symmetric key with recipient's public key
Transmit both the encrypted message and the encrypted key

## Setting up an encrypted communication channel:

Negotiate a symmetric key using RSA
Use the symmetric key for subsequent communication
PKCS: Public-Key Cryptography Standards (\#1-\#15)
Make different implementations interoperable
Avoid various known pitfalls in commonly used schemes

## Forward Secrecy

Threat: capture encrypted traffic now, use it in the future
Private keys may be compromised later on (e.g., infiltrate system)
A cryptanalytic breakthrough may be achieved
FS: Even if current keys are leaked, past encrypted traffic cannot be decrypted
Generate random secret keys without using a deterministic algorithm
Cannot read old messages
Cannot forge a message and claim that it was sent in the past

## Support

IPsec, SSH, Off-the-Record messaging (OTR), TLS (Diffie-Hellman instead of RSA key exchange)
Not a panacea
Ephemeral keys may be kept in memory for hours
Server could be forced to record all session keys
TLS session resumption needs careful treatment

## Elliptic Curve Cryptography

Proposed in 1985, but not used until 15 years later
Relies on the intractability of a different mathematical problem: "elliptic curve discrete logarithm"

Main benefit over RSA: shorter key length
Example: a 256-bit elliptic curve public key is believed to provide comparable security to a 3072-bit RSA public key

## Endorsed by NIST

Key exchange: elliptic curve Diffie-Hellman (ECDH)
Digital signing: elliptic curve digital signature algorithm (ECDSA)

Commercial National Security Algorithm
Suite and Quantum Computing FAQ

Q: What is the Commercial National Security Algorithm Suite? A: The Commercial National Security Algorithm Suite is the suite of algorithms identified in CNSS Advisory Memorandum 02-15 for protecting NSS up to and including TOP SECRET classification. This suite of algorithms will be incorporated in a new version of the National Information Assurance Policy on the Use of Public Standards for the Secure Sharing of Information Among National Security Systems (CNSSP-15 dated October 2012). The Advisory

| Algorithm | Usage |
| :--- | :--- |
| RSA 3072-bit or larger | Key Establishment, Digital Signature |
| Diffie-Hellman (DH) 3072-bit or larger | Key Establishment |
| ECDH with NIST P-384 | Key Establishment |
| ECDSA with NIST P-384 | Digital Signature |
| SHA-384 | Integrity |
| AES-256 | Confidentiality |

## Cryptographic Hash Functions

Hash functions that are considered practically impossible to invert


Properties of an ideal cryptographic hash function
Easy to compute the hash value for any given message
Infeasible to generate a message that has a given hash
Infeasible to modify a message without changing the hash
Infeasible to find two different messages with the same hash
Many-to-one function: collisions can happen

## Cryptographic Hash Function Properties

## Pre-image resistance

Given a hash value $\boldsymbol{h}$, it should be computationally infeasible to find any input $\boldsymbol{m}$ such that $\boldsymbol{h}=\boldsymbol{\operatorname { h a s h }}(\boldsymbol{m})$
Example: break a hashed password

## Second pre-image resistance

Given an input $\boldsymbol{m}_{\boldsymbol{1}}$, it should be computationally infeasible to find another input $\boldsymbol{m}_{\mathbf{2}}$ such that $\boldsymbol{m}_{\boldsymbol{1}} \neq \boldsymbol{m}_{\mathbf{2}}$ and $\operatorname{hash}\left(\boldsymbol{m}_{\boldsymbol{1}}\right)=\operatorname{hash}\left(\boldsymbol{m}_{\mathbf{2}}\right)$
Example: forge an existing certificate

## Collision Resistance

It should be computationally infeasible to find two different inputs $\boldsymbol{m}_{\boldsymbol{1}}$ and $\boldsymbol{m}_{\boldsymbol{2}}$ such that $\operatorname{hash}\left(\boldsymbol{m}_{\boldsymbol{1}}\right)=\operatorname{hash}\left(\boldsymbol{m}_{\boldsymbol{2}}\right)$ (collision)
Example: prepare two contradicting versions of a contract

## Birthday Paradox

How many people does it take before the odds are 50\% or better of having
... another person with the same birthday as you? 253
Second pre-image resistance
... two people with the same birthday? 23

## Collision resistance



## Uses of Cryptographic Hash Functions

## Data integrity

Digital signatures
Message authentication
User authentication
Timestamping
Certificate revocation management

## Common Hash Functions

## MD5: 128-bit output

1993: Boer and Bosselaers, "pseudo-collision" in which 2 different IVs produce an identical digest
1996: Dobbertin, collision of the MD5 compression function
2004: Wang, Feng, Lai, and Yu, collisions for the full MD5
2005: Lenstra, Wang, and de Weger, construction of X. 509 certs with different public keys but same hash
2008: Sotirov, Stevens, Appelbaum, Lenstra, Molnar, Osvik, de Wege, creation of rogue CA certificates
Use it? NO, it's unsafe

## SHA-1: 160-bit output

2005: Rijmen and Oswald, attack on a reduced version of SHA1 (53 out of 80 rounds)
2005: Wang, Yao, and Yao, lowered the complexity for finding a collision to $2^{63}$
2006: Rechberger, attack with $2^{35}$ compression function evaluations
) Example Domain
コ ヘix https://www.example.com

2015: Stevens, Karpman, and Thomas, freestart collision attack
2017: SHAttered attack, generated two different PDF files with the same SHA-1 hash
2020: Leurent and Peyrin, chosen-prefix collision attack with a complexity of 263.4 ( $\sim 45 \mathrm{~K}$ USD per collision)
Use it? NO, use SHA-256 or better instead

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TechNet Blogs = Security Research \& Defense * Flame malware collision attack explained

## Flame malware collision attack explained

swiat $\quad 6$ Jun 2012 9:57 AM 0

Since our last MSRC blog post, we've received questions on the nature of the cryptographic attack we saw in the complex, targeted malware known as Flame. This blog summarizes what our research revealed and why we made the decision to release Security Advisory 2718704 on Sunday night PDT. In short, by default the attacker's certificate would not work on Windows Vista or more recent versions of Windows. They had to perform a collision attack to forge a certificate that would be valid for code signing on Windows Vista or more recent versions of Windows. On systems that pre-date Windows Vista, an attack is possible without an MD5 hash collision. This certificate and all certificates from the involved certificate authorities were invalidated in Security Advisory 2718704 . We continue to encourage all customers who are not installing updates automatically to do so immediately.

## Mysterious Missing Extensions

When we first examined the Flame malware, we saw a file that had a valid digital signature that chained up to a Microsoft Root authority. As we reviewed this certificate, we noticed several irregularities. First, it had no X. 509 extension fields, which was not consistent with the certificates we issued from the Terminal Server licensing infrastructure. We expected to find a Certificate Revocation List (CRL) Distribution Point (CDP) extension, an Authority Information Access (AIA) extension, and a "Microsoft Hydra" critical extension. All of these were absent.

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## NOVEMBER 4, 2015 10:00 AM

## SHA-1 Deprecation Update

By Kyle Pflug / Program Manager, Microsoft Edge

In a previous update on TechNet, we announced that Windows will block SHA-1 signed TLS certificates starting on January 1, 2017. In light of recent advances in attacks on the SHA-1 algorithm, we are now considering an accelerated timeline to deprecate $\mathrm{SHA}-1$ signed TLS certificates as early as June 2016.

Mozilla recently announced a similar intent on the Mozilla Security Blog. We will continue to coordinate with other browser vendors to evaluate the impact of this timeline based on telemetry and current projections for feasibility of SHA-1 collisions.

For more details on our schedule, please see Windows Enforcement of Authenticode Code Sianing and Timestamping on Technet, or

## RELATED POSTS

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## Message Authentication Codes (MACs)

Verify both message integrity and authenticity


## MAC = H(key || message)

|| denotes concatenation
Problem: easy to append data to the message without knowing the key and obtain another valid MAC
Length-extension attack: calculate $\mathrm{H}\left(m_{1} \| m_{2}\right)$ for an attacker-controlled $m_{2}$ given only $\mathrm{H}\left(m_{1}\right)$ and the length of $m_{1}$
Keyed-hash message authentication code (HMAC)
$\operatorname{HMAC}(K, m)=\mathrm{H}((K \oplus$ opad $) \| \mathrm{H}(K \oplus$ ipad $\| m))$

## opad/ipad: outer/inner padding

Impossible to generate the HMAC of a message without knowing the secret key
Double nesting prevents various forms of length-extension attacks

## Order of Encryption and MACing

Encrypted data usually must be protected with a MAC
Encryption alone protects only against passive adversaries

## Different options:

## MAC-and-Encrypt $\quad E(P) \| M(P)$

No integrity of the ciphertext

## MAC-then-Encrypt $\quad E(P \| M(P))$

No integrity of the ciphertext (have to decrypt it first)
Encrypt-then-MAC $\quad E(P) \| M(E(P))$
Provides integrity of the ciphertext
Preferable option - always MAC the ciphertext

## Digital Signatures

## Use RSA backwards:

Sign (encrypt) with the private key
Verify (decrypt) with the public key
Ownership of a private key turns it into a digital signature
Anyone can verify that a message was signed by its owner $\rightarrow$ Non-repudiation
Again, too expensive to sign the whole message
Calculate a cryptographic hash of the message and then sign the hash
What if a private key was stolen or deliberately leaked?
All signatures (past and future) of that signer become suspect
The signer might know which signatures were issued legitimately, but there is no way for the verifier to distinguish between them

## Digital Signatures



## Hashes vs. MACs vs. Digital Signatures

|  | Hash | MAC | Signature |
| ---: | :---: | :---: | :---: |
| Integrity | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Authentication |  | $\checkmark$ | $\checkmark$ |
| Non-repudiation |  | $\checkmark$ |  |
| Keys | None | Symmetric | Asymmetric |

## Public Key Authenticity

Authentication without confidence in the keys used is pointless
Need to obtain evidence that a given public key is authentic
It is correct and belongs to the person or entity claimed
Has not been tampered with or replaced by an attacker
Different ways to establish trust (future lecture)
TOFU: trust on first use (e.g., SSH)
Web of trust: decentralized trust model (e.g., PGP)
PKI: public key infrastructure (e.g.,TLS)

Adi Shamir: Crypto is typically bypassed, not penetrated


