Bit Error Analysis

- Assume $s_1 = \sqrt{\mathcal{E}_b}$ and $s_2 = -\sqrt{\mathcal{E}_b}$ where $\mathcal{E}_b$ is energy per bit. These two states indicate information symbol 1 and 0, respectively.
- Note in this case each symbol is one bit.

Impact of Noise

- Noise (*n*) adds to the received signal. Assume noise is Gaussian with zero mean and variance = $\sigma_n^2 = N_0/2$
- More variance means more noise power.
- Thus, assuming s1 is transmitted, the received signal (after demodulation) is:

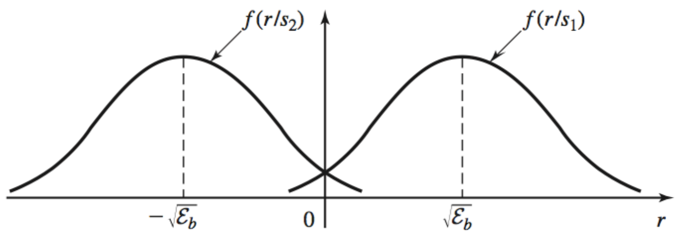
$$r = s_1 + n = \sqrt{\mathcal{E}_b} + n$$

Impact of Noise

- Assume, decision threshold is 0. Two conditional PDFs of r:

$$f(r \mid s_1) = \frac{1}{\sqrt{\pi N_0}} e^{-(r-\sqrt{\mathcal{E}_b})^2/N_0}$$

$$f(r \mid s_2) = \frac{1}{\sqrt{\pi N_0}} e^{-(r+\sqrt{\mathcal{E}_b})^2/N_0}$$



Analysis

- Assume, s1 was transmitted. Then prob of bit error is equal to prob that r < 0.

$$P(e \mid s_1) = \int_{\infty}^{0} p(r \mid s_1)\, dr$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-(r-\sqrt{\mathcal{E}_b})^2/N_0}\, dr$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\sqrt{2\mathcal{E}_b/N_0}} e^{-x^2/2}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2\mathcal{E}_b/N_0}}^{\infty} e^{-x^2/2}\, dx$$

$$= Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

Note: Q function is the tail prob of standard Normal dist. Related to error function and complementary error function
https://en.wikipedia.org/wiki/Q-function

Bit Error Rate (BER)

- Prob of bit error:

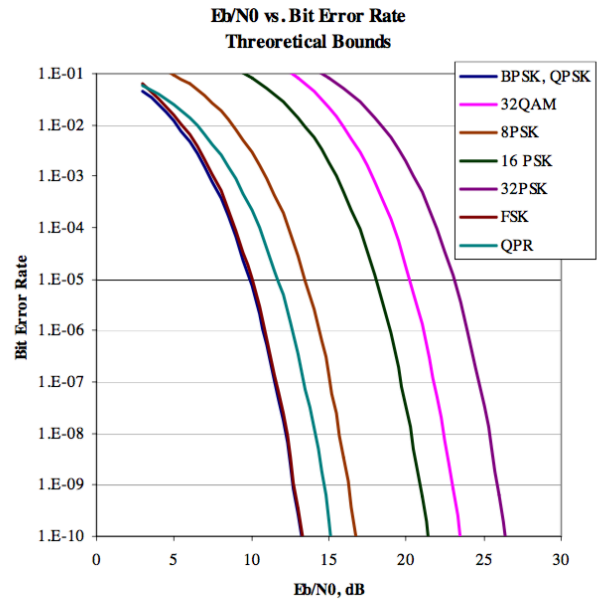
$$P_b = \frac{1}{2}P(e\,|\,s_1) + \frac{1}{2}P(e\,|\,s_2)$$
$$= Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) = \frac{1}{2}erfc(\sqrt{\frac{\varepsilon_b}{N_0}})$$

- What is $\mathcal{E}_b/N_0$?
  - Recall, ε_b is energy per bit
  - Recall $\frac{N_0}{2}$ is noise variance. This is also same as noise power per unit bandwidth (watts/Hz). Also, called *noise power spectral density.*

https://en.wikipedia.org/wiki/Eb/N0

What Happens for M-ary Modulations?

- Slightly more complex analysis as symbols now comprise of multiple ($\log_2(M)$) bits.
- Depends also on how bits are laid out on symbols. Use of Gray coding is popular so that nearest symbols differ just by 1 bit.
- Generally speaking, with the same power levels, more symbols make inter symbol distance smaller. This leads to larger bit errors.



Example: 802.11b (the older standard) BER vs. SNR charts

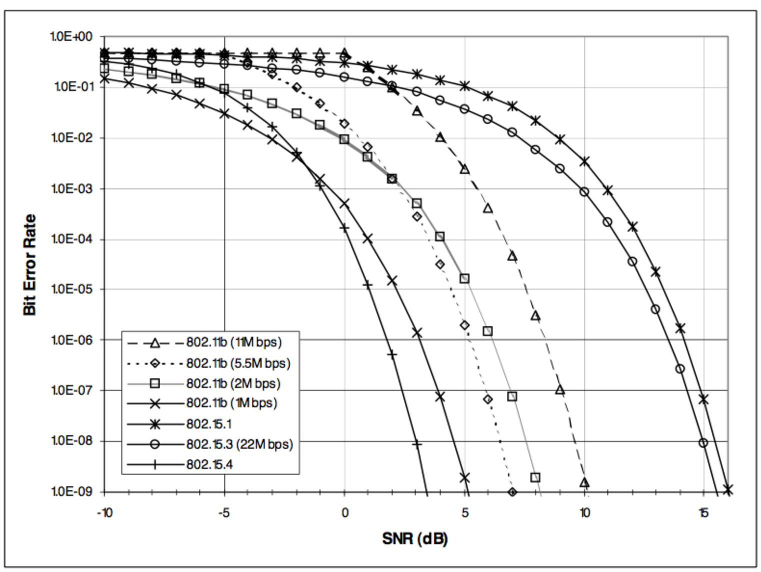
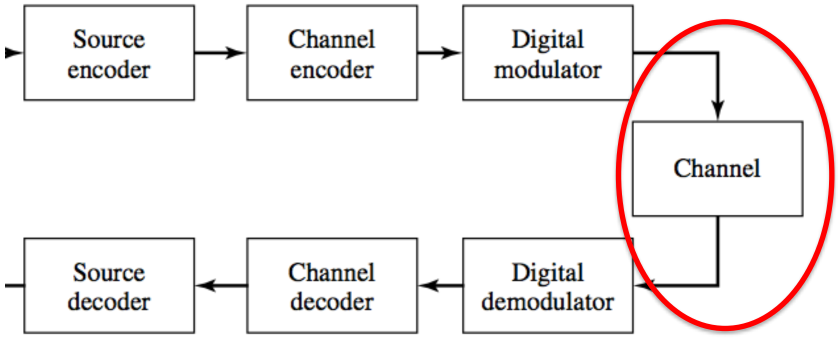


Figure E.2—BER Results for IEEE 802.11, IEEE 802.15.1, IEEE P802.15.3 and IEEE 802.15.4

Takeaways

- Fundamental tradeoffs
  - Bit error rate is related to SNR. Higher SNR means lower BER.
  - Bit error rate is also related to #bits/symbol. More #bits/symbol increases bit rate, but also increases BER.

Modeling Wireless Channel



Path Gain and Path Loss

- Path gain = ratio of received and transmit powers
- Path loss = 1 / path gain
- Note, received power is always less than transmit power. So gain is always < 1.

Modeling Path Loss

- Large scale path loss
  - Models average channel condition
- Small scale path loss
  - Models short term variations due to a phenomenon called “fading.”
- We will mostly limit ourselves to large scale losses.

Free Space Path Loss Model

- Simplest model – assumes free space. Modeled using *Friis equation*.
  https://en.wikipedia.org/wiki/Friis_transmission_equation
- Power decays as inverse of square of distance *d* from transmitter

$$P_r \propto \frac{P_t}{d^2} \quad \text{or} \quad P_r = K\frac{P_t}{d^2}$$

- The constant *K* is related to wavelength λ and transmit and receive antenna gains G_t and G_r

$$K = G_t G_r \left(\frac{\lambda}{4\pi}\right)^2$$

Free Space Path Loss (contd.)

- This formula is valid only when $d >> \lambda$
- Note that path loss depends on frequency.
- Path gain in dB :

$$10\log\frac{P_r}{P_t} = G_t(\text{in db}) + G_r(\text{in dB}) + 20\log\left(\frac{\lambda}{4\pi d}\right)$$

- Path loss in dB is simply –ve of path gain in dB.
- Example: 3dB antennas, WiFi frequency (2.4GHz), what is the path loss in dB at 100m?

Alternative Representations

- Use a reference distance d_0. Then for any $d \geq d_0$,

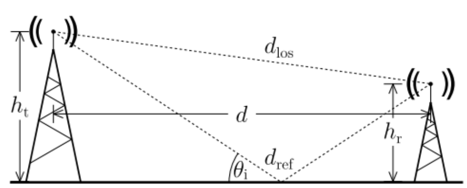
$$P_r(d) = P_r(d_0)\frac{{d_0}^2}{d^2}$$

- Path loss in dB:

$$PL(d) = PL(d_0) + 20\log\frac{d}{d_0}$$

Two Ray Ground Propagation Model

- Antennas at a height over the ground. Two rays one direct and one reflected adding up at the receiver.



$P_r = K\frac{P_t}{d^4}$, where $K = G_t G_r {h_t}^2 {h_r}^2$

- Valid only at a long distances.

[http://en.wikipedia.org/wiki/Two-ray_ground-reflection_model]

Generalizing the Exponent

- Extensive measurement experience has shown that the path loss exponent is very sensitive to surrounding environment.
- Often the exponent (α) is instantiated after actual measurement. Usually 2—6.
- This gives the Log-distance path loss model:

$$P_r = K\frac{P_t}{d^\alpha} \quad \text{or} \quad P_r(d) = P_r(d_0)\frac{{d_0}^\alpha}{d^\alpha}$$

or path loss in dB $PL(d) = PL(d_0) + 10\alpha\log\frac{d}{d_0}$

Log Normal Shadowing Model

- In reality, the power attenuation with distance cannot be constant in every direction as radio obstructions (shadowing) in every direction cannot be the same.
- This is captured by randomness:

$$PL(d) = \overline{PL}(d) + X_\sigma$$

where $\overline{PL}(d)$ is the average path loss at distance *d* and X_σ is a Gaussian (Normal) random variable with zero mean and std. deviation σ.

Models Vs Reality

- Model is an approximation of reality. The models presented so far only makes a crude approximation.
- More sophisticated models are available – many of them calibrated by actual measurements.

SNR and SINR

- *SNR* = Signal to Noise Ratio. This is signal power divided by noise power. The signal power is the received signal power.
- *SINR* = Signal to Interference plus Noise Ratio. Same as above expect noise is replaced by noise + interference.

$$SNR = \frac{S}{N} \qquad SINR = \frac{S}{I+N}$$

S and I are same as P_r

Bit Error Rate (BER)

- Fraction of bits received incorrectly. Also, called bit error probability P_b.
- BER depends on
  - SNR (or SINR)
  - Modulation technique
  - Bit rate
- BER can be analytically computed assuming noise (or interference) to have AWGN property.

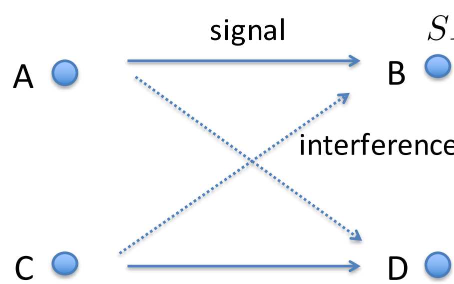
SINR vs. BER Fundamentals

- Higher SINR -> lower BER. This means *I* and *N* remaining equal we want to increase *S* for lower BER. *S* and *N* remaining equal we want to reduce I.
- Higher bit rate -> higher BER.
- Packer error rate (PER) depends also on packet size. Larger packet size will have higher PER. Why?

What does it mean by a "link"?

- It is wireless. Where is the "link"?
- Wireless link is a logical concept in most part.
- Assume, for a given bit rate/modulation BER < Threshold to keep PER at acceptable level.
- This will define a Threshold on SINR (say, β).
- We say that there is a link between Tx and Rx if SINR $\geq \beta$.
- This is sometimes called "SINR Threshold Model".

Example



$$SINR = \frac{\frac{KP_A}{d_{AB}^{\alpha}}}{N + \frac{KP_C}{d_{CB}^{\alpha}}}$$

$$SINR = \frac{\frac{KP_C}{d_{CD}^{\alpha}}}{N + \frac{KP_A}{d_{AD}^{\alpha}}}$$

P_A etc are transmit powers. d_{AB} are distances. Log-distance path loss model is used. There is a link between AB or CD if the corresponding SINR $\geq \beta$.

Understand Common Terms

- (Transmission) Range
- Bandwidth
- Capacity
- Bit rate
- Bit error rate
- Throughput

Packet Error Rate

- Packet error rate (PER) is different from bit error rate (BER).
  - Depends on coding
  - Depends on packet size

What is a “Wireless Link”?

- Wireless link is an abstraction. There is no physical link.
- Power goes everywhere – just reduces in magnitude at larger distances. Thus, one can expect E_b/N_0 to reduce at greater distance and thus BER to increase.
- Typically, we use a “threshold model” for convenience.
  - SNR threshold beyond which PER or BER is too high.