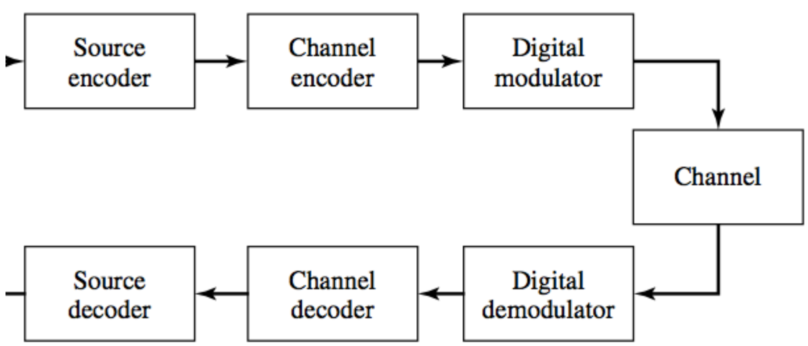
# Wireless Physical Layer

## Basic Elements of a Digital Communication System



## Basic Elements Contd.

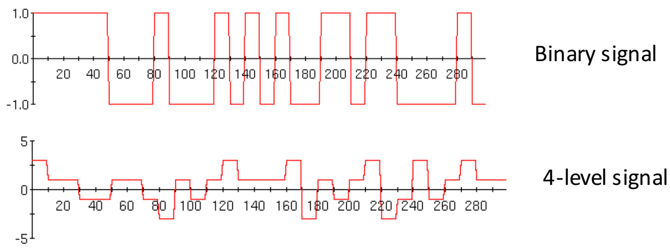
- Source Encoding
  - Encode information into bits.
- Channel Encoding
  - Add enough redundancy so that some basic reliability is achieved.
- Modulation
  - Map the digital info on signal waveform
- Channel
  - Physical medium that carries the signal. Characterized by noise.

## Noise in Wireless Channel

- Noise causes signal degradation via a variety of mechanisms. Eventually causes bit errors.
- Various sources of noise
  - Quantization noise at the receiver
  - Thermal noise at the receiver
  - Radio signals already present in nature
- Noise is additive meaning adds to the received signal.
- Typically modeled as a random phenomenon.
  - Additive White Gaussian Noise (AWGN) is a very common model.

# Basics of Signal

- `Signal'
  - Physical quantity that varies over time.
  - Conveys information.
- Information signal
  - Signal representing information to be transmitted.



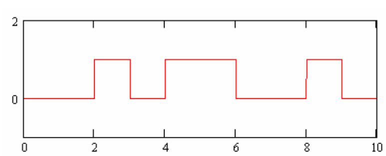

# Carrier

- Information signal often can't be transmitted directly over RF.
  - Frequency is often low. Hard to transmit low frequency with enough power. May need impractically large antenna.
  - All frequencies do not propagate well in a given environment.
  - Interference avoidance will be hard as similar information signals have similar frequencies.
- Information signal (also called 'baseband') is thus carried over a 'carrier' signal.
  - Carrier is just a sinusoid of a much higher frequency.
  - Frequency carefully chosen so that it is allowed by regulation, propagation is good, antennas can be reasonable, etc.

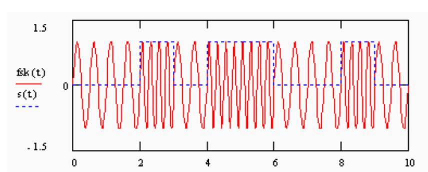
# Modulation

- Modulation is the process of mapping baseband signal on the carrier.
- Basic approach – vary a parameter of the carrier sinusoid to represent baseband signal.
- Three basic choices
  - Amplitude-Shift Keying (ASK)
  - Frequency-Shift Keying (FSK)
  - Phase-Shift Keying (PSK)

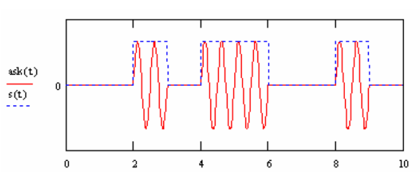
# Examples



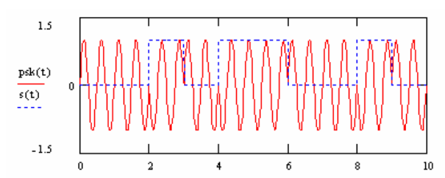

Baseband signal: 0010110010




Binary FSK




Binary ASK




Binary PSK

# Frequency and Bandwidth

- Modulation is like multiplication

$$w_i(t) \quad = \quad s_i(t) \;\; \cos(2\pi f_c t)$$

Modulated signal | Baseband signal | Carrier signal

- Question: What is the frequency of the modulated signal? What is the bandwidth?
- Note that carrier frequency is $f_c$ and bandwidth is zero.

# Concept of Time and Frequency Domains

# Fourier Series

- Fourier series – Any periodic function can be written as an infinite sum of sinusoids.
- If periodic function f(t) has a period 2L, then

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{L}\right)$$

- Fourier series interactive demo:

http://www.intmath.com/fourier-series/fourier-graph-applet.php

# Time and Frequency Domain

- If periodic function f(t) has a period *2L*, then

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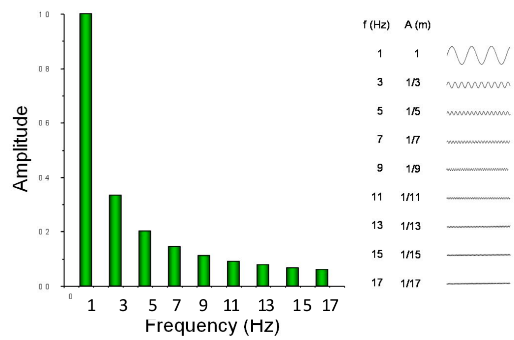
- Then, we can represent *f(t)* in the frequency domain:
  - At each harmonic, there is some “amount” of signal. The amount of represented as amplitude.

# Square Wave Example

Frequency domain representation of a square wave of frequency 1 Hz. Amplitudes are shown normalized to the first harmonic.




$$x_{\text{square}}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi(2k-1)ft\right)}{(2k-1)}$$
$$= \frac{4}{\pi}\left(\sin(2\pi ft) + \frac{1}{3}\sin(6\pi ft) + \frac{1}{5}\sin(10\pi ft) + \cdots\right)$$

# Takeaway

- Unless the signal is a pure sinusoid, there are many frequencies in the signal.
- So far, we considered only periodic signals. But most signals are not periodic.
- An extension of this technique (Fourier Transform) can also be used for non-periodic signals.
  – The math assumes that period tends to infinity.

# Fourier Transform

- Need to know a little about complex numbers.
- Use Euler formula: $e^{j\theta} = \cos\theta + j\sin\theta$
- Get a different but equivalent representation of what we had before for periodic functions:

$$f(t) = \sum_{n=-\infty}^{n=\infty} A_n e^{j(2\pi n t/L)}$$

- $A_n$ has a simple relationship with coefficients $a_n$ and $b_n$ that we have seen earlier.
- See http://mathworld.wolfram.com/FourierSeries.html

# Fourier Transform (contd)

- $A_n$ is easily computed given the periodic signal

$$A_n = \frac{1}{L}\int_{-L/2}^{L/2} f(t)e^{-j(2\pi n t/L)}dt$$

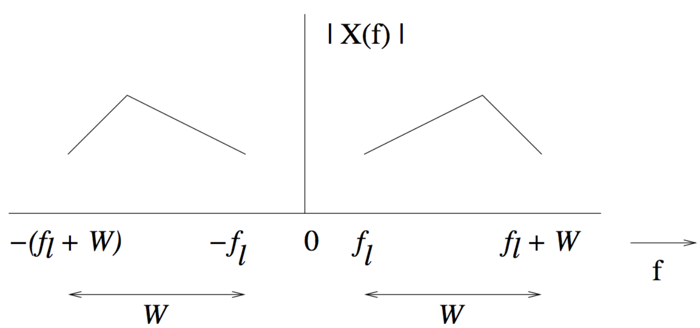
- $A_n$ is the "amount" of signal at frequency $2\pi nt/L$
- When signal is not periodic, imagine $L \to \infty$. $A_n$ becomes continuous.

# Fourier Transform (contd)

$$F(f) = \int_{-\infty}^{\infty} f(t)e^{-j(2\pi ft)}dt$$

- *F(f)* is the Fourier Transform of the time domain signal *f(t)*.
- *F(t)* is the time domain representation, *F(f)* is the frequency domain representation.
- See: http://mathworld.wolfram.com/FourierTransform.html

# Bandwidth




Frequency range (W) for which the Fourier transform values are non-zero.

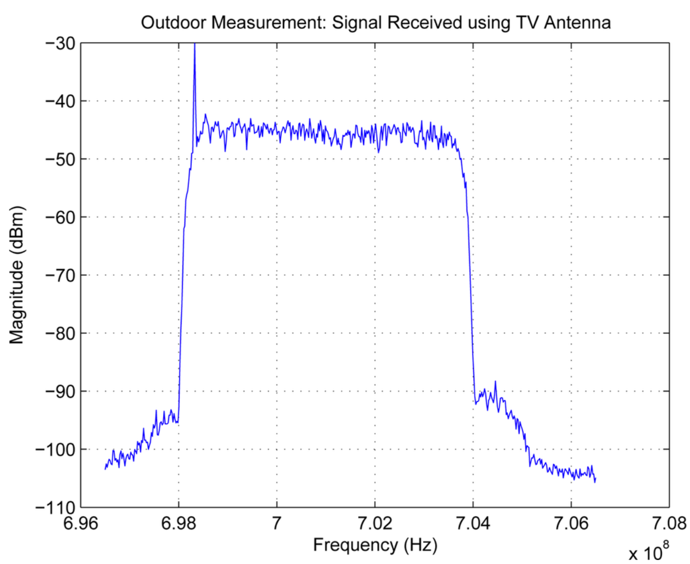
# Power and Energy

Energy content of signal $E = \int_{t_1}^{t_2} |f(t)|^2 dt$

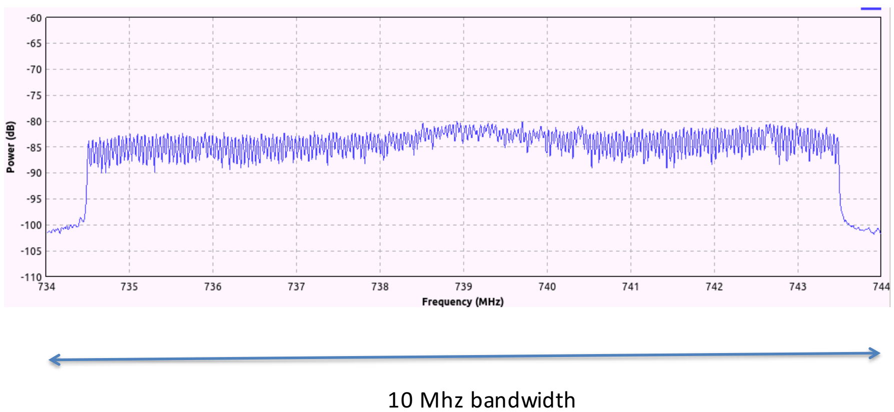
Divide by time to determine power.

One can determine these quantities via the Fourier transform as well.

# Example: ATSC TV signal Spectrum



# Example: Verizon's LTE signal 734 MHz – 744 MHz




# Concepts to be familiar with

- Band or channel – a specific range in the RF spectrum (i.e., range of frequencies) allocated to a specific technology or used for specific communication.
- Bandwidth of a signal – the range of frequencies in the frequency domain representation of the signal that has non-zero value.
- Power –Signal amplitude squared (averaged over time for average power).
- Energy – signal amplitude squared integrated over time. (Note power and energy are related concepts. Power is energy per unit time.

## Decibel Notation

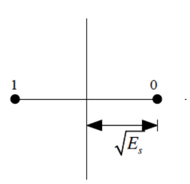
- Power is often expressed in decibel notation.
- dB is ratio of two powers in identical units expressed as

$$10\log_{10}\frac{P_1}{P_2}$$

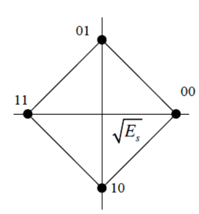
- But this only provides a ratio of two powers.
- For use in expressing absolute power, use a reference $P_2 = 1$ mW. Then $P_1$ is in dBm.

## M-ary Modulation

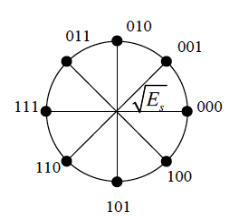
- In binary modulation each `symbol' sends 1 bit. But the idea can be extended to M bits per symbol.
- Take example of PSK. Create M phases by equally dividing $2\pi$ .




BPSK




QPSK




8-PSK

# Mathematically ..

$$s_i(t) = \underbrace{\sqrt{\frac{2E_s}{T}}}_{Constant} \cos\left( \underbrace{2\pi f_c t}_{\substack{Changing \\ with\ time}} + \underbrace{\frac{2\pi i}{M}}_{\substack{Changing \\ with\ info}} \right) \qquad i = 0,1,....M$$

Constant amplitude

*fc* is carrier frequency

Phase depending on information bits represented by *i*

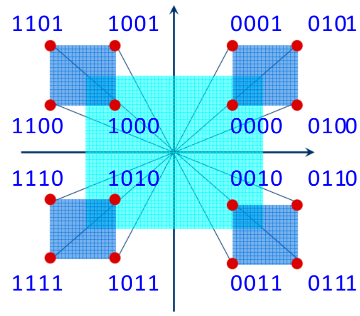
*T* is the 'symbol time'.
*Es* is the 'energy per symbol'.

'Energy' = signal 'power' integrated over time
'Power' = signal squared

# Back to Modulation

# More Complex Modulations

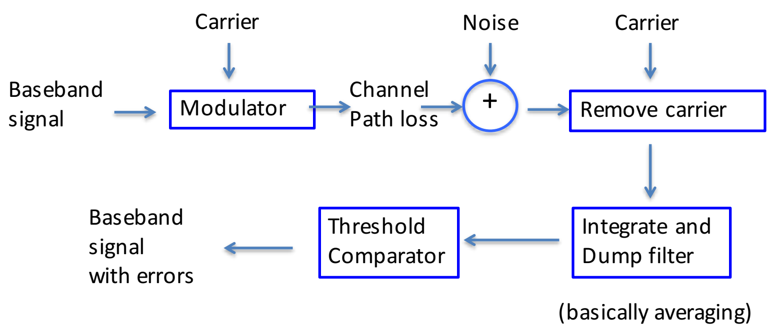
- Mix both phase and amplitude modulations – quadrature amplitude modulation (QAM).
- Example: 16-QAM
  - 4 bits per symbol
  - Symbols are mapped in a Gray code
- Higher order QAMs possible, such as 64-QAM, 256-QAM.
- Technology example: 802.11n supports BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM




16-QAM
3 amplitudes, 12 phases

# Noise Causes Bit Errors

- We use a simple example – BPSK modulation



# Example for BPSK