

Moderately Complex Paxos Made Simple: High-Level Specification of Distributed Algorithms*

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Abstract

This paper presents simpler specifications of more complex variants of the Paxos algorithm for distributed consensus, as a case study of high-level specification of distributed algorithms. The development of the specifications uses a method and language for expressing complex control flows and synchronization conditions precisely at a high level.

We show that English and pseudocode descriptions of algorithms can be captured precisely at a high level, yielding clearer and simpler specifications than ever before. The resulting specifications have allowed us to easily discover a main liveness violation that was unknown in a previous specification. We also show that the resulting specifications can be executed directly and optimized cleanly, yielding drastic performance improvement. Finally, we show that the resulting specifications can be formally verified using a proof system, with proofs an order of magnitude smaller than prior proofs, and allowing us to detect and fix a subtle safety violation that was unknown in an early specification.

1 Introduction

Distributed algorithms are increasingly important as distributed applications are increasingly developed. At the same time, distributed algorithms are often difficult to understand, even if they might appear simple. The most important and well-known of such algorithms is Paxos [Lam98, Lam01, vRA15] for distributed consensus—a set of distributed processes trying to agree on a single value or a continuing sequence of values, called single-value consensus or multi-value consensus, respectively.

Since Paxos was first developed by Lamport [Lam98, Lam17], there has been a long series of studies of it, from optimizations and extensions, especially for use in practical systems, e.g., [Bur06, CGR07], to more variations and expositions, especially with effort for better understanding, e.g., [DPLL00, Lam01, OO14, vRSS15], and for formal verification, e.g., [WWP⁺15, WWA⁺16, CLS16]. This series has finally come to a better comprehensive understanding of Paxos, as presented in Paxos Made Moderately Complex by van Renesse and Altinbuken [vRA15], starting from its simpler core, as presented in Paxos Made Simple by Lamport [Lam01].

This paper presents simpler specifications of more complex variants of the Paxos algorithm for distributed consensus, as a case study of high-level specification of distributed algorithms. The development of the specifications uses a method and language for expressing complex control flows

*This work was supported in part by NSF under grants CCF-1414078, CCF-1248184, CNS-1421893, and ONR under grant N000141512208. Contact Author: Y. Annie Liu, Computer Science Department, Stony Brook University, Stony Brook, NY 11794-2424, U.S.A. liu@cs.stonybrook.edu.

and synchronization conditions precisely at a high level. The specifications are based on Lamport’s English description of Paxos for single-value consensus [Lam01], which we call Basic Paxos, and van Renesse and Altinbuken’s pseudocode for Paxos for multiple-value consensus with preemption and reconfiguration [vRA15], which we call vRA Multi-Paxos. Our contributions include the following:

- We show that English and pseudocode descriptions of algorithms, as for Basic Paxos and vRA Multi-Paxos as examples, can be captured precisely at a high level, yielding clearer and simpler specifications than ever before, in one page or less for Paxos.
- The resulting specifications have allowed us to easily discover a main liveness violation that was unknown in the original vRA Multi-Paxos specification.
- We show that the resulting specifications can be executed directly and optimized cleanly, yielding drastic performance improvement.
- We show that the resulting specifications can be formally verified using a proof system, with proofs an order of magnitude smaller than prior proofs, and allowing us to detect and fix a subtle safety violation that was unknown in an early specification.

Our specifications are executable in DistAlgo [Dis16], an extension of the Python programming language. Our proofs are mechanically checked using TLAPS [Mic15], a proof system for TLA+, Lamport’s Temporal Logic of Actions [Lam02]. Our specifications in DistAlgo and proofs in TLAPS are available at darlab.cs.stonybrook.edu/paxos.

There have been continuous studies of specification and verification of distributed algorithms, especially of Paxos, as discussed in Section 6. However, with the exception of vRA Multi-Paxos, no previous published papers present complete precise specification of Multi-Paxos. Previous formal specifications are only for Basic Paxos or are too long or too complicated to present in papers and, to the best of our knowledge, no previous efforts of proofs for those specifications reported finding any correctness violations.

2 Basic Paxos and high-level specification

We describe our language and method of specification using Basic Paxos as an example.

2.1 Basic Paxos

Figure 1 shows Lamport’s description of Basic Paxos [Lam01]. It discusses the algorithm for (1) the proposer and acceptor—the two phases, and (2) the learner—the obvious algorithm.

2.2 High-level specification

From the description of Basic Paxos, one can see that high-level specification of distributed algorithms needs four main components:

Distributed processes that can send messages. In Figure 1, there are proposer, acceptor, and learner processes, *prepare* and *accept* messages from a proposer to an acceptor, response messages back from an acceptor, and messages for accepted proposals from an acceptor to a learner.

Putting the actions of the proposer and acceptor together, we see that the algorithm operates in the following two phases.

Phase 1. (a) A proposer selects a proposal number n and sends a *prepare* request with number n to a majority of acceptors.

(b) If an acceptor receives a *prepare* request with number n greater than that of any *prepare* request to which it has already responded, then it responds to the request with a promise not to accept any more proposals numbered less than n and with the highest-numbered proposal (if any) that it has accepted.

Phase 2. (a) If the proposer receives a response to its *prepare* requests (numbered n) from a majority of acceptors, then it sends an *accept* request to each of those acceptors for a proposal numbered n with a value v , where v is the value of the highest-numbered proposal among the responses, or is any value if the responses reported no proposals.

(b) If an acceptor receives an *accept* request for a proposal numbered n , it accepts the proposal unless it has already responded to a *prepare* request having a number greater than n .

To learn that a value has been chosen, a learner must find out that a proposal has been accepted by a majority of acceptors. The obvious algorithm is to have each acceptor, whenever it accepts a proposal, respond to all learners, sending them the proposal. ...

Figure 1: Lamport's description of Basic Paxos in English [Lam01].

Control flows for handling received messages. In Figure 1, messages can be received by acceptors, by proposers, and by learners asynchronously at any time, but processes must synchronize by testing and waiting for different conditions on the received messages. Capturing such complex control flows is the most difficult part.

High-level queries for synchronization conditions. In Figure 1, the conditions for Phases 1b, 2a, 2b, and learner before taking actions involve sets of many messages sent and received. Capturing such conditions at a high-level is essential to making control flows much clearer and easier to understand.

Configuration for setting up and running. This is often implicit in descriptions of distributed algorithms. In Figure 1, each process needs to be set up and get a hold of other processes with which it needs to communicate. In general, there may also be special configuration requirements, such as use of reliable communication channels.

Figure 2 shows a high-level specification of Basic Paxos in DistAlgo, explained below. Figure 1 corresponds to the body of `run` in `Proposer`, the two `receive` definitions in `Acceptor`, and the condition of `await` in `Learner`, 20 lines total, including selecting a proposal number, in Phase 1a, to be self, and taking any value, in Phase 2a, to be an integer in the range of 1..100. The rest, 18 lines total, shows how to set up processes, start to run them, and output the result.

Besides the language constructs explained below, we use commonly used notations in high-level languages for no operation (`pass`), assignments ($v := e$), etc. We use indentation for scoping, ':' for separation, and '#' for comments.

```

1 process Proposer:
2   def setup(acceptors):
3     self.n := undefined
4     self.majority := acceptors
5   def run():
6     n := self
7     send ('prepare',n) to majority
8     await count {a: received ('respond',=n,_) from a}
9     > (count acceptors)/2:
10    v := any ({v: received ('respond',=n,(n2,v)),
11              n2 = max {n2: received ('respond',=n,(n2,_))} }
12            or {any 1..100})
13    responded := {a: received ('respond',=n,_) from a}
14    send ('accept',n,v) to responded

15 process Acceptor:
16 def setup(learners): pass
17 def run(): await false
18 receive ('prepare',n) from p:
19   if each sent ('respond',n2,_) has n > n2:
20     max_prop := any {(n,v): sent ('accepted',n,v),
21                          n = max {n: sent ('accepted',n,_)}}
22     send ('respond',n,max_prop) to p
23 receive ('accept',n,v):
24   if not some sent ('respond',n2,_) has n2 > n:
25     send ('accepted',n,v) to learners

26 process Learner:
27 def setup(acceptors): pass
28 def run():
29   await some received ('accepted',n,v) has
30     count {a: received ('accepted',=n,=v) from a}
31     > (count acceptors)/2:
32   output('learned',n,v)

33 def main():
34   acceptors := 3 new Acceptor
35   proposers := 3 new Proposer(acceptors)
36   learners := 3 new Learner(acceptors)
37   acceptors.setup(learners)
38   (acceptors + proposers + learners).start()

```

Figure 2: A high-level specification of Basic Paxos in DistAlgo, including setting up and running 3 each of Proposer, Acceptor, and Learner processes and outputting the result.

Distributed processes that can send messages A type P of distributed processes can be defined by `process P: body`, e.g., lines 1-14 in Figure 2. The process body may contain

- a `setup` definition for taking in and setting up the values used by a type P process, e.g., lines 2-4,
- a `run` definition for carrying out the main control flow of the process, e.g., lines 5-14, and
- `receive` definitions for handling received messages, e.g., lines 18-22 and lines 23-25.

A process can refer to itself as `self`. `self.name` (or `name` when there is no ambiguity) refers to the value of `name` in the process. `ps := n new P(args)` creates n new processes of type P , optionally passing in values of `args` to `setup`, and assigns the new processes to `ps`, e.g., lines 34 and 35. `ps.setup(args)` sets up processes `ps` using values of `args`, e.g., line 37, and `ps.start()` starts `run` of `ps`, e.g., line 38.

Processes can easily send messages: `send m to ps` sends message m to processes `ps`, e.g., line 7.

Control flow for handling received messages Received messages can be handled both asynchronously, using yield points and `receive` definitions, and synchronously, using `await` statements.

- A yield point, `-- l`, with optional label l , specifies a point where control flow can yield to handling of un-handled messages and resume afterward. There is an implicit yield point, if not an explicit one, before each `await` statement, e.g., line 8, for handling messages while waiting.
- A definition, `receive m from p at l1, ..., lj: body`, handles, at yield points l_1, \dots, l_j , un-handled messages that match m from p , e.g., lines 18-22. The `from` and `at` clauses are optional.
- A statement, `await cond1: stmt1 or ... or condk: stmtk timeout t: stmt`, waits for one of $cond_1, \dots, cond_k$ to be true or a timeout after period t , and then nondeterministically selects one of $stmt_1, \dots, stmt_k, stmt$ whose conditions are true to execute, e.g., lines 8-14. Each branch is optional. So is the statement in `await` with a single branch.

High-level queries for synchronization conditions High-level queries can be used over message histories, and patterns can be used for matching the messages.

- Histories of messages sent and received by a process are kept in `sent` and `received`, respectively. `sent` is updated at each `send` statement, by adding each message sent. `received` is updated at the next yield point if there are un-handled messages, by adding un-handled messages before executing any matching `receive` definitions.

Expression `sent m to p` is equivalent to `m to p in sent`. It returns true iff a message that matches `m to p` is in `sent`. The `to` clause is optional. `received m from p` is similar.

- A pattern can be used to match a message, in `sent` and `received`, and by a `receive` definition. A constant value, such as `'response'`, or a previously bound variable, indicated with prefix `=`, in the pattern must match the corresponding components of the message. An underscore matches anything. Previously unbound variables in the pattern are bound to the corresponding components in the matched message.

For example, `received ('response',=n,_) from a` on line 8 matches every triple in `received` whose first two components are `'response'` and the value of `n`, and binds `a` to the sender.

A query can be a comprehension, aggregation, or quantification over sets or sequences.

- A comprehension, `{e: v1 in s1, ..., vk in sk, cond}`, where v_i can be a pattern, returns the set of values of e for all combinations of values of variables that satisfy all v_i in s_i clauses and condition `cond`, e.g., the comprehension on line 8.

- An aggregation, *agg s*, where *agg* is an aggregation operator such as `count` or `max`, returns the value of applying *agg* to the set value of *s*, e.g., the `count` query on line 8.
- An existential quantification, `some v_1 in s_1 , ..., v_k in s_k has cond`, returns true iff for some combinations of values of variables that satisfy all `v_i in s_i` clauses, *cond* holds, e.g., the `some` query on line 24. When the query returns true, all variables in the query are also bound to a combination of satisfying values, called a witness, e.g., `n` and `v` on lines 29-31.
- A universal quantification, `each v_1 in s_1 , ..., v_k in s_k has cond`, returns true iff for all combinations of values of variables that satisfy all `v_i in s_i` clauses, *cond* holds, e.g., the `each` query on line 19.

Other operations on sets can also be used, in particular:

`any s` returns any element of set *s* if *s* is not empty, or a special value `undefined` otherwise.

`n_1 .. n_2` returns the set of integers ranging from n_1 to n_2 for $n_1 \leq n_2$.

`$s_1 + s_2$` returns the union of sets s_1 and s_2 . `s_1 or s_2` returns s_1 if s_1 is not empty, or s_2 otherwise.

2.3 Precise understanding of Basic Paxos

We can now precisely follow Phases 1a and 1b and Phases 2a and 2b in Figure 2, and see how it exactly follows Lamport’s description in Figure 1.

Phase 1a (lines 6-7) is simple, straightforwardly following the description in Figure 1.

In Phase 1b (lines 18-22), when an acceptor receives a `prepare` with a proposal number larger than all numbers in its previous responses, it responds with this proposal number and with the accepted `(n,v)` where `n` is maximum among all accepted it has sent; note if it has not sent any accepted, `undefined` is in the response instead of some `(n,v)`.

In Phase 2a (lines 8-14), when a proposer receives responses to its proposal number `n` from a majority of acceptors, it takes `v` in the `(n2,v)` that has the maximum `n2` in all responses to `n`, or any value in `1..100` if the responses have no `(n2,v)` pairs but only `undefined`; note in the latter case, the set on lines 10-11 is empty because `undefined` does not match `(n2,v)`. The proposer then sends `accept` for a proposal with number `n` and value `v` to acceptors that responded to `n`.

Phase 2b (23-25) is also simple, directly following the description in Figure 1.

In particular, the specification in Figure 2 helps make the “promise” in Phase 1b of Figure 1 precise—the “promise” refers to what are to be accepted later in Phase 2b.

Indeed, this is the hardest part for understanding Paxos, because understanding an earlier phase requires understanding a later phase, which requires understanding the earlier phases. The key idea is that the later phase to be understood is for a smaller (or equal¹) proposal number, and the later phase for before the smallest proposal number needs to provide nothing, i.e., provide only `undefined`.

We can see that the use of high-level control flows and high-level queries allow our Basic Paxos specification to be at the same high level as Lamport’s English description, while making everything completely precise. With also precise support for setting up and running, the complete specification is both directly executable, by automatic compilation to a language like Python, and ready to be verified, by translation to a language like TLA+.

¹In the case of “equal”, which happens only when messages are duplicated and is rare, it is safe to either accept, i.e., send “accepted”, or not. Sending helps if the sent accepted was lost but is a waste otherwise.

Configuration for setting up and running Configuration for requirements such as reliable channels and logical clocks can be specified in a `main` definition, e.g., lines 33-38. Basic Paxos does not have special configuration requirements, besides setting up and running the processes by calling `new`, `setup`, and `start` as already described. In general, `new` can have an additional clause `at node` specifying remote nodes where the created processes will run; the default is the local node.

3 Multi-Paxos with preemption and reconfiguration, and improved specification

Multi-Paxos extends Basic Paxos to reach consensus on a continuing sequence of values, instead of a single value. It is significantly more sophisticated than running Basic Paxos for each slot in the sequence, because proposals must be made continuously for each next slot, with the proposal number, also called ballot number or simply ballot, incremented repeatedly in new rounds if needed, and the ballot is shared for all the slots for obvious efficiency reasons.

Preemption allows a proposer, also called leader, to be preempted by another leader that has a larger ballot. That is, if a leader receives a message with a larger ballot than its own ballot, it abandons its own ballot, and starts a larger ballot later.

Reconfiguration allows a set of new leaders to be used during the execution of the algorithm. The slot in which the change of leaders is to happen must be agreed on by the old leaders. This is done by taking the change as one of the values, also called commands, to be agreed on.

We describe a simplified specification of Multi-Paxos with preemption and reconfiguration, which has also allowed us to cleanly remove an unnecessary delay and easily discover a main liveness violation.

3.1 vRA Multi-Paxos pseudocode

vRA Multi-Paxos gives complete pseudocode for Multi-Paxos with preemption and reconfiguration [vRA15, Figures 1,4,6,7 and definitions of *pmax* and \triangleleft]. The core is the same as Basic Paxos. However, except for Acceptor, all the names used are either changed, with Leader also spawning two new types of processes, Scout and Commander:

Basic Paxos	Proposer & Learner	proposal number	value	prepare	respond	accept	accepted
vRA Multi-Paxos	Leader	ballot number	command	1a	1b	2a	2b

or new:

vRA Multi-Paxos	Replica	slot	request	response	propose	decision	preempt
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A Replica process keeps the state of an application, e.g., a bank. It continuously receives requested commands ('request') from clients, and sends proposed slots for the commands ('propose') to leaders; it also receives decisions about slots for the commands ('decision') from leaders, applies the operations in the commands to the state, and sends the results ('response') to the clients. A slot is just a component in a proposal or decision in Leader and Acceptor, but is realized in Replica as 'slot_in', for the next slot to send a proposal, and 'slot_out', for the next slot to apply a decision; a window between these two is used so that a decided reconfiguration takes effect at the slot at the end of the window, and other commands can still be proposed and decided within the window while the reconfiguration proposal is being considered. A command is a triple of client id, command id, and operation, and the operation for reconfiguration holds the set of new leaders.

```

1 process Replica:
2   def setup(leaders, state):
3     self.slot_in, self.slot_out := 1, 1
4   def run():
5     while true:
6       await slot_in < slot_out + WINDOW and
7         some received ('request',c) has
8         each sent ('propose',s,c) has
9         some received ('decision',=s,c2) has c2 != c:
10      if some received ('decision', slot_in - WINDOW, (_,_,op)) has is_reconfig(op):
11        leaders := op.leaders
12      if not some received ('decision',=slot_in,_):
13        send ('propose', slot_in, c) to leaders
14        slot_in := slot_in + 1
15      or some received ('decision', =slot_out, c):
16        client, cmd_id, op := c
17        if not (some received ('decision',s,c) has s < slot_out) and not is_reconfig(op):
18          state, result := apply(op, state)
19          send ('response', cmd_id, result) to client
20          slot_out := slot_out + 1

21 process Leader:
22   def setup(acceptors, replicas):
23     self.ballot := (0, self)
24   def run():
25     while true:
26       send ('1a', ballot) to acceptors
27       await count {a: received ('1b',=ballot,_) from a} > (count acceptors)/2:
28         for (s,c) in pmax({t: received ('1b',=ballot,accepted), t in accepted}):
29           send ('2a', ballot, s, c) to acceptors
30         while true:
31           await some received ('propose',s,c) has not some sent ('2a',=ballot,=s,_):
32             send ('2a', ballot, s, c) to acceptors
33         or some received ('2b',=ballot,s,c) has
34           count {a: received ('2b',=ballot,=s,c) from a} > (count acceptors)/2:
35             send ('decision', s, c) to replicas
36         or some received ('preempt',(r2,leader2)) has (r2,leader2) > ballot: break
37         or some received ('preempt',(r2,leader2)) has (r2,leader2) > ballot: pass
38         ballot := (r2+1, self)
39   def pmax(pvals):
40     return {(s,c): (b,s,c) in pvals, b = max {b: (b,=s,_) in pvals}}

41 process Acceptor:
42   def run(): await false
43   receive ('1a', b) from leader:
44     max_b := max {b: received ('1a',b)}
45     if b = max_b:
46       accepted := {(b,s,c): received ('2a',b,s,c)}
47       send ('1b', max_b, accepted) to leader
48     else: send ('preempt', max_b) to leader
49   receive ('2a', b, s, c) from leader:
50     max_b := max {b: received ('1a',b)}
51     if b = max_b:
52       send ('2b', max_b, s, c) to leader
53     else: send ('preempt', max_b) to leader

```

Figure 3: A high-level specification of vRA Multi-Paxos in DistAlgo.

3.2 Simplified specification

Figure 3 shows a simplified high-level specification of vRA Multi-Paxos. It corresponds to over 100 lines of pseudocode and over 300 lines of Python (that runs only threads for processes, not distributed processes over networks) in [vRA15]. The simplifications and new organizations are as follows.

Scout and Commander are removed, their main roles for the two phases are merged into Leader, and their roles for determining preemption are merged into Acceptor.

Leader uses two `count` queries (lines 27 and 34), instead of repeated updates of two `waitfor` sets in Scout and Commander, to collect majority votes from Phases 1b and 2b; uses two queries to collect previously accepted and newly proposed proposals (lines 28 and 31) instead of updating a `proposals` set; and uses two simple queries for preemption (lines 36 and 37).

The rest are simple `send` statements for `1a`, `2a`, and `decision` (lines 26, 29, 32, and 35), `while true` for repeatedly incrementing `ballot` for Phase 1a (lines 25 and 38), and `while true` for repeatedly processing new proposals in Phase 2a and learning accepted proposals (lines 30 and 33).

Note that the simplified specification is now similar to Basic Paxos, except that a `1b` message contains a set of triples instead of a single pair, a `2a` message is sent for a proposal (`ballot`, `slot`, `command`) for each slot instead of a single proposal (`number`, `value`), and a `2b` message follows with an additional slot component too, all put together with two `while true` loops.

Acceptor uses queries to find the maximum ballot (lines 44 and 50) and the set of accepted triples (line 46), instead of maintaining them by repeated updates, and takes the additional role of determining preemption and informing the leader (lines 48 and 53) instead of always replying with set of accepted triples for Phase 1b and the received slot and command for Phase 2b.

Note that the set of accepted triples is a superset of the set sent in the original pseudocode; the latter cannot be defined as a function of `received`. Also note that determining and replying with preemption in our specification is more efficient than always replying as normal to a scout or commander in the original pseudocode, especially by omitting the set of accepted triples for Phase 1b, because the set is large and is ignored by the leader anyway.

Replica uses two clearly separated conditions, one for proposing commands based on requests, one for applying commands based on decisions, instead of keeping and updating `requests`, `proposals`, and `decisions` in sophisticated control flows.

3.3 Unnecessary delay, liveness violation, and fixes

Specification of Replica is arguably the most complex among all types of processes in vRA Multi-Paxos, due to the need of mediation between clients and leaders, while supporting also consensus for change of leaders upon reconfiguration.

Our effort to create a high-level specification of Replica allowed us to notice and fix an unnecessary delay in processing requests. Note that our first `await` condition allows any received request, for which all sent proposals for a slot have that slot taken by a different command in received decisions, to be detected immediately. The original pseudocode delays the detection until `slot_out` equals the taken slot.

Our high-level specification also allowed us to discover a main violation of liveness in the original pseudocode: if there is no decision made for a slot, e.g., due to lost `propose` messages from all replicas proposing for that slot, all replicas will stop applying decisions from that slot on, so `slot_out` will

stop incrementing; furthermore, due to the limited `WINDOW` used for incrementing `slot_in`, all sending of proposals will stop after the `WINDOW` is used up. So all replicas will be completely stuck, and the entire system will stop making progress. A fix could be to propose for that slot again after a timeout. The leader can work on deciding for that slot if a decision for it has not been made; otherwise, it can send back the decision for that slot.

Note that the liveness violation must be fixed, because the `propose` messages are internal to the vRA Multi-Paxos mechanism and may be lost even if no processes fail. The unnecessary delay may look minor, but realizing it and removing it also helped us develop our simpler specification than the original pseudocode, especially in terms of control flows, which subsequently allowed us to easily discover the liveness violation.

Both problems in Replica are difficult to detect in the original pseudocode due to complex control flows. Developing higher-level specifications, especially using nondeterministic `await` with high-level synchronization conditions, helped us understand the algorithm better and discover these problems easily.

4 Further optimizations and merging processes

High-level specifications also allow additional optimizations and extensions to be done more easily. We describe the two most important ones suggested for vRA Multi-Paxos [vRA15], and discuss a general method for merging processes that supports a range of additional optimizations.

4.1 State reduction

The most serious efficiency problem of the algorithm in Figure 3 is the fast growing set `accepted`, which quickly chokes any execution of the algorithm that does real message passing. The solution is to not keep all triples received in `2a` messages, as in Figure 3, or those that have the maximum ballot when the triple was received in a `2a` message, as in the original pseudocode [vRA15], but keep only triples with the maximum ballot for each slot, so there is at most one triple for each slot. This is done by changing line 46 to

```
46         accepted := {(b,s,c): received ('2a',b,s,c), b = max {b: received ('2a',b,s,_)}}
```

This drastically reduces not only the state space of acceptors, but also leaders, which receive messages containing `accepted`.

4.2 Failure detection

Failure detection addresses the next most serious problem: leaders compete unnecessarily to become the leader with the highest ballot, leaving little or no time for proposals to be decided. Adding failure detection uses ping-pong after preemption: in `Leader`, after exiting the outer `await` following preemption and before incrementing `ballot`, periodically ping the leader `leader2` that has the larger round number `r2` in the ballot and wait for replies, by inserting

```
37.1         while each sent('ping',r2,t) to =leader2 has received ('pong',r2,t) from leader2:
37.2             send ('ping', r2, logical_time()) to leader2
37.3             await timeout TIMEOUT
```

and adding the following `receive` definition after the `run` definition:

```
38.1 receive ('ping', r2, t) from leader2:
38.2     send ('pong', r2, t) to leader2
```

4.3 Merging processes

High-level specifications in DistAlgo allow different types of processes that run at the same time to be merged easily, even if they interact with each other in sophisticated ways, provided they together have one main flow of control. There are two cases:

1. A process P that has only `await false` in `run` can be merged easily with any process Q . For example, for vRA Multi-Paxos in Figure 3, `Acceptor` can be merged with `Leader` by adding the `receive` definitions of `Acceptor` to the body of `Leader`.
2. A process P that has only `while true: await...` in `run`, with no `timeout` in the `await`, can be merged easily with any process Q . For example, for vRA Multi-Paxos in Figure 3, `Replica` can be merged with `Leader` by adding, for each branch `cond: stmt` of `await` of `Replica`, a `receive _: if cond: stmt` definition to the body of `Leader`.

Process setups can be transformed accordingly. Details are omitted because they are less important. These transformations are easy to automate. Inversely, independent `receive` definitions can be easily put into separate processes. For vRA Multi-Paxos in Figure 3, all three types of processes, or any two of them, can be merged, giving a total of 4 possible merged specifications.

Merging supports colocation of processes cleanly, and allows a range of optimizations, e.g., garbage collection of states of leaders and acceptors, for decided slots already learned by all replicas [vRA15]. Furthermore, communication between processes that are merged no longer needs real message passing but can be done more efficiently through shared memory. Also, because the actions are independent, lightweight threads can be used to make each process more efficient.

With a few more small variations to vRA Multi-Paxos, merging `Replica`, `Leader`, and `Acceptor` into one process type yields essentially the 'Replica' in Chubby [Bur06], Google's distributed lock service that uses Paxos, and the 'Server' in Raft [OO14], a pseudocode for the main features of Chubby. In general, separate processes provide modularity, and merged processes provide efficiency. Being able to merge separate processes easily allows one to obtain the benefits of both.

4.4 Complete executable specification

A `main` definition can set up a number of `Replica`, `Leader`, and `Acceptor` processes, or their merged versions, and some `Client` processes that send `request` messages to `Replica` processes and receive `response` messages, and define parameters `WINDOW` and `TIMEOUT`. We summarize the results of running DistAlgo specifications:

- DistAlgo specifications can be run directly. For example, a complete specification of vRA Multi-Paxos with state reduction and failure detection in file `spec.da` (available at `darlab.cs.stonybrook.edu/paxos`) can be run directly with Python (3.5 or higher), by executing the commands `pip install pyDistAlgo` followed by `python -m da spec.da`.
- vRA Multi-Paxos as in Figure 3, without the state reduction in Section 4.1, almost immediately overflows the default message buffer size of 4KB, yielding `Could not send message due to:MessageTooBigException('** Outgoing message object too big to fit in buffer, dropped.')`
- vRA Multi-Paxos with state reduction, without the failure detection in Section 4.2, runs continuously but most times stops making progress for 3 leaders, 3 acceptors, and 3 replicas, serving 10 client requests, and was killed after 200 rounds (200–600 ballots) have been attempted.

- vRA Multi-Paxos with both state reduction and failure detection runs smoothly. For example, setting up 10 processes (3 leaders, 3 acceptors, 3 replicas, and 1 client) and processing 10 requests takes 84.117 milliseconds (ranging from 70.158 to 96.903), averaged over 10 runs, on a Intel Core i7-6650U CPU @ 2.20GHz with 16 GB RAM, running Windows 10 and Python 3.5.

Many additional optimizations and experiments could be carried out though they are beyond the scope of this paper. Our experience is that precise high-level specifications and optimizations, formally verified as discussed next, allow us to understand the algorithms much better and to significantly improve efficiency as well as correctness much more easily than possible before.

5 Correctness and formal verification

The delay and liveness problems discovered for Replica in vRA Multi-Paxos do not affect the safety of vRA Multi-Paxos. However, even the safety is not easy to understand.

We have developed formal proofs of safety of the complete specification of vRA Multi-Paxos in Figure 3, and of the one extended with state reduction and the one further extended with failure detection as described in Section 4. The safety property ensures that, for each slot, only a single command may be decided and it must be one of the commands proposed.

The proofs are done by first translating the specifications into TLA+; this is currently done manually, but an automatic translator is being developed. The high-level nature of our specifications makes the translation simple in principle: each type of data in DistAlgo corresponds a type of data in TLA+, and each expression and statement in DistAlgo corresponds to a conjunction of equations in TLA+. The three specifications in TLA+ are 154, 157, and 217 lines.

5.1 Proofs in TLAPS

The proofs in TLAPS are 4959, 5005, and 7006 lines, respectively. The proofs are much more complex and longer than the previous proof of 1033 lines for Multi-Paxos with preemption [CLS16], because of the additional details captured in vRA Multi-Paxos, and the extensions for state reduction and failure detection. Appendix A contains additional details about our mechanically checked proofs.

Compared to the other existing mechanically checked proofs for Multi-Paxos and variants, namely, a proof of Multi-Paxos in Dafny from the IronFleet project [HHK⁺15] and a proof of Raft in Coq from the Verdi project [WWA⁺16], our proofs in TLAPS are an order of magnitude smaller, as summarized in Section 6. We believe that this is at least partly due to the fact that we started with higher-level specifications that are simpler. The shorter proofs are not only much easier to understand and maintain, but also much easier and faster to check automatically. Both are significant advantages for practical development cycles of specifications, programs, and proofs.

Our longest proof checking time, for our largest proof in TLAPS, is 13 minutes, as shown in Appendix A. No proof checking time is reported for the proof from Verdi [WWA⁺16], but we were able to run proof check for the proof after solving some version mismatch problems, and it took 29 minutes to run on the same machine as our proof. The proof checking times for the proofs from IronFleet are reported to be 147 minutes for the protocol-level proof and 312 minutes including also the implementation-level proof [HHK⁺15]; we have not been able to run proof check for their proofs on our machine due to an error from a build file.

5.2 Safety violation and fix

Our development of formal proofs also allowed us to discover and fix a safety violation in an earlier version of our specification for vRA Multi-Paxos, where acceptors always reply with `1b` and `2b` messages, not `preempt` messages, as in the original pseudocode, and leaders try to detect preemption. It is incorrect because the ballot number in leaders may increase after a `1a` or `2a` message is sent. The fix of having acceptors detect preemption and inform the leader also makes the algorithm much more efficient in the case of preemption upon receiving `1a` messages: a simple `preempt` message is sent instead of a `1b` message with an entire `accepted` set.

The earlier incorrect version was used in distributed algorithms and distributed systems courses for several years, with dozens of course projects and homeworks having used it, including ones directed specifically at testing and even modeling using TLA+ and model checking using TLC [Mic16]. However, this safety violation was never found, because it requires delays of many messages, extremely unlikely to be found by testing or model checking.

6 Related work and conclusion

Paxos and its variants, especially including their specification and verification, have seen a long series of studies, as introduced in Section 1. van Renesse and Altinbuken’s pseudocode [vRA15] is by far the most concise specification of a more realistic version of Paxos among existing works. For example, a specification of Raft in Verdi is reported to be 530 lines [WWA⁺16]. Even Basic Paxos in various languages, e.g., IOA, Overlog, and Bloom, are 145–230 lines, as summarized previously [LSLG12]. Our specification is significantly simpler and captures the control flows and synchronization conditions at a higher level. It has helped tremendously in teaching and allowed us to easily discover the main liveness violation discussed. An earlier, incomplete specification [LSL12] has a similar liveness violation, and a delay in applying decisions by not using `await`.

The language we use is DistAlgo [LSLG12, LSL17], presented concisely. It is high-level, as pseudocode languages or even English, precise, as specification languages, and executable, as programming languages. In particular, it is significantly higher-level and easier to read and write than conventional programming languages, including concise languages like Erlang and Python, and formal specification languages, such as PlusCal and IOA, as discussed previously [LSLG12]. DistAlgo has been implemented in Python [LSLG12] and given an operational semantics [LSL17]. Straightforward execution when message histories and high-level queries are used can be extremely inefficient; optimization by incrementalization generates efficient code that maintains query results incrementally as messages are sent and received [LSLG12, LSL17, LBSL16].

Formal verification of Paxos has mostly been for Basic Paxos only or by model checking, as summarized previously [CLS16]. Model checking Paxos works for only a minimum number of processes and even just a single slot as in, e.g., DeMeter-MaceMC and DeMeter-MoDist [GWZ⁺11], even after exponential state-space reduction. The only prior proofs for Multi-Paxos and variants are proofs from IronFleet [HHK⁺15], a proof from Verdi [WWP⁺15], and Chand et al.’s proof of Multi-Paxos with preemption using TLAPS [CLS16]. IronFleet is superior in that it also proves liveness properties, but it is a complex system, with 3 levels and many components of specifications, over 1000 lines, and proofs, 30,000 lines, in Dafny [HHK⁺15], with the essence of the proof hard to find and understand. Verdi’s proof of Raft in Coq is 50,000 lines [WWA⁺16]. Our proofs in TLAPS are an order of magnitude smaller, but larger than Chand et al.’s proof [CLS16] due to the additional details modeled and the extensions. Shorter proofs are not only much easier to understand and

maintain, but also much easier and faster to check automatically, as discussed in Section 5.

There are many directions for future research: better high-level specifications of more variants of Paxos and other important distributed algorithms, further optimizations for generating efficient implementations, and better and more automated proofs.

Acknowledgment

We thank Leslie Lamport and Robbert van Renesse for their clear explanations and helpful discussions about Paxos. We thank Bo Lin for his robust DistAlgo compiler with excellent support.

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A Mechanically checked proofs in TLAPS

We manually translated the specification of vRA Multi-Paxos in Figure 3 and the two extensions in Section 4 to TLA+. We specified and proved safety for three versions: vRA Multi-Paxos, vRA Multi-Paxos with state reduction, and vRA Multi-Paxos with state reduction and failure detection. The high-level nature of our DistAlgo specifications makes the translation relatively simple.

We developed inductive proofs of safety for all three versions. The proofs, like the proof for Multi-Paxos from [CLS16], are based on several invariants which together imply safety. The proofs involve three types of invariants: (1) type safety invariants, stating that as the system progresses, all data in the system have the expected types, (2) invariants about local data of processes, for example, about the values of `ballot`, `accepted`, and `max_b`, and (3) invariants about the global data of the system, in particular, about the messages sent in the system.

Our proofs for vRA Multi-Paxos differ from the proof for Multi-Paxos from [CLS16] for several reasons, including differences between the algorithms themselves. For example, the `accepted` set in vRA Multi-Paxos contains all triples for which a `2a` message was sent and received and may contain a triple for which a `2b` message was not sent, whereas in Multi-Paxos in [CLS16], the `accepted` set would only keep a triple if a `2b` message was sent containing that triple. Also, to keep our specification in TLA+ close to the specification in DistAlgo, we model ballots as tuples containing a natural number and a process ID, as opposed to modeling them as natural numbers as in [CLS16]. This modeling difference has huge impact on the proof, because comparison operators like `>` and `≥` on natural numbers are built-in operators in TLAPS, and it can reason about them automatically, but comparison operators on tuples need to be defined using predicates, and all of their properties, including fundamental properties like transitivity and non-commutativity, need to be explicitly stated in lemmas and proved. In addition, we specify and prove safety of three versions of Multi-Paxos, all of which are variations not considered in [CLS16].

Figure 4 presents the results about our specifications and proofs of vRA Multi-Paxos and its extensions, and the specifications and proofs of Multi-Paxos from [CLS16] and Basic Paxos from [LMD14]. First, we compare the specifications and proofs of vRA Multi-Paxos and its extensions with each other:

- The specification size grows by only 3 lines (1.9%) from 154 when we add state reduction, but by 60 more lines (38%), for the new actions added, when we add failure detection.
- The proof size grows by only 46 lines (0.9%) from 4959 when we add state reduction, but by 2001 more lines (40%) when we add failure detection, roughly proportional to the increase in specification size.
- The maximum level and degree of proof tree nodes remain unchanged when state reduction is added. When failure detection is added, the maximum level of proof tree nodes remains unchanged, but the maximum degree of proof tree nodes increases by 20 (71%), from 28 to 48, due to more complex proofs for the new actions added for failure detection.

Metric	Basic Paxos	Multi-Paxos		vRA Multi-Paxos & extensions		
		Multi-Paxos	Multi- w/ Preemption	w/ details & reconfig.	w/ also st. reduct.	w/ also fail. detect.
Spec size excl. comments	56	81	97	154	157	217
Spec size incl. comments	115	133	158	249	254	347
Proof size excl. comments	306	1003	1033	4959	5005	7006
Proof size incl. comments	423	1106	1136	5256	5301	7384
Max level of proof tree nodes	7	11	11	12	12	12
Max degree of proof tree nodes	3	17	17	28	28	48
# lemmas	4	11	12	24	23	23
# continuity lemmas	1	5	6	8	8	8
# uses of continuity lemmas	8	27	29	76	76	76
# proofs by induction on set increment	0	4	4	30	30	30
# proofs by contradiction	1	1	1	14	16	17
# obligations in TLAPS	239	918	959	4364	4517	5580
TLAPS check time (seconds)	24	128	94	590	569	781

Figure 4: Comparison of results for safety proofs of Basic Paxos from [LMD14], Multi-Paxos from [CLS16], and vRA Multi-Paxos. Spec size and proof size are measured in lines. An obligation is a condition that TLAPS checks. The time to check is on an Intel i7-4720HQ 2.6 GHz CPU with 16 GB of memory, running Ubuntu 16.04 LTS and TLAPS 1.5.2.

- An interesting decrease of one lemma is seen after state reduction is added. The lemma states that the maximum of a set is one of the maximums of its two partitions. This lemma was needed in the case when all triples in $2a$ messages are kept by the acceptors. However, owing to state reduction, only triples with the maximum ballots are kept, making the proofs simpler. The number of continuity lemmas and their uses remain unchanged when we add extensions. A *continuity lemma* is a lemma asserting that a predicate continues to hold (or not hold) as the system goes from one state to the next in a single step.
- The number of proofs by induction on set increment remains unchanged when we add extensions. The number of proofs by contradiction increases; in those cases, constructive proofs were more challenging.
- The number of obligations, i.e., conditions that TLAPS proves, increases by 153 (3.5%) from 4364 when state reduction is added, and by 1063 (24%) more when failure detection is added, contributing to the increase in proof size.
- The proof check time decreases by 21 seconds (3.6%) from 590 to 569 when state reduction is added. This was expected because, with state reduction, for each slot, only the triple with the maximum ballot is kept. Upon receiving a triple with a larger ballot, only the new triple is kept, and the maximum of a singleton set is the item itself, making the proof time decrease. The proof check time increases by 212 seconds (37%) when failure detection is added. This is expected, because there are more proof obligations (24%) and the proof is larger (40%).

Finally, we compare our TLA+ specification and proof of vRA Multi-Paxos (without state reduction or failure detection) with those of Multi-Paxos with preemption from [CLS16].

- The specification of vRA Multi-Paxos, excluding comments, is 154 lines, which is 59% more. This increase is because our specification is intentionally kept very close to the specification in Figure 3 which models additional details.
- The proof of vRA Multi-Paxos, excluding comments, is 4959 lines, which is 380% more. This increase is due to many factors, including more actions (for sending `2a` messages in two cases and for sending decisions), more invariants (about the looser `accepted` set and about program points for the additional actions), and representing ballots as tuples instead of natural numbers, as mentioned above.
- The proof tree for vRA Multi-Paxos is more complex, as shown by the 65% increase, from 17 to 28, in the maximum degree of proof tree nodes and 9% increase, from 11 to 12, in the maximum level of proof tree nodes.
- Twice as many lemmas are needed for vRA Multi-Paxos, 24 compared with 12, because properties of operations on tuples need to be explicitly stated in lemmas and proved, as mentioned above.

We prove 2 more continuity lemmas for vRA Multi-Paxos for the additional actions. The number of uses of continuity lemmas increases by 47 (162%), from 29 to 76, because of the additional actions and the larger number of invariants.

- The number of proofs by induction on set increment increases by 26 (650%), from 4 to 30, the number of proofs by contradiction increases from 1 to 14 (1300%), the number of obligations increases by 3405 (355%), from 959 to 4364, and the proof check time increases by 496 seconds (527%), from 94 seconds to 590 seconds, all due to increased complexity in the specification, more actions, and more invariants.