# Theory of Computational Complexity

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## *Errata*

Page 21, line 4: "$A \in DTIME(c{\cdot}t(n))$" should read "$L(M) \in DTIME(c{\cdot}t(n))$"
Page 47, line 11 from the bottom: "*EXP*" should read "*EXPPOLY*"
Page 56, line 13: Both occurrences of "$x[j]$" should be changed to "$x[k]$"
Page 67, line 24: Delete "at most $3K_T/2$ and, hence,"
Page 87, line 17: Add the following sentence at the end:

> We also let 3-CNF-Tau$_k$ (3-DNF-Tau$_k$) denote the problem of determining whether a given 3-CNF (3-DNF, respectively) formula $F$ satisfies
>
> $$(\forall \tau_1 : X_1 \to \{0,1\})(\exists \tau_2 : X_2 \to \{0,1\} \cdots \\ (Q_{k+1}\tau_k : X_k \to \{0,1\})\ F|_{\tau_1,\tau_2\ldots,\tau_k} = 1,$$
>
> where $Q_k = \exists$ if $k$ is odd, and $Q_k = \forall$ if $k$ is even.

Page 87, Corollary 3.14: The second half of part (a) should read

> "*and* 3-DNF-Tau$_k$ *is* $\leq_m^P$*-complete for* $\Pi_k^P$."

The second half of part (b) should read

> "*and* 3-CNF-Tau$_k$ *is* $\leq_m^P$*-complete for* $\Pi_k^P$."

Page 110, Exercise 3.13: This exercise should be changed to the following:

> **3.13** Prove that the game Geography played on undirected graphs is solvable in polynomial time.

Page 114, line 8 from the bottom: "In particular, $|S_{n-1}| = \varphi(n-1) > 0$" should be replaced by the following paragraph (in the following, $a|b$ denotes that $a$ divides $b$):

> Since, for all $d$ that do not divide $n-1$, we have $S_d = \emptyset$, we get $\sum_{d|(n-1)} |S_d| = |Z_n^*| = n-1$. In the above, we have proved that if $d$ divides $n-1$, then either $S_d = \emptyset$ or $|S_d| = \varphi(d)$. Thus, by Euler's Theorem, for each $d$ that divides $n-1$, we must have $|S_d| = \varphi(d)$, for otherwise $\sum_{d|(n-1)} |S_d|$ would be less than $n-1$. It follows that $|S_{n-1}| = \varphi(n-1) > 0$.

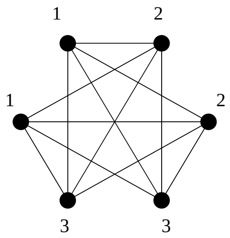
Page 115, line 18: "less than $n-1$" should read "less than or equal to $(n-1)/2$"

Page 121, line 8 from the bottom: "approximation to $f$" should read "approximation to $f^{-1}$"

Page 204, line 13: "$\langle 0, w\rangle \neq B$" should read "$\langle 0, w\rangle \notin B$"

Page 204, line 16: "then $\langle \psi, w\rangle \in B$" should read "then $\langle \phi, w\rangle \in B$"

Page 206, Figure 6.5: The figure at the right-hand side should be changed to




Page 218, line 1: "Each subcircuit $C_i$" should read "Each subcircuit $C_{i+1}$"

Page 239, line 9: "$\mathcal{D} = P^{\mathcal{C}}$" should read "$\mathcal{D} = P^{\mathcal{C}}$ for some class $\mathcal{C}$ that contains $P$ as a subclass"

Page 258, line 22: "there is a co-sparse set" should read "there is no co-sparse set"

Page 258, line 23: "is also em no" should read "is also NO"

Page 291, Lemma 8.5: Add two more items to Lemma 8.5:

*(c) For all* $x, y > 1$ *such that* $\gcd(x, n) = \gcd(y, n) = 1$, $(\frac{x}{n}) \cdot (\frac{y}{n}) = (\frac{xy}{n})$.

*(d) For all odd* $n > 2$, $(\frac{2}{n}) = (-1)^{(n^2-1)/8}$.

Page 291, line 14 from the bottom: "Lemma 8.5(b) above shows" should read "Lemma 8.5 above shows"

Page 306, line 12: "$F|_{x=1} \in \textsc{Sat}$" should read "$F|_{x_1=1} \in \textsc{Sat}$"

Page 333, line 11: "a cover of $H_k$" should read "the path $(u, u_1, v_1, v)$ and a cycle cover of $H_k - \{u, u_1, v_1, v\}$"

Page 421: The first half of the proof of Theorem 11.24 has an error and should be changed as follows:

> Lines 26 to 27: Replace "We assume" by "We assume that $\Phi$ has $k(h+1)^m$ variables and $(h+1)^m$ clauses. In addition, we assume"
>
> Line 27: Replace "where $\Pi$" by "where each $f_i$, $1 \le i \le k$, defines Boolean assignments to $(h+1)^m$ variables, and $\Pi$"
>
> Lines 31 to 33: Replace the two sentences "Then, $V_k$ simulates $V$ ... uses $f_i(\vec{a})$ for it." by "Then, $V_k$ simulates $V$ with the following modification: It uses $\Pi_{k+1}$ as the additional proof, and replaces $\bar{G}^f(\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{v})$ by

$$\bar{G}_1^{f_1,\ldots,f_k}(\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{v}) = \prod_{j=1}^{3}\prod_{i=1}^{k} \bar{\theta}_{ij}(\vec{u}_j, \vec{v})(\bar{\varphi}_j(\vec{v}) - f_i(\vec{u}_j)),$$

where $\theta_{ij}(s,t)$ is equal to 1 if the $s$th variable in the $i$th block is the $j$th variable in $C_t$, and equal to 0 otherwise. Note that the sum check proof system with respect to function $\bar{G}_1^{f_1,\ldots,f_k}$ now requires three values of $f_i(s)$ for $1 \le i \le k$ and $s \in \mathcal{F}^m$."

Page 432, line 6 from the bottom: "and $F$ has" should read "and $F_x$ has"

Page 432, line 5 from the bottom: "total of $d_1 n^c$ clauses)." should read "total of $d_1 n^c$ clauses) if $x \notin L$."

Page 436, line 13: Delete the period at the end of the formula for $S'$.

Page 465, lines 26 to 28: Change this reference item to

Kozen, D. [1977], Lower bounds for natural proof systems, *Proceedings of the 18th IEEE Symposium on Foundations of Computer Science*, IEEE Computer Society Press, Los Angeles, pp. 254–266.

Updated June 30, 2006