Fractional Cascading in Wireless Sensor Networks

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Sensor Networks

• Large number of small devices for environment monitoring
My recent work

- Lightweight, distributed algorithms
  - Routing
  - Network localization
  - Data aggregation/processing
  - Resource management
  - Network hole detection
  - Mobility issues
How much info should a node know about the network?

• Each node knows the entire network
  – Size of $O(n)$, not scalable
• Each node only knows its local neighbors
  – Difficult to achieve global objectives
• **Fractional cascaded information**: more knowledge of nearby nodes, less information of faraway nodes.
  – $O(\log n)$ size information
  – Support near optimal routing schemes.
Problem I: routing

• Find a near shortest path from s to t.
• Suppose each node knows the location of itself, its neighbors and the destination.
• Geographical greedy forwarding: send to a neighbor closer to the destination.
Routing local minimum

• A lot of literature on “how to get out of the local minimum.”
  – Most of them do not guarantee path quality.

With only local information it is difficult to tell which path is shorter.
Routing local minimum

• The geographical distance $|st|$ is not an accurate measure of the graph hop count distance $\sigma(s, t)$.

• Our problem: given an approximate distance oracle, build routing tables s.t. some greedy method achieves $1+\epsilon$ routing.

• Approach: each node stores paths to selected other nodes, called long links.

• Challenge: what long links to choose?
Build long links

• Each node build “long links” to all other nodes.
  – Achieves shortest path routing
  – Routing table size = $\Theta(n)$.

• Goal: select a few long links
  – Achieves 1+\(\varepsilon\) routing.
  – Routing table size is \(o(n)\).
What is next?

• I’ll first explain a local algorithm for routing in the plane.
  – With Euclidean coordinates, how to route to a destination?

• Then extend to the case of sensor networks when
  – The Euclidean distance only approximates the (unknown) hop count distance.
An easier case: routing in the plane

- Choose an intermediate node towards destination $t$
  - $p$ should be closer to $t$.
    \[ |pt| \leq \beta \cdot |st|, \beta \leq 1. \]
  - $p$ should be near the shortest path.
    \[ |sp| + |pt| \leq \gamma \cdot |st|, \gamma \geq 1. \]
Near shortest path routing

• Recursively, we get $1+\varepsilon$ path:

$$P(s, t) \leq (1+\varepsilon) \cdot |st|, \text{ if } (\gamma-1)/(1-\beta) \leq \varepsilon$$

• Only requirement: choose the next hop with the *forwarding region*.

  - $|pt| \leq \beta \cdot |st|, \beta \leq 1.$
  - $|sp| + |pt| \leq \gamma \cdot |st|, \gamma \geq 1.$
Forwarding region

- Near shortest path routing, if the next hop is within the forwarding region.

Boundary is defined by the intersection of a ball: $|pt| \leq \beta |st|$ and an ellipse: $|sp| + |pt| \leq \gamma |st|$

Local routing algorithm achieving a global objective.
Forwarding region properties

• The shape of the forwarding region is scale invariant.

• The larger \( \varepsilon \), the fatter.
In the case of sensor networks

• Define forwarding region in the graph setting.
  – Replace Euclidean distance $|\cdot|$ by graph distance $\sigma$
  – Ball: $\sigma(p, t) \leq \beta \cdot \sigma(s, t)$, $\beta \leq 1$.
  – Ellipse: $\sigma(s, p) + \sigma(p, t) \leq \gamma \cdot \sigma(s, t)$, $\gamma \geq 1$.

• Problem: graph distance $\sigma$ is unknown.
  – Approximated by Euclidean distances.
  – Approximated by landmark-based distances.

[Kleinberg, Slivkins and Wexler, FOCS 04]
Euclidean distance = approximate distance oracle

• Assumption: \( \delta_1 |st| \leq \sigma(s, t) \leq \delta_2 |st| \), \( \delta_1, \delta_2 \) are two positive constants.
  – Local disturbances due to low sensor density
  – Global disturbances due to (fat) network holes
Landmark-based approximate distance oracle

- When Euclidean distances are not available
  - Each node measures hop counts to some landmarks \( L \) in the network
  - Use **triangle inequality** to bound \( \sigma(s, t) \).

\[
\max_j |\sigma(s, l_j) - \sigma(l_j, t)| \leq \sigma(s, t) \leq \min_j \sigma(s, l_j) + \sigma(l_j, t)
\]

Kleinberg et.al. [FOCS 04] proved that with \( O(\log n) \) landmarks this gives \( 1+\varepsilon \) approximation for most pairs.
Forwarding region w/ approximate distance oracle

• Approximate distance $\delta_1 |st| \leq \sigma(s, t) \leq \delta_2 |st|$
  
  – Ball: $\delta_2 |pt| \leq \beta \cdot \delta_1 |st|$
  – Ellipse: $\delta_2 |sp| + \delta_2 |pt| \leq \gamma \cdot \delta_1 |st|$

• Smaller.
• Does not include source s.
Select long links in forwarding regions

- Select long links such that every forwarding region contains one link.

- Intuition:
  - select more links nearby (smaller forwarding regions)
  - and fewer links faraway (larger forwarding regions).
Solution: spatial distribution

- $p$ selects a link $p \rightarrow q$ with probability $\sim 1/|pq|^2$

  Take the largest annulus not including $t$.

  Inside this annulus there is a large ball $B \sim \Theta(|st|)$ inside the forwarding region.

  Geometric packing: Each annulus can be covered by $O(1)$ such balls.

Balls of radius $2^i$ partitions into $O(\log n)$ annuli.
Spatial distribution

• $p$ selects a link $p \rightarrow q$ with probability $\sim \frac{1}{|pq|^2}$

Each annulus has equal probability of getting a link. The ball $B$ has prob $\sim 1/\log n$ of getting a link.

Balls and Bins: Select $O(\log^2 n)$ links such ball $B$ has a long link with prob $\sim 1 - 1/n^2$
Routing path

- Routing path: put the long links together.

The first long link gets closer to \( t \) by a constant fraction.

The path consists of \( O(\log n) \) long links.

The path of \( O(\log n) \) long links exists with prob \( \sim 1 - 1/n \).
Graph setting

• Assumption: the graph has bounded growth rate ≈ sensors are deployed w/ uniform density.
• # nodes of r hops away from u = \( \Theta(r) \)
• # nodes within r hops from u = \( \Theta(r^2) \)
• Similar packing argument holds.
Routing scheme put together

• Preprocessing:
  – Each node selects $O(\log^2 n)$ long links
  – The paths realizing these long links are stored at the routing tables of the nodes on the paths.
  – Randomized scheme, no global coordination.

• Routing:
  – Recursively select a long link in the forwarding region.
  – Local routing algorithm.
Select long links

• Euclidean distance oracle
  – Sample a point with spatial distribution.
  – Round to the closest sensor node.

• Landmark-based distance oracle
  – $O(\log^2 n)$ random landmarks flood the network
  – Each node selects long links on the paths to these landmarks.
Construction of the routing tables

• Compute shortest paths
• Or, bootstrapping process:
  – All nodes first build the short “long links”
  – Use the short links to build longer “long links” (the same with standard routing operation)
Simulations

• Three topologies

• Compare with
  – Virtual Ring Routing (VRR): select long links uniformly randomly. [Caesar, et.al, SIGCOMM 06]
  – S4: stretch-3 landmark-based routing (first route to the landmark closest to t, when getting close, use routing table). [Mao, et.al, NSDI 07]
Our scheme v.s. VRR

- VRR: select randomly “long links”.
- We have smaller routing table, better stretch.
Our scheme v.s. VRR

- VRR: select randomly “long links”.
- We have smaller routing table, better stretch.

On average our “long links” are shorter.

Going “too far” hurts path stretch.
Routing stretch

- Comparable with S4 in stretch, with smaller routing table
- At a cost of small prob of delivery failure (i.e., need flooding)
Summary of Part I

• Fractional cascaded information at each node to help routing.

• Strikes on scalability & locality while achieving globally optimal objectives.

• Part II: fractional cascaded information for data aggregation.
Multi-Resolution Representations

- Sensor nodes start with no knowledge of the global picture.

- Problem: Find aggregates (max, min, avg) for exponentially larger neighborhoods surrounding each node
Multi-resolutional data aggregation

• Each node keeps the aggregated information from all sensors within $2^i$ hops, $i=1, 2, ..., \log n$.
  – Locally relevant picture of the network
  – Data validation (is it an outlier?)
  – Support range queries

• How to construct it?

• Flooding works, with total $\Theta(n^2)$ messages.

• Our solution: use spatial gossip.
Gossip algorithms

• In a round, each node:
  – Selects another node randomly
  – Exchanges information via multi-hop routing
  – Repeats every round

• Simple
• Distributed
• Robust to link dynamics, transmission errors
Types of Gossip

• **Uniform/Geographic gossip**
  
  – Select a node q \textit{uniformly randomly} and gossip

  [Dimakis, Sarwate, Wainwright, IPSN 06]

• **Spatial Gossip**

  – Select a node q at distance r with probability \(1/r^\alpha\).

  [Kempe, Kleinberg, Demers, STOC 01]
Communication Cost

- Uniform/Geographic gossip
  - Cost per step ~ $O(n\sqrt{n})$
  - # rounds for a message to reach everyone ~ $O(\log n)$

- Spatial Gossip
  - $\text{Prob} = \frac{1}{r^2}$, cost per step ~ $O(n\sqrt{n})$
  - $\text{Prob} = \frac{1}{r^3}$, cost per step ~ $O(n\log n)$
  - # rounds for a message to reach everyone ~ $O(\log n)$
Spatial Gossip

Expanding Neighborhood

Useful for multi-resolution aggregates
Using spatial gossip for aggregation

- Total $\log n$ phases
- In phase $i$ we build aggregates for neighborhood of size $2^i$ around each node

$$V(D) = v(D) + v(B) + v(A) + v(C)$$

- $v(B) = v(B) + v(A)$
- $v(C) = v(C)$
- $v(A) = v(A)$
- $1/r^3$
Double counting?

\[ v(B) = v(B) + v(A) \]

\[ v(C) = v(C) + v(A) \]

\[ v(D) = v(D) + v(B) + v(C) + v(A) + v(A) \]
Use Order and Duplicate Insensitive Aggregation

\[ V(B) = v(B) + v(A) \]
\[ V(C) = v(C) + v(A) \]
\[ V(D) = v(D) + v(B) + v(C) + v(A) + v(A) = v(D) + v(B) + v(C) + v(A) \]
Order and Duplicate Insensitive Synopses

• The final aggregation does not change if a data value is aggregated multiple times.
• Min, Max are natural ODI aggregates
• ODI Synopses exist for other aggregates like sum, average, count..
  – Use probabilistic counting

Ref : Nath, Gibbons, Seshan, Anderson, SenSys 04
Considine, Kollios, Byers, ICDE 04
Data Distribution in Phases

Phases 1 to 6
Property 1 of Algorithm: Information Spreads Fast

- In phase $i$, $i=1, 2, ..., \log n$, with $O(i^{3.4})$ rounds, information spreads to every node within $2^i$ distance, with high probability.

Ref: Kempe, Kleinberg, Demers STOC 01
Property 2: Information Does Not Spread Too Far

• In phase $i$, $i=1, 2, \ldots, \log n$, with $O(i^{3.4})$ rounds, information does not spread beyond distance $O(i^{3.4}2^i)$ for sure, and

• Does not spread beyond $O(i^{2.4}2^i)$ with high probability
Overall Efficiency

• $O(\log^{4.4} n)$ rounds.

• Total communication cost $O(n \log^{5.4} n)$.

• $O(\log n)$ aggregates per node simultaneously.
The Price of Accurate Computation

• Our aggregates are on fuzzy neighborhoods.
• Sharp multi-resolution aggregate computation requires a high communication cost of $\Omega(n\sqrt{n})$

Reduce the communication cost substantially by sacrificing a little on accuracy!
Range Queries

- User supplies a region, and asks for aggregate

We pick a suitable node

And a suitable resolution level to cover the region
Range Queries

• Complex region
• Total communication cost \( \sim \) perimeter
• Cost of flooding \( \sim \) area of the region
Summary

• Information is most relevant in the spatial-temporal locale.

• Fractional cascading principle: diffuse information in a way cascaded with distances.

• Develop near optimal algorithms with low communication cost in a resource constrained network.
Questions or comments?

- Joint work with my students Rik Sarkar and Xianjin Zhu.
- Spatial distributions in routing table design for sensor networks, INFOCOM’09, mini-conference.
- Hierarchical spatial gossip for multi-resolution Representations in Sensor Networks, IPSN’07.
- Available at http://www.cs.sunysb.edu/~jgao