Supplementary Material: Transfer Learning Based Visual Tracking with Gaussian Process Regression

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Abstract. In this supplementary document, we provide proofs of Proposition 1 (Section 2) and Proposition 2 (Section 3), and plots for the **OPE** performance of the top ten trackers on the attribute subsets of the CVPR2013 Visual Tracker Benchmark (Section 4). Firstly, we begin with an introduction of partitioned matrix inversion theorem in Section 1, which is crucial to the proofs.

1 Partitioned Matrix Inversion Theorem

Recall in the main paper that, $\mathbf{G}_{\text{all}} = \begin{pmatrix} \mathbf{G}_{LL} & \mathbf{G}_{LU} \\ \mathbf{G}_{UL} & \mathbf{G}_{UU} \end{pmatrix}$ and $\mathbf{G}_{\text{all}}^{-1} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathrm{T}} & \mathbf{M} \end{pmatrix}$ are the $(n_L+n_U)\times(n_L+n_U)$ Gram matrix (symmetric, non-singular) and its inverse of all the training (auxiliary and target) samples and unlabeled samples. Actually, $\mathbf{G}_{\text{all}}$ and $\mathbf{G}_{\text{all}}^{-1}$ are partitioned in different ways. $\mathbf{G}_{LL}$ and $\mathbf{G}_{UU}$ are the $n_L \times n_L$ and $n_U \times n_U$ matrices respectively; while $\mathbf{A}$ and $\mathbf{M}$ are the $n_T \times n_T$ and $(n_A + n_U) \times (n_A + n_U)$ matrices respectively. For the convenience of the using of partitioned matrix inversion theorem in Proposition 1 and Proposition 2 respectively, we additionally use two different ways to partition $\mathbf{G}_{\text{all}}$ and $\mathbf{G}_{\text{all}}^{-1}$.

As for Proposition 1, let $\mathbf{G}_{\text{all}} = \begin{pmatrix} \mathbf{G}_{TT} & \mathbf{G}_{TZ} \\ \mathbf{G}_{ZT} & \mathbf{G}_{ZZ} \end{pmatrix}$, where $\mathbf{G}_{TT}$ and $\mathbf{G}_{ZZ}$ are the $n_T \times n_T$ and $(n_A+n_U)\times(n_A+n_U)$ matrices respectively. From partitioned matrix inversion theorem,

$$\mathbf{M} = \left(\mathbf{G}_{ZZ} - \mathbf{G}_{ZT}\mathbf{G}_{TT}^{-1}\mathbf{G}_{TZ}\right)^{-1} , \tag{1}$$

$$\mathbf{B} = -\mathbf{G}_{TT}^{-1}\mathbf{G}_{TZ}\mathbf{M} , \tag{2}$$

where $\left(\mathbf{M}^{-1}\right)^{\mathrm{T}} = \mathbf{M}^{-1}$.

As for Proposition 2, let $\mathbf{G}_{\text{all}}^{-1} = \begin{pmatrix} \mathbf{A}_L & \mathbf{B}_L \\ \mathbf{B}_L^{\mathrm{T}} & \mathbf{M}_L \end{pmatrix}$, where $\mathbf{A}_L$ and $\mathbf{M}_L$ are the $n_L \times n_L$ and $n_U \times n_U$ matrices respectively. From partitioned matrix inversion theorem,

$$\mathbf{M}_L = \left(\mathbf{G}_{UU} - \mathbf{G}_{UL}\mathbf{G}_{LL}^{-1}\mathbf{G}_{LU}\right)^{-1} , \tag{3}$$

$$\mathbf{B}_L = -\mathbf{G}_{LL}^{-1}\mathbf{G}_{LU}\mathbf{M}_L , \tag{4}$$

$$\mathbf{A}_L = \mathbf{G}_{LL}^{-1} + \mathbf{B}_L\mathbf{M}_L^{-1}\mathbf{B}_L^{\mathrm{T}} . \tag{5}$$

2 Proof of Proposition 1

Proposition 1 *By defining the prior Gram matrix $\boldsymbol{G}_{\text{all}}$ over all the training and unlabeled samples, we can hence determine $\boldsymbol{\mu}$ and $\boldsymbol{G}$ in Eq. (8) of the main paper for our GPR based observation model inference as follows: $\boldsymbol{\mu} = -\boldsymbol{M}^{-1}\boldsymbol{B}^{\mathrm{T}}\mathbf{y}_T$, $\boldsymbol{G} = \boldsymbol{M}^{-1}$.*

Proof. In Eq. (10) of the main paper,

$$\begin{aligned} Q_2(\mathbf{z}_A, \mathbf{z}_U) &= -\frac{1}{2}\left(\ln(2\pi)^{n_A+n_U} + \ln|\mathbf{G}| + (\mathbf{z}-\boldsymbol{\mu})^{\mathrm{T}}\,\mathbf{G}^{-1}\,(\mathbf{z}-\boldsymbol{\mu})\right) \\ &= -\frac{1}{2}\left(\ln|\mathbf{G}_{\text{all}}| + \left(\mathbf{y}_T^{\mathrm{T}}\ \mathbf{z}^{\mathrm{T}}\right)\mathbf{G}_{\text{all}}^{-1}\begin{pmatrix}\mathbf{y}_T \\ \mathbf{z}\end{pmatrix}\right) + c_1 \ , \end{aligned} \tag{6}$$

where $\mathbf{z} = \begin{pmatrix}\mathbf{z}_A \\ \mathbf{z}_U\end{pmatrix}$ and $\mathbf{z}^{\mathrm{T}} = \left(\mathbf{z}_A^{\mathrm{T}}\ \mathbf{z}_U^{\mathrm{T}}\right)$. Recall $\mathbf{G}_{\text{all}}^{-1} = \begin{pmatrix}\mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathrm{T}} & \mathbf{M}\end{pmatrix}$ in the main paper, then

$$\left(\mathbf{y}_T^{\mathrm{T}}\ \mathbf{z}^{\mathrm{T}}\right)\mathbf{G}_{\text{all}}^{-1}\begin{pmatrix}\mathbf{y}_T \\ \mathbf{z}\end{pmatrix} = \mathbf{y}_T^{\mathrm{T}}\mathbf{A}\mathbf{y}_T + \mathbf{z}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{y}_T + \mathbf{y}_T^{\mathrm{T}}\mathbf{B}\mathbf{z} + \mathbf{z}^{\mathrm{T}}\mathbf{M}\mathbf{z}. \tag{7}$$

Because

$$(\mathbf{z}-\boldsymbol{\mu})^{\mathrm{T}}\,\mathbf{G}^{-1}\,(\mathbf{z}-\boldsymbol{\mu}) = \boldsymbol{\mu}^{\mathrm{T}}\mathbf{G}^{-1}\boldsymbol{\mu} - \boldsymbol{\mu}^{\mathrm{T}}\mathbf{G}^{-1}\mathbf{z} - \mathbf{z}^{\mathrm{T}}\mathbf{G}^{-1}\boldsymbol{\mu} + \mathbf{z}^{\mathrm{T}}\mathbf{G}^{-1}\mathbf{z} \ , \tag{8}$$

when we set $\boldsymbol{\mu} = -\mathbf{M}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{y}_T$ and $\mathbf{G} = \mathbf{M}^{-1}$,

$$\begin{aligned} & Q_2(\mathbf{z}_A, \mathbf{z}_U) = -\frac{1}{2}\left(\ln(2\pi)^{n_A+n_U} + \ln|\mathbf{G}| + (\mathbf{z}-\boldsymbol{\mu})^{\mathrm{T}}\,\mathbf{G}^{-1}\,(\mathbf{z}-\boldsymbol{\mu})\right) \\ &= -\frac{1}{2}\left(\ln|\mathbf{G}_{\text{all}}| - \boldsymbol{\mu}^{\mathrm{T}}\mathbf{G}^{-1}\mathbf{z} - \mathbf{z}^{\mathrm{T}}\mathbf{G}^{-1}\boldsymbol{\mu} + \mathbf{z}^{\mathrm{T}}\mathbf{G}^{-1}\mathbf{z} + \boldsymbol{\mu}^{\mathrm{T}}\mathbf{G}^{-1}\boldsymbol{\mu} + \ln|\mathbf{G}| - \ln|\mathbf{G}_{\text{all}}| + \ln(2\pi)^{n_A+n_U}\right) \\ &= -\frac{1}{2}\left(\ln|\mathbf{G}_{\text{all}}| + \mathbf{y}_T^{\mathrm{T}}\mathbf{B}\mathbf{z} + \mathbf{z}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{y}_T + \mathbf{z}^{\mathrm{T}}\mathbf{M}\mathbf{z} + \mathbf{y}_T^{\mathrm{T}}\mathbf{B}\mathbf{M}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{y}_T + \ln|\mathbf{G}| - \ln|\mathbf{G}_{\text{all}}| + \ln(2\pi)^{n_A+n_U}\right) \\ &= -\frac{1}{2}\left(\ln|\mathbf{G}_{\text{all}}| + \mathbf{y}_T^{\mathrm{T}}\mathbf{A}\mathbf{y}_T + \mathbf{z}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{y}_T + \mathbf{y}_T^{\mathrm{T}}\mathbf{B}\mathbf{z} + \mathbf{z}^{\mathrm{T}}\mathbf{M}\mathbf{z}\right) + c_1 \\ &= -\frac{1}{2}\left(\ln|\mathbf{G}_{\text{all}}| + \left(\mathbf{y}_T^{\mathrm{T}}\ \mathbf{z}^{\mathrm{T}}\right)\mathbf{G}_{\text{all}}^{-1}\begin{pmatrix}\mathbf{y}_T \\ \mathbf{z}\end{pmatrix}\right) + c_1 \ , \end{aligned} \tag{9}$$

where $c_1 = -\frac{1}{2}\left(\mathbf{y}_T^{\mathrm{T}}(\mathbf{B}\mathbf{M}^{-1}\mathbf{B}^{\mathrm{T}} - \mathbf{A})\mathbf{y}_T + \ln|\mathbf{G}| - \ln|\mathbf{G}_{\text{all}}| + \ln(2\pi)^{n_A+n_U}\right)$. □

3 Proof of Proposition 2

Proposition 2 *The optimal value $\hat{z_A}$ is formally given by:*

$$\begin{aligned} \hat{z_A} &= \underset{\mathbf{z}_A\in\mathbb{R}^{n_A}}{\arg\max}\, Q_1 + Q_2 \\ &= \underset{\mathbf{z}_A\in\mathbb{R}^{n_A}}{\arg\max} \sum_{j=n_T+1}^{n_L} \ln\left(\mathrm{P_r}\left(y_i|z_i\right)\right) - \frac{1}{2}\left(y_T^{\top}\ z_A^{\top}\right)\boldsymbol{G}_{LL}^{-1}\begin{pmatrix}y_T \\ z_A\end{pmatrix} + c_2 \ , \end{aligned} \tag{10}$$

where $Q_1(\mathbf{z}_A) = \sum_{j=n_T+1}^{n_L} \ln\left(\mathrm{P_r}\left(y_i|z_i\right)\right)$ and $c_2 = c_1 - \frac{1}{2}\ln|\boldsymbol{G}_{\text{all}}|$.

Proof. As for Q_1, recall $Q_1(\mathbf{z}_A) = \ln\left(\mathrm{P_r}\left(\mathbf{y}_A|\mathbf{z}_A\right)\right)$ in Eq. (9) of the main paper, then

$$\begin{aligned} Q_1(\mathbf{z}_A) &= \ln\left(\mathrm{P_r}\left(\mathbf{y}_A|\mathbf{z}_A\right)\right) \\ &= \ln\left(\prod_{j=n_T+1}^{n_L} \mathrm{P_r}\left(y_i|z_i\right)\right) \\ &= \sum_{j=n_T+1}^{n_L} \ln\left(\mathrm{P_r}\left(y_i|z_i\right)\right) \ . \end{aligned} \tag{11}$$

As for Q_2, recall Eq. (11) of the main paper

$$\begin{aligned} Q_2(\mathbf{z}_A, \mathbf{z}_U) &= -\frac{1}{2}\left(\ln(2\pi)^{n_A+n_U} + \ln|\mathbf{G}| + (\mathbf{z}-\boldsymbol{\mu})^{\mathrm{T}}\,\mathbf{G}^{-1}\,(\mathbf{z}-\boldsymbol{\mu})\right) \\ &= -\frac{1}{2}\left(\ln|\mathbf{G}_{\mathrm{all}}| + \left(\mathbf{y}^{\mathrm{T}}\ \mathbf{z}_U^{\mathrm{T}}\right)\mathbf{G}_{\mathrm{all}}^{-1}\begin{pmatrix}\mathbf{y}\\ \mathbf{z}_U\end{pmatrix}\right) + c_1 \ , \end{aligned} \tag{12}$$

where $\mathbf{y} = \begin{pmatrix}\mathbf{y}_T\\ \mathbf{z}_A\end{pmatrix}$ and $\mathbf{y}^{\mathrm{T}} = \left(\mathbf{y}_T^{\mathrm{T}}\ \mathbf{z}_A^{\mathrm{T}}\right)$. Recall Eq. (12) of the main paper, let

$$\mathbf{z}_U = \mathbf{G}_{UL}\mathbf{G}_{LL}^{-1}\begin{pmatrix}\mathbf{y}_T\\ \mathbf{z}_A\end{pmatrix} = \mathbf{G}_{UL}\mathbf{G}_{LL}^{-1}\mathbf{y} \ . \tag{13}$$

Then

$$\begin{aligned} &\left(\mathbf{y}^{\mathrm{T}}\ \mathbf{z}_U^{\mathrm{T}}\right)\mathbf{G}_{\mathrm{all}}^{-1}\begin{pmatrix}\mathbf{y}\\ \mathbf{z}_U\end{pmatrix} = \mathbf{y}^{\mathrm{T}}\mathbf{A}_L\mathbf{y} + \mathbf{z}_U^{\mathrm{T}}\mathbf{B}_L^{\mathrm{T}}\mathbf{y} + \mathbf{y}^{\mathrm{T}}\mathbf{B}_L\mathbf{z}_U + \mathbf{z}_U^{\mathrm{T}}\mathbf{M}_L\mathbf{z}_U \\ &= \mathbf{y}^{\mathrm{T}}\left(\mathbf{G}_{LL}^{-1} + \mathbf{B}_L\mathbf{M}_L^{-1}\mathbf{B}_L^{\mathrm{T}}\right)\mathbf{y} - 2\mathbf{y}^{\mathrm{T}}\mathbf{G}_{LL}^{-1}\mathbf{G}_{LU}\mathbf{M}_L\mathbf{G}_{UL}\mathbf{G}_{LL}^{-1}\mathbf{y} + \mathbf{y}^{\mathrm{T}}\mathbf{G}_{LL}^{-1}\mathbf{G}_{LU}\mathbf{M}_L\mathbf{G}_{UL}\mathbf{G}_{LL}^{-1}\mathbf{y} \\ &= \mathbf{y}^{\mathrm{T}}\left(\mathbf{G}_{LL}^{-1} + \mathbf{B}_L\mathbf{M}_L^{-1}\mathbf{B}_L^{\mathrm{T}}\right)\mathbf{y} - \mathbf{y}^{\mathrm{T}}\mathbf{G}_{LL}^{-1}\mathbf{G}_{LU}\mathbf{M}_L\mathbf{G}_{UL}\mathbf{G}_{LL}^{-1}\mathbf{y} \\ &= \mathbf{y}^{\mathrm{T}}\mathbf{G}_{LL}^{-1}\mathbf{y} + \mathbf{y}^{\mathrm{T}}\mathbf{B}_L\mathbf{M}_L^{-1}\mathbf{B}_L^{\mathrm{T}}\mathbf{y} - \mathbf{y}^{\mathrm{T}}\left(\mathbf{G}_{LL}^{-1}\mathbf{G}_{LU}\mathbf{M}_L\right)\mathbf{M}_L^{-1}\left(\mathbf{G}_{LL}^{-1}\mathbf{G}_{LU}\mathbf{M}_L\right)^{\mathrm{T}}\mathbf{y} \\ &= \mathbf{y}^{\mathrm{T}}\mathbf{G}_{LL}^{-1}\mathbf{y} + \mathbf{y}^{\mathrm{T}}\mathbf{B}_L\mathbf{M}_L^{-1}\mathbf{B}_L^{\mathrm{T}}\mathbf{y} - \mathbf{y}^{\mathrm{T}}\mathbf{B}_L\mathbf{M}_L^{-1}\mathbf{B}_L^{\mathrm{T}}\mathbf{y} \\ &= \mathbf{y}^{\mathrm{T}}\mathbf{G}_{LL}^{-1}\mathbf{y} \\ &= \left(\mathbf{y}_T^{\mathrm{T}}\ \mathbf{z}_A^{\mathrm{T}}\right)\mathbf{G}_{LL}^{-1}\begin{pmatrix}\mathbf{y}_T\\ \mathbf{z}_A\end{pmatrix} \ . \end{aligned} \tag{14}$$

So Q_2 can be written as

$$\begin{aligned} Q_2(\mathbf{z}_A, \mathbf{z}_U) &= -\frac{1}{2}\left(\mathbf{y}_T^{\mathrm{T}}\ \mathbf{z}_A^{\mathrm{T}}\right)\mathbf{G}_{LL}^{-1}\begin{pmatrix}\mathbf{y}_T\\ \mathbf{z}_A\end{pmatrix} + c_1 - \frac{1}{2}\ln|\mathbf{G}_{\mathrm{all}}| \\ &= -\frac{1}{2}\left(\mathbf{y}_T^{\mathrm{T}}\ \mathbf{z}_A^{\mathrm{T}}\right)\mathbf{G}_{LL}^{-1}\begin{pmatrix}\mathbf{y}_T\\ \mathbf{z}_A\end{pmatrix} + c_2 \ , \end{aligned} \tag{15}$$

where $c_2 = c_1 - \frac{1}{2}\ln|\mathbf{G}_{\mathrm{all}}|$. $\square$

4 Attribute-based OPE Performance on the CVPR2013 Visual Tracker Benchmark

The plots for the **OPE** performance of the top ten trackers on the attribute subsets are shown from Figure 1 to Figure 3. From these figures, we can see that our proposed new tracker TGPR outperforms the state-of-the-arts in most attribute subsets. In the fast motion subset, low resolution subset, scale variation subset and out of view subset, TGPR also achieves comparable tracking results with the state-of-the-arts.

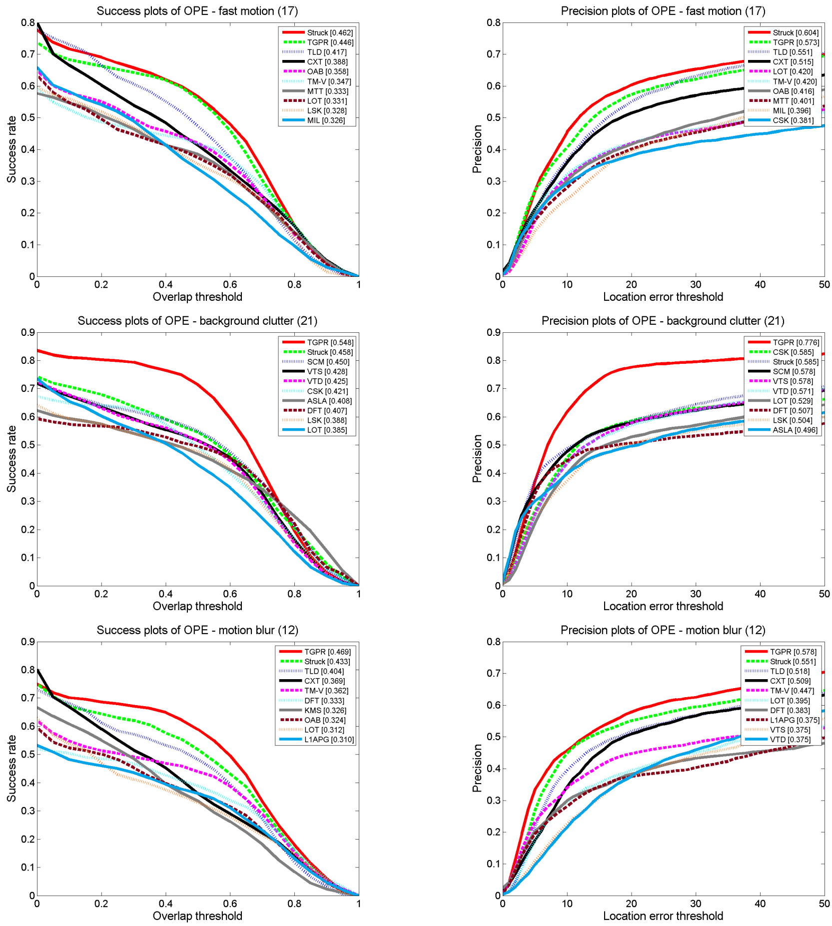

Fig. 1: The value appears in the title is the number of sequences in that sub-dataset

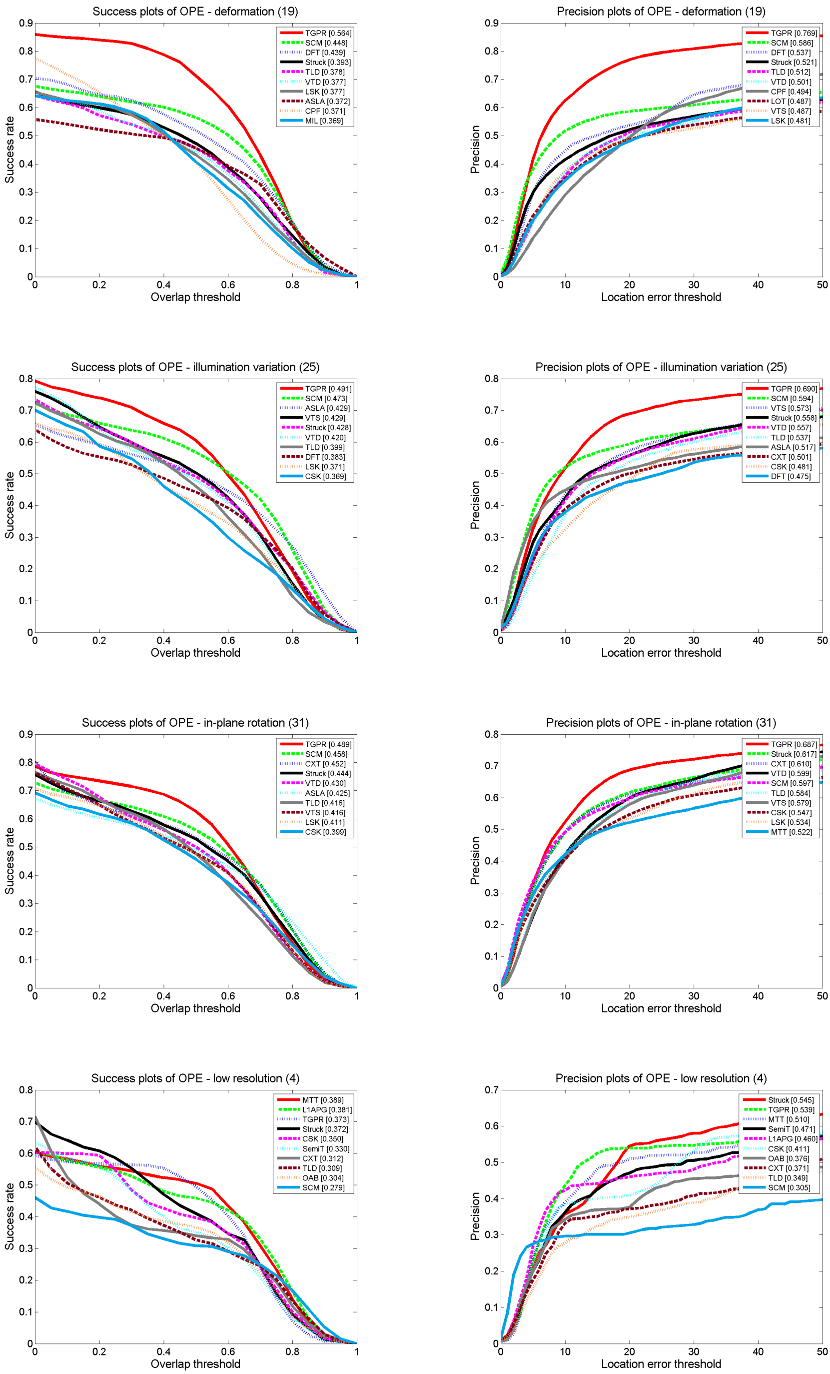

Fig. 2: The value appears in the title is the number of sequences in that sub-dataset

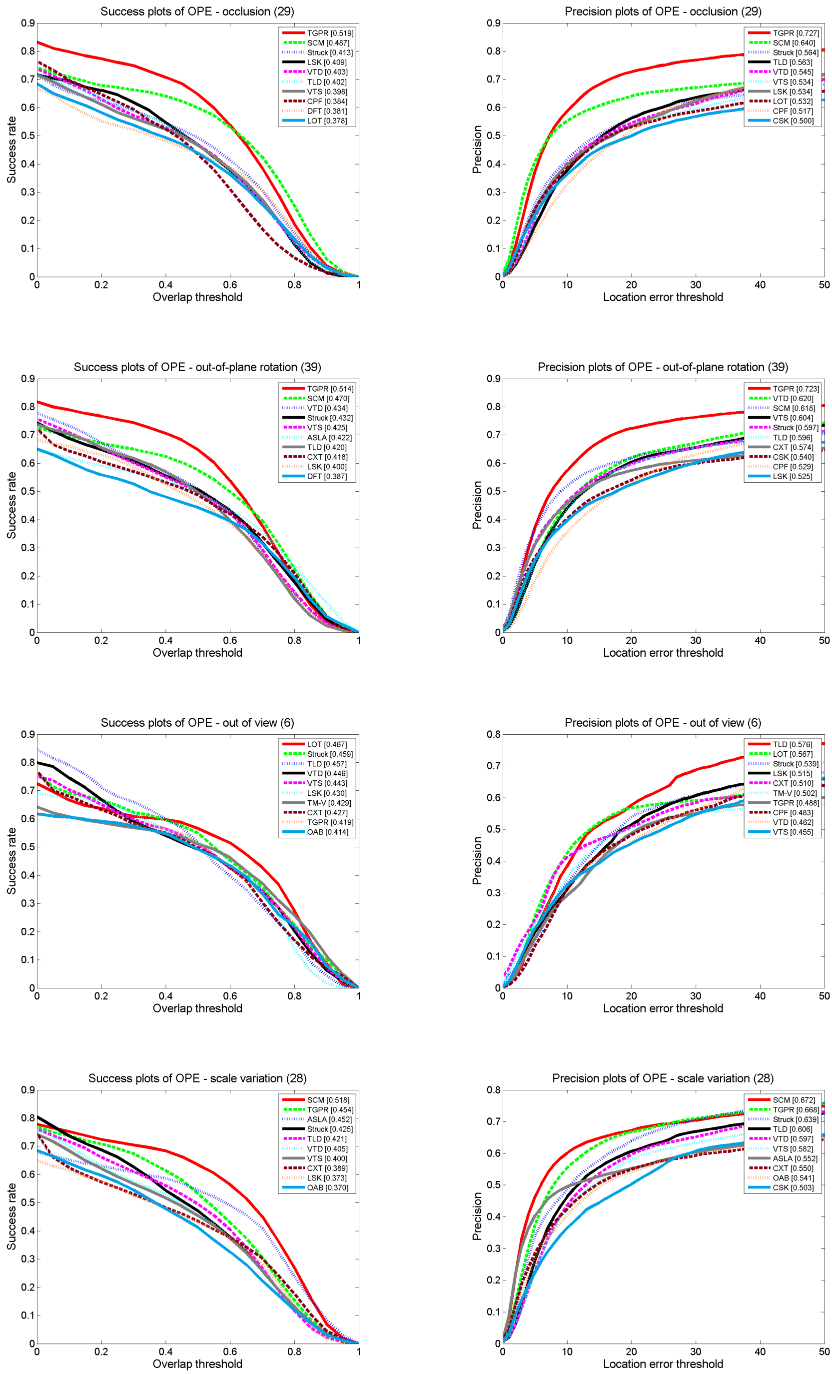

Fig. 3: The value appears in the title is the number of sequences in that sub-dataset