Language Modeling

CSE392 - Spring 2019
Special Topic in CS
Task

- Language Modeling (auto-complete)
  how?
- Probabilistic Modeling
  - ML: Logistic Regression
  - Probability Theory
Language Modeling

-- assigning a probability to sequences of words.

Version 1: Compute $P(w_1, w_2, w_3, w_4, w_5) = P(W)$
:probability of a sequence of words
Language Modeling

-- assigning a probability to sequences of words.

**Version 1:** Compute \( P(w_1, w_2, w_3, w_4, w_5) = P(W) \)
:probability of a sequence of words

**Version 2:** Compute \( P(w_5| w_1, w_2, w_3, w_4) \)
\( = P(w_n| w_1, w_2, ..., w_{n-1}) \)
:probability of a next word given history
Language Modeling

Version 1: Compute $P(w_1, w_2, w_3, w_4, w_5) = P(W)$
  :probability of a sequence of words
  $P(\text{He ate the cake with the fork}) = ?$

Version 2: Compute $P(w_5| w_1, w_2, w_3, w_4)$
  $= P(w_n| w_1, w_2, ..., w_{n-1})$
  :probability of a next word given history
  $P(\text{fork} | \text{He ate the cake with the}) = ?$
Language Modeling

Applications:

- Auto-complete: What word is next?
- Machine Translation: Which translation is most likely?
- Spell Correction: Which word is most likely given error?
- Speech Recognition: What did they just say? “eyes aw of an” (example from Jurafsky, 2017)
Language Modeling

Version 1: Compute $P(w_1, w_2, w_3, w_4, w_5) = P(W)$
- probability of a sequence of words

$P(\text{He ate the cake with the fork}) = ?$

Version 2: Compute $P(w_5 \mid w_1, w_2, w_3, w_4)$

$= P(w_n \mid w_1, w_2, \ldots, w_{n-1})$

- probability of a next word given history

$P(\text{fork} \mid \text{He ate the cake with the}) = ?$
Simple Solution

Version 1: Compute $P(w_1, w_2, w_3, w_4, w_5) = P(W)$

:probability of a sequence of words

$P(\text{He ate the cake with the fork}) = \frac{\text{count(He ate the cake with the fork)}}{\text{count(* * * * * * * *)}}$
Simple Solution: The Maximum Likelihood Estimate

Version 1: Compute $P(w_1, w_2, w_3, w_4, w_5) = P(W)$: probability of a sequence of words

$P(He \text{ ate the cake with the fork}) = \frac{\text{count}(He \text{ ate the cake with the fork})}{\text{count}(\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_)}$
Simple Solution: The Maximum Likelihood Estimate

\[ P(\text{He ate the cake with the fork}) = \frac{\text{count(He ate the cake with the fork)}}{\text{count( * * * * * * * *)}} \]

\[ P(\text{fork } | \text{ He ate the cake with the}) = \frac{\text{count(He ate the cake with the fork)}}{\text{count(He at the cake with the)}} \]
Simple Solution: The Maximum Likelihood Estimate

**Problem:** even the Web isn’t large enough to enable good estimates of most phrases.

\[
P(\text{He ate the cake with the fork}) = \frac{\text{count}(\text{He ate the cake with the fork})}{\text{count}(\text{* * * * * * * * * * * *})}
\]

\[
P(\text{fork} \mid \text{He ate the cake with the}) = \frac{\text{count}(\text{He ate the cake with the fork})}{\text{count}(\text{He at the cake with the})}
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**Problem:** even the Web isn’t large enough to enable good estimates of most phrases.

**Solution:** Estimate from shorter sequences, use more sophisticated probability theory.
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\[ P(B|A) = \frac{P(B, A)}{P(A)} \iff P(A)P(B|A) = P(B, A) = P(A, B) \]

Example from (Jurafsky, 2017)
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**The Chain Rule:**

\[
P(X_1, X_2..., X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2)...P(X_n|X_1, ..., X_{n-1})
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\[
P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i|X_1, X_2, ..., X_i)
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\]

Markov Assumption: \[
P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i|X_{i-k}, X_{i-(k-1)}, \ldots, X_i)
\]

\[
P(X_n|X_1\ldots, X_{n-1}) \approx P(X_n|X_{n-k}, \ldots, X_{n-1}) \text{ where } k < n
\]
**Solution:** Estimate from shorter sequences, use more sophisticated probability theory.

**Problem:** Even the Web isn't large enough to enable good estimates of most phrases.

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P(B|A) = \frac{P(B, A)}{P(A)}
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Markov Assumption:

\[
P(X_n|X_1, \ldots, X_{n-1}) \approx P(X_n|X_{n-k}, \ldots, X_{n-1}) \quad \text{where} \quad k < n
\]

What about Logistic Regression? \(Y = \text{next word}\)

\[
P(Y|X) = P(X_n | X_1, X_2, X_3, \ldots)
\]

Not a terrible option, but \(X_1\) through \(X_{n-1}\) would be modeled as independent dimensions. Let's revisit later.
**Markov Assumption:**

\[ P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_{i-k}, X_{i-(k-1)}, \ldots, X_i) \]

\[ P(X_n | X_1, \ldots, X_{n-1}) \approx P(X_n | X_{n-k}, \ldots, X_{n-1}) \quad \text{where } k < n \]

**Unigram Model:** \( k = 0; \)

\[ P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i) \]

**Problem:**

Even the Web isn't large enough to enable good estimates of most phrases.

\[ P(B | A) = \frac{P(B, A)}{P(A)} \iff P(A)P(B | A) = P(B, A) = P(A, B) \]

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**The Chain Rule:**

\[ P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_1, X_2, \ldots, X_i) \]

\[ P(X_1, X_2 \ldots, X_n) = P(X_1)P(X_2 | X_1)P(X_3 | X_1, X_2) \ldots P(X_n | X_1, \ldots, X_{n-1}) \]
Markov Assumption: $P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i|X_{i-k}, X_{i-(k-1)}, ..., X_i)$

$P(X_n|X_1..., X_{n-1}) \approx P(X_n|X_{n-k}, ..., X_{n-1})$ where $k < n$

Bigram Model: $k = 1$;

$$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i|X_{i-1})$$

Example generated sentence:

outside, new, car, parking, lot, of, the, agreement, reached

$P(X_1 = \text{"outside"}, X_2 = \text{"new"}, X_3 = \text{"car"}, ....) \approx P(X_2 = \text{"new"}|X_1 = \text{"outside"}) \times P(X_3 = \text{"car"} | X_2 = \text{"new"}) \times ...$

Example from (Jurafsky, 2017)
Language Modeling

Building a model (or system / API) that can answer the following:

- A sequence of natural language
- How common is this sequence?
- What is the next word in the sequence?
Language Modeling

Building a model (or system / API) that can answer the following:

- *a sequence of natural language*
- Language Model
- How to build?
- How common is this sequence?
- What is the next word in the sequence?
Language Modeling

Building a model (or system / API) that can answer the following:

- a sequence of natural language
- How common is this sequence?
- What is the next word in the sequence?

How to build?

Training Corpus

training (fit, learn)
Language Modeling

Building a model (or system / API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

Inputs:
- A sequence of natural language
- Training Corpus

Process:
- Training (fit, learn)
Language Modeling

Building a model (or system / API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

**Training Corpus**

*example from (Jurafsky, 2017)*

<table>
<thead>
<tr>
<th>first word</th>
<th>i</th>
<th>want</th>
<th>to</th>
<th>eat</th>
<th>chinese</th>
<th>food</th>
<th>lunch</th>
<th>spend</th>
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<tbody>
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Language Modeling

Building a model (or system / API) that can answer the following:

- How common is this sequence?
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### Training Corpus

**Bigram Counts**

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**Training (fit, learn)**

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**Bigram model:**

\[
P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_{i-1})
\]

**Need to estimate:**

\[
P(X_i | X_{i-1}) = \frac{\text{count}(X_{i-1} X_i)}{\text{count}(X_{i-1})}
\]
## Language Modeling

Building a model (or system/API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

### Training Corpus

Training (fit, learn) first word ($X_{i-1}$) and second word ($X_i$) with

$$P(X_i | X_{i-1}) = \frac{\text{count}(X_{i-1} X_i)}{\text{count}(X_i)}$$

**Example from (Jurafsky, 2017)**

#### Bigram model:

$$P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_{i-1})$$

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- A sequence of natural language

**Language Model**

- How common is this sequence?
- What is the next word in the sequence?

**Training Corpus**

Training (fit, learn)

First word ($X_{i-1}$)  

<table>
<thead>
<tr>
<th>first word ($X_{i-1}$)</th>
<th>i</th>
<th>want</th>
<th>to</th>
<th>eat</th>
<th>chinese</th>
<th>food</th>
<th>lunch</th>
<th>spend</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>0.002</td>
<td>0.33</td>
<td>0</td>
<td>0.0036</td>
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<td>0.0022</td>
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<td>0.0011</td>
<td>0.0065</td>
<td>0.0065</td>
<td>0.0054</td>
<td>0.0011</td>
</tr>
</tbody>
</table>


**Example from (Jurafsky, 2017)**

**Bigram model:**

Need to estimate:  

$$P(X_i \mid X_{i-1}) = \frac{\text{count}(X_{i-1} X_i)}{\text{count}(X_i)}$$
Language Modeling

Building a model (or system/API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

Training Corpus

training (fit, learn)

Example from (Jurafsky, 2017)

Bigram model:

\[ P(X_i | X_{i-1}) = \frac{\text{count}(X_{i-1} X_i)}{\text{count}(X_{i-1})} \]

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<thead>
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<tr>
<td>to</td>
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<td>0.0017</td>
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<td>0.00083</td>
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<td>0.0025</td>
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<td>0.00092</td>
<td>0.0037</td>
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</tbody>
</table>
Language Modeling

Building a model (or system / API) that can answer the following:

- a sequence of natural language
- Trained Language Model
- How common is this sequence?
- What is the next word in the sequence?

Training Corpus

training (fit, learn)
Language Modeling

Building a model (or system / API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

Train the model (fit, learn) using a training corpus.

Example:

- Food

Diagram:

- Training Corpus
- Trained Language Model
- How common is this sequence?
- What is the next word in the sequence?

Graph:

- Possible sequences and their probabilities.
Language Modeling

Building a model (or system / API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

```
a sequence of natural language
```

```
Training Corpus
```

```
Test?
```

```
(fit, learn)
```
Language Modeling

Building a model (or system/API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

Training Corpus

Test Corpus

Test:
Feed the model $X_1...X_{i-1}$ and see how well it predicts $X_i$. 

Trained Language Model
Language Modeling

Building a model (or system / API) that can answer the following:

- How common is this sequence?
- What is the next word in the sequence?

**Training Corpus**

**Test Corpus**

**Test:**
Feed the model $X_1...X_{i-1}$ and see how well it predicts $X_i$.

**Perplexity**

$$PP(W) = \frac{1}{N} \log \frac{1}{P(w_1w_2...w_N)}$$
Evaluation

Test Corpus → Trained Language Model → What is the next word in the sequence?

Perplexity

\[ PP(W) = \frac{1}{P(w_1w_2...w_N)^\frac{1}{N}} \]

\[ = \sqrt[N]{\frac{1}{P(w_1w_2...w_N)}} \]
Evaluation

What is the next word in the sequence?

Test Corpus → Trained Language Model

Apply Chain Rule:

\[ PP(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}} \]

\[ PP(W) = \frac{1}{P(w_1 w_2 \ldots w_N)^{\frac{1}{N}}} \]

\[ = \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}} \]
Evaluation

Test Corpus → Trained Language Model → What is the next word in the sequence?

Perplexity

Apply Chain Rule:

\[ PP(W) = \sqrt[1]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_1 \ldots w_{i-1})}} \]

Thus, PP for Bigrams:

\[ PP(W) = \sqrt[1]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_{i-1})}} \]

\[ PP(W) = \frac{1}{N} \prod_{i=1}^{N} \frac{1}{P(w_1w_2\ldots w_N)} \]
Coding Example: Modeling Tweets from POS data

1. Count unigrams, bigrams, and trigrams
2. Train probabilities for unigram, bigram, and trigram models (over training)
3. Generate language

   Trigram model when good evidence (high counts)
   Backing off to bigram or even unigram
Coding Example: Modeling Tweets from POS data

Practical Considerations:

- Use log probability to keep numbers reasonable and save computation. (uses addition rather than multiplication)

- Out-of-vocabulary (OOV)
  Choose minimum frequency and mark as <OOV>

- Sentence start and end: <s> this is a sentence </s>
Zeros and Smoothing

Example from (Jurafsky, 2017)

<table>
<thead>
<tr>
<th></th>
<th>i</th>
<th>want</th>
<th>to</th>
<th>eat</th>
<th>chinese</th>
<th>food</th>
<th>lunch</th>
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</table>
Zeros and Smoothing

Laplace ("Add one") smoothing: add 1 to all counts

<table>
<thead>
<tr>
<th>first word \ second word</th>
<th>i</th>
<th>want</th>
<th>to</th>
<th>eat</th>
<th>chinese</th>
<th>food</th>
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<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>1</td>
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Zeros and Smoothing

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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Unsmoothed probs

The table shows the probabilities $P(X_i | X_{i-1})$ for the second word given the first word in a sequence. The table is an example from (Jurafsky, 2017).

<table>
<thead>
<tr>
<th>first word ($X_{i-1}$)</th>
<th>second word ($X_i$)</th>
<th>want</th>
<th>to</th>
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<tr>
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<td>chinese</td>
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<td>0.0027</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Smoothed

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

(vocabulary size)

Example from (Jurafsky, 2017)

<table>
<thead>
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<td>0.00058</td>
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</tr>
</tbody>
</table>
Why Smoothing? Generalizes

Original

With Smoothing

(Example from Jurafsky / Originally Dan Klein)
Why Smoothing? Generalizes

Add-one is blunt: can lead to very large changes.

Better Smoothing:

- Good-Turing
- Kneser-Nay

These are outside scope of course because we will eventually cover, even stronger, deep learning based models.
Language Modeling Summary

- Two versions of assigning probability to sequence of words
- Applications
- The Chain Rule, The Markov Assumption: \( P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i|X_{i-k}, X_{i-(k-1)}, \ldots, X_i) \)
- Training a unigram, bigram, trigram model based on counts
- Evaluation: Perplexity
- Zeros, Low Counts, and Generalizability
- Add-one smoothing