Shape-based Diffeomorphic Registration on Hippocampal Surfaces Using Beltrami Holomorphic Flow

Abstract. Finding meaningful 1-1 correspondences between hippocampal (HP) surfaces is an important but difficult problem in computational anatomy. Unless high-field imaging is used, there are no well-defined anatomical features on the HP that can be used as landmark constraints, so defining meaningful registrations between HP surfaces is challenging. Here we developed a new algorithm to automatically register HP surfaces with complete geometric matching, avoiding the need to manually label landmark features. A good registration depends on a reasonable choice of shape energy that measures the dissimilarity between surfaces. In our algorithm, we first propose a complete shape index using the Beltrami coefficient and curvatures, which measures subtle local differences. The proposed shape energy is zero if and only if two shapes are identical up to a rigid motion. We then seek the best surface registration by minimizing the shape energy. We propose a simple representation of surface diffeomorphisms using Beltrami coefficients, which simplifies the optimization process. We then iteratively minimize the shape energy using the Beltrami Holomorphic flow (BHF) method introduced in this paper. Experimental results on 212 HP of normal and diseased (Alzheimer's disease) subjects show our proposed algorithm is effective in registering HP surfaces with complete geometric matching. The proposed shape energy can also capture local shape differences between HP, for shape analysis in Alzheimer's disease, schizophrenia and epilepsy.

1 Introduction

The hippocampus(HP) is an important subcortical structure of the human brain that plays a key role in long-term memory and spatial navigation. Surface-based shape analysis is commonly used to study local changes of HP surfaces due to pathologies such as Alzheimer disease (AD), schizophrenia and epilepsy[11]. When comparing data on two anatomical surfaces, a 1-1 correspondence must be computed to register one surface nonlinearly onto the other. On HP surfaces, there are no well-defined anatomical landmark features that can be used as a constraint to establish good correspondences. High-field structural or functional imaging, where discrete cellular fields are evident [15], is still not routinely used. Finding meaningful registrations between HP surfaces becomes challenging. Inaccuracies in shape analysis are often introduced due to incorrect registrations. In fact, shape analysis and surface registration are closely related. The results of shape analysis can be highly affected by the registration, but a good registration depends largely on the appropriate choice of shape measure that captures dissimilarities between surfaces. Therefore, it is of utmost importance to combine the two processes and define a suitable shape measure to drive the registration.

Here we developed an algorithm to automatically register HP surfaces with complete geometric matching, avoiding the need to manually label landmark features. We first propose a complete shape index using the Beltrami coefficient (BC) and curvatures, which measures subtle local differences. It can be proven that the proposed shape energy is identically zero if and only if two shapes are equal up to a rigid motion. We then minimize the shape energy to obtain the best surface registration with complete geometric matching. We propose a simple representation of surface diffeomorphisms using BCs, which simplifies the optimization. We then optimize the shape energy using the Beltrami Holomorphic flow (BHF) method introduced in this paper. The optimal shape energy obtained may also be used to measure local shape differences across subjects or time.

2 Related work

Surface registration has been studied extensively. Conformal or quasi-conformal surface registration is commonly used [4, 5, 14], and gives a parameterization minimizing angular distortions. However, it cannot guarantee the matching of geometric information such as curvature across subjects. Landmark-based diffeomorphisms are often used to compute, or adjust, cortical surface parameterizations [3, 6, 12]. For example, Glaunes et al.[3] proposed to generate large deformation diffeomorphisms of the sphere onto itself, given the displacements of a finite set of template landmarks. Leow et al.[6] proposed a level-set based approach to match different types of features, including points and 2D or 3D curves represented as implicit functions. These methods provide good registrations when corresponding landmark points on the surfaces can be labeled in advance. It is, however, difficult for HP surfaces on which there are no well-defined anatomical landmarks. Some authors have proposed driving features into correspondence based on shape information. Lyttelton et al. [8] computed surface parameterizations that match surface curvature. Fischl et al. [1] improved the alignment of cortical folding patterns by minimizing the mean squared difference between the average convexity across a set of subjects and that of the individual. Wang et al. [13] computed surface registrations that maximize the mutual information between mean curvature and conformal factor maps across subjects. Lord et al. [7] matched surfaces by minimizing the deviation from isometry. The shape indices that drive the registration process in these approaches are not complete shape measurements and do not capture shape differences completely. There are cases when two different surfaces might have the same shape value. This could lead to inaccurate registration results.

3 Theoretical background and definitions

Given two Riemann surfaces M and N, a map $f : M \to N$ is *conformal* if it preserves the surface metric up to a multiplicative factor. One generalization of conformal maps is the *quasi-conformal* maps, which are orientation-preserving homeomorphisms between Riemann surfaces with bounded conformality distortion, in the sense that their first order approximations takes small circles to small ellipses of bounded eccentricity [2]. Thus, a conformal homeomorphism that maps a small circle to a small circle may also be regarded as quasi-conformal. Mathematically, $f\colon \mathbb{C} \to \mathbb{C}$ is quasi-conformal if it satisfies the

Beltrami equation: $\frac{\partial f}{\partial \bar{z}} = \mu(z)\frac{\partial f}{\partial z}$, for some complex valued function μ satisfying $||\mu||_\infty < 1$. μ is called the *Beltrami coefficient* (BC), which is a measure of non-conformality. In particular, the map f is conformal around a small neighborhood of p when $\mu(p) = 0$. From $\mu(p)$, we can determine the angles of the directions of maximal magnification and shrinking and the amount of them as well. Specifically, the angle of maximal magnification is $\arg(\mu(p))/2$ with magnifying factor $1 + |\mu(p)|$; The angle of maximal shrinking is the orthogonal angle $(\arg(\mu(p)) - \pi)/2$ with shrinking factor $1 - |\mu(p)|$. The distortion or dilation is given by: $K = 1 + |\mu(p)|/1 - |\mu(p)|$.

4 Proposed model

4.1 A complete shape index

As discussed earlier, a good registration depends greatly on the appropriate choice of a shape measure to capture dissimilarities between surfaces. It is therefore important to look for a good shape measure. We propose a complete shape index E_{shape} using the Beltrami coefficient and curvatures, which measures subtle local changes completely. Given two HP surfaces S_1 and S_2. Let $f : S_1 \to S_2$ be a registration between S_1 and S_2. The complete shape index E_{shape} is defined as follow:

$$E_{shape}(f) = \alpha|\mu|^2 + \beta(H_1 - H_2(f))^2 + \gamma(K_1 - K_2(f))^2 \tag{1}$$

where μ is the Beltrami coefficient of f; $H_1,\ H_2$ are the mean curvatures on S_1 and S_2 respectively; and $K_1,\ K_2$ are the Gaussian curvatures. The first term measures the conformality distortion of the surface registration. The second and third terms measure the curvature mismatch. It turns out E_{shape} is a complete shape index that measures subtle shape differences between two surfaces. It can be proven that $E_{shape}(f) = 0$ if and only if S_1 and S_2 are equal up to a rigid motion. Also, by adjusting the parameters (i.e., α, β and γ), E_{shape} can be made equivalent to other existing shape indices. For example, when $\beta = 0$, E_{shape} is equivalent to the isometric shape index; when $\alpha = 0$, E_{shape} is equivalent to the curvature index; when $\beta = \gamma = 0$, E_{shape} measures the conformality distortion. In our work, we set $\alpha,\ \beta,\ \gamma \neq 0$ to measure complete shape changes.

We can now minimize E_{shape} to obtain the optimized surface map $\widetilde{f}$ that best matches the geometry. One advantage of using E_{shape} is that it can be defined in the space of BCs. The space of BCs is a simple functional space, which makes the optimization much easier.

4.2 Surface map representation using Beltrami Coefficients

Surface registration often involves an optimization process that minimizes an energy functional. Surface registration is commonly parameterized using 3D coordinate functions in $\mathbb{R}^3$. This representation is difficult to manipulate. For example, the 3D coordinate functions have to satisfy certain constraints on the Jacobian J (namely, $J > 0$), to preserve the 1-1 correspondence of the surface maps. Enforcing this constraint adds extra difficulty in manipulating and optimizing surface maps. The diffeomorphic property is often lost during the optimization. To tackle this, we propose a simple representation of surface diffeomorphisms using Beltrami coefficients (BCs). Fixing any 3 points on a pair of surfaces, there is a 1-1 correspondence between the set of surface diffeomorphisms between them and the set of BCs on the source domain. Hence, every bijective surface map can be represented by a unique BC.

Suppose S_1 and S_2 are both either genus 0 closed surfaces or simply connected open surfaces. Let $f\colon S_1 \to S_2$, and given 3 point correspondences, S_1 and S_2 can be conformally parameterized with a global patch D. Denote the parameterizations by $\phi_1\colon S_1 \to D$ and $\phi_2\colon S_2 \to D$. Now, we can compute the Beltrami coefficient μ_f associated uniquely to f to represent f (See Figure 1). The Beltrami coefficient μ_f can be computed by considering the composition map $\widetilde{f} = \phi_2 \circ f \circ \phi_1^{-1}\colon D \to D$. Mathematically, μ_f is given by the following formula: $\mu_f = \frac{\partial \widetilde{f}}{\partial \bar{z}} / \frac{\partial \widetilde{f}}{\partial z} = \frac{1}{2}(\frac{\partial \widetilde{f}}{\partial x} + \sqrt{-1}\frac{\partial \widetilde{f}}{\partial y}) / \frac{1}{2}(\frac{\partial \widetilde{f}}{\partial x} - \sqrt{-1}\frac{\partial \widetilde{f}}{\partial y})$.

The space of BCs is a simple functional space. There are no restrictions on μ that it has to be 1-1, surjective or satisfy some constraints on the Jacobian. Using the Beltrami representation makes the optimization process of surface maps much easier.

4.3 Optimized surface registration matching the geometry

E_{shape} gives us a complete shape index which measures local dissimilarities between two surfaces. Specifically, $E_{shape}(f) = 0$ if and only if S_1 and S_2 are equal up to a rigid motion. Therefore, the surface map f minimizing $E_{shape}(f)$ is the best registration that matches the geometric information as far as possible. Given two HP surfaces S_1 and S_2. We propose to find $f : S_1 \to S_2$ that minimizes $E = \int E_{Shape}(f)$. To simplify the computation, we can conformally parameterize S_1 and S_2 onto the parameter domain D. So, all computations are carried out on the simple domain D. By representing surface maps with Beltrami coefficients μ, we can define the energy on the space of BCs - a much simpler functional space for the optimization process. Mathematically, the compound energy E can be written with respect to μ as:

$$E(\mu) = \int_D \alpha|\mu|^2 + \beta(H_1 - H_2(f^\mu))^2 + \gamma(K_1 - K_2(f^\mu))^2 \tag{2}$$

The variation of f^μ under the variation of μ can be expressed explicitly. Suppose $\widetilde{\mu}(z) = \mu(z) + t\nu(z) + \mathcal{O}(t^2)$. Then, $f^{\widetilde{\mu}(z)}(w) = f^\mu(w) + tV(f^\mu, \nu)(w) + \mathcal{O}(t^2)$, where

$$V(f^\mu, \nu)(w) = -\frac{f^\mu(w)(f^\mu(w) - 1)}{\pi} \int_D \frac{\nu(z)(f_z^\mu(z))^2 dxdy}{f^\mu(z)(f^\mu(z) - 1)(f^\mu(z) - f^\mu(w))}. \tag{3}$$

Using the variational formula, we can derive the Euler-Lagrange equation of Equation 2 with respect to μ easily. Specifically, we can minimize $E(\mu)$ by the following iterative scheme:

$$\mu^{n+1} - \mu^n = -2(\alpha\mu^n - \int_z [(\beta\widetilde{H}^n + \gamma\widetilde{K}^n) \cdot G^n, \mathbf{det}(\beta\widetilde{H}^n + \gamma\widetilde{K}^n, G^n)]\ )dt \tag{4}$$

where $\int_w \bullet := \int_D \bullet \, dw$ and $\int_z \bullet := \int_D \bullet \, dz$ is defined as the integral over the variable w and z respectively; $\widetilde{H} := (H_1 - H_2(f^\mu))\nabla H_2(f^\mu)$; $\widetilde{K} := (K_1 - K_2(f^\mu))\nabla K_2(f^\mu)$; $\mathbf{det}(a, b)$ is the determinant of the 2 by 2 matrix or equivalently, the norm of the cross product of a and b.

We call this iterative algorithm the *Beltrami Holomorphic flow* (BHF). During the BHF process, the maps are guaranteed to be diffeomorphic and are holomorphic in t. The detailed computational algorithm is as follows:

Algorithm 1. *BHF Surface Registration*
Input: Hippocampal surfaces S_1 and S_2, step length dt, threshold ϵ
Output: Geometric matching registration f^μ and the shape index $E(f^\mu)$

1. *Compute the conformal parameterizations of S_1 and S_2. Denote them by $\phi_1 : S_1 \to D$ and $\phi_2 : S_2 \to D$*
2. *Set $\varphi^0 := \mathbf{Id} : D \to D$ and $n = 0$.*
3. *Compute the Beltrami coefficient μ^n_φ of φ^n_i (e.g. $\mu^0_\varphi = 0$). Update μ^{n+1}_φ by Equation 4.*
4. *Compute: $\boldsymbol{V}_n = F(\mu^{n+1}_\varphi - \mu^n_\varphi)$ using Equation 3. Let $\varphi^{n+1} = \varphi^n + \boldsymbol{V}_n$. Set $n = n{+}1$.*
5. *Repeat Step 3 to Step 5. If $|E(\mu^{n+1}_\varphi) - E(\mu^n_\varphi)| < \epsilon$,* **Stop**.

5 Experimental results

We tested our algorithm on 212 HP surfaces automatically extracted from 3D brain MRI scans with a validated algorithm [9]. Scans were acquired from normal and diseased (AD) elderly subjects at 1.5 T (on a GE Signa scanner). Experimental results show our proposed algorithm is effective in registering HP surfaces with geometric matching. The proposed shape energy can also be used to measure local shape difference between HPs.

Figure 1 shows the Beltrami representations of bijective surface maps. The left column shows the bijective surface maps of the HP surfaces and cortical surfaces. The middle column shows the Beltrami (BC) representations of the maps. The right column shows the reconstruction of surface maps from their BCs. The reconstructed maps closely resemble the original maps, meaning that BCs can effectively represent bijective surface maps.

Figure 2(A) shows two different HP surfaces. They are registered using our proposed BHF algorithm with geometric matching. The registration is visualized using a grid map and texture map, which shows a smooth 1-1 correspondence. The optimal shape index E_{shape} is plotted as colormap in (B). E_{shape} effectively captures the local shape difference between the surfaces. (C) shows the shape energy in each iteration. With the BHF algorithm, the shape energy decreases as the number of iterations increases. (D) shows the curvature mismatch energy $(E = \int \beta(H_1 - H_2(f))^2 + \gamma(K_1 - K_2(f))^2)$. It decreases as the number of iterations increases, meaning that the geometric matching improves. (E) shows the Beltrami coefficient of the map in each iteration, which shows the conformality distortion of the map. Some conformality is intentionally lost to allow better geometric matching.

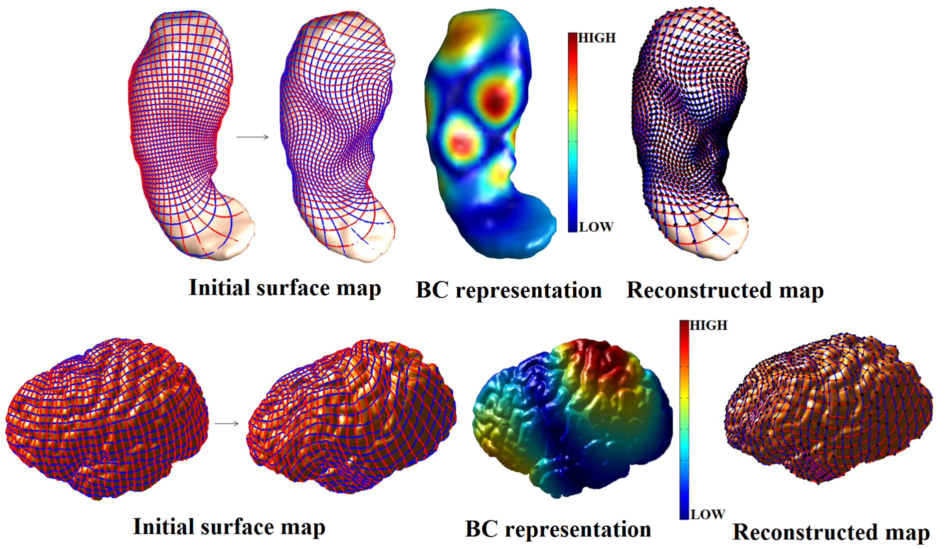

Fig. 1. Representation of surface registration using Beltrami Coefficients

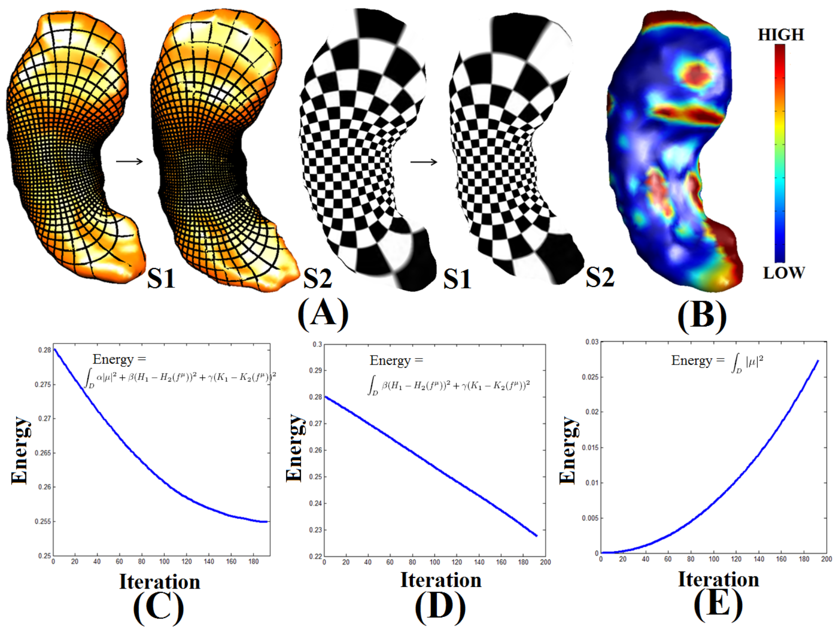

Fig. 2. Shape registration with geometric matching using Beltrami Holomorphic flow (BHF).

Figure 3 shows the BHF registration between two normal HPs. The complete shape index E_{shape} is plotted as colormap on the right. Again, E_{shape} can accurately capture local shape differences between the normal HP surfaces.

Figure 4 shows the BHF hippocampal registrations between normal elderly subjects and subjects with Alzheimer's disease. The BHF registrations give smooth 1-1 correspondences between the HP surfaces. We can use the complete shape index E_{shape} to detect local shape differences between healthy and unhealthy subjects.

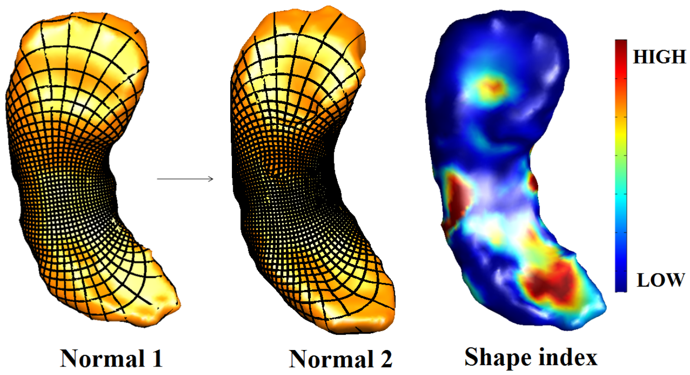

Fig. 3. BHF registration between two normal subjects. The shape index E_{shape} is plotted on the right, which captures local shape differences.

We also study the temporal shape changes of normal and AD HP surfaces, as shown in Figure 5. For each subject, we compute the deformation pattern of its HP surfaces measured at time = 0 and time = 12 Months (see [10] for longitudinal scanning details). The left two panels show the temporal deformation patterns for two normal subjects. The middle two panels show the temporal deformation patterns for two AD subjects. The last column shows the statistical significance p-map measuring the difference in the deformation pattern between the normal (n=47) and AD (n=53) groups, plotted on a control HP. The deep red color highlights regions of significant statistical difference. This method can be potentially used to study factors that influence brain changes in AD.

6 Conclusion and future work

We developed an algorithm to automatically register HP surfaces with complete geometric matching, avoiding the need for manually-labeled landmark features. We did this by defining a complete shape index that measures subtle shape differences and we used it to drive the registration. Experimental results on 212 HP surfaces from normal and diseased (Alzheimer's) subjects show our proposed algorithm is effective in registering HP surfaces over time and across subjects, with complete geometric matching. The proposed shape energy can also capture local shape differences between HPs, which may be useful for shape analysis in Alzheimer's disease, schizophrenia and epilepsy. In future, we will use the BHF algorithm to register more HP surfaces and systematically study their local shape differences and factors that affect deformation patterns between normal and AD subjects.

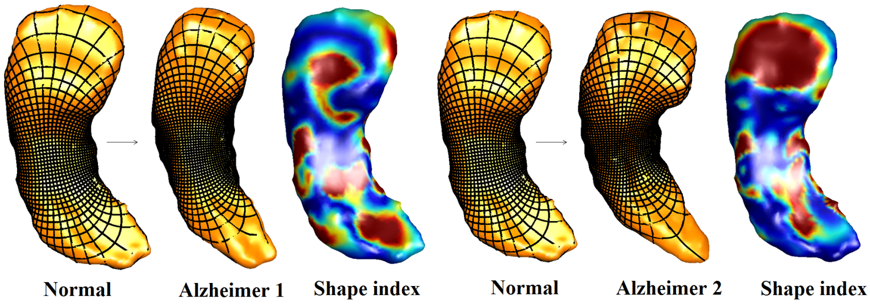

Fig. 4. BHF registration between normal subjects and subjects with Alzheimer's disease. Their local shape differences are captured by E_{shape}.

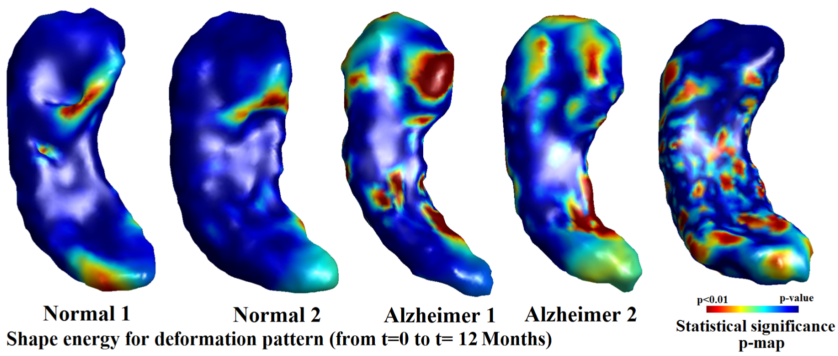

Fig. 5. Temporal hippocampal shape changes of normal and subjects with Alzheimer's disease.

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