Assignment Five: 4D Convex Hull Construction

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Convex Hull

The input to the 4D convex hull algorithm is a set of 3D points

$$P = \{p_1, p_2, \dots, p_n\}$$

The output is the 3D Delaunay triangulation of the point set P.

Input

The input points are randomly generated within the unit sphere.

Output

The Delaunay triangulation is represented as a tetrahedral mesh, using Dart data structure to store.

Convex Hull

Algorithm Pipeline

- Lift each point $p_i = (x_i, y_i, z_i)$ to the parabola,

$$q_i = (x_i, y_i, z_i, w_i), \quad w_i = 1/2(x_i^2 + y_i^2 + z_i^2),$$

- Pick four points to form two tetrahedra with opposite orientations, and glue them to form a topological ball, and assign the ball as the initial convex hull of $\{q_i\}$, denoted as C;
- select a point q_l, which is as far as possible from the current C;
- For each face on the hull C, test the visibility with respect to q_l;
- Remove all the visible tetra from C;
- For each face $[q_i, q_j, q_k]$ on the contour (the surface separating the visible and invisible parts of C), connect the face with the point q_l to form a triangle $[p_i, p_j, p_k]$, add the face to C;
- Repeat step 2 through 5, until all the points have been processed.

Convex Hull

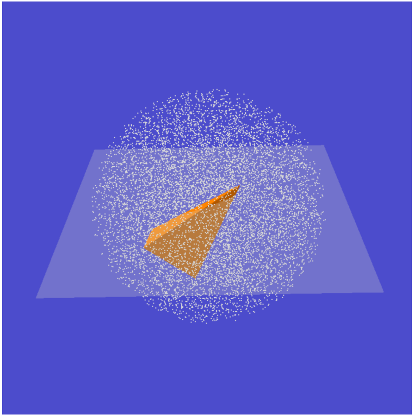
Visibility Testing

Given a tetrahedron $[p_i, p_j, p_k, p_l]$ and the new point p_m, the visibility testing is equivalent to compute the volume of the 5-simple $[p_i, p_j, p_k, p_l, p_m]$, which is given by

$$\begin{vmatrix} x_i - x_m & y_i - y_m & z_i - z_m & w_i - w_m \\ x_j - x_m & y_j - y_m & z_j - z_m & w_j - w_m \\ x_k - x_m & y_k - y_m & z_k - z_m & w_k - w_m \\ x_l - x_m & y_l - y_m & z_l - z_m & w_l - w_m \end{vmatrix}$$

and check whether is the volume is positive or not.

Example



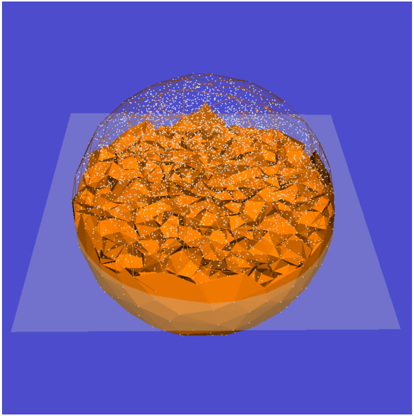


Figure: Convex hull computation process.

Example

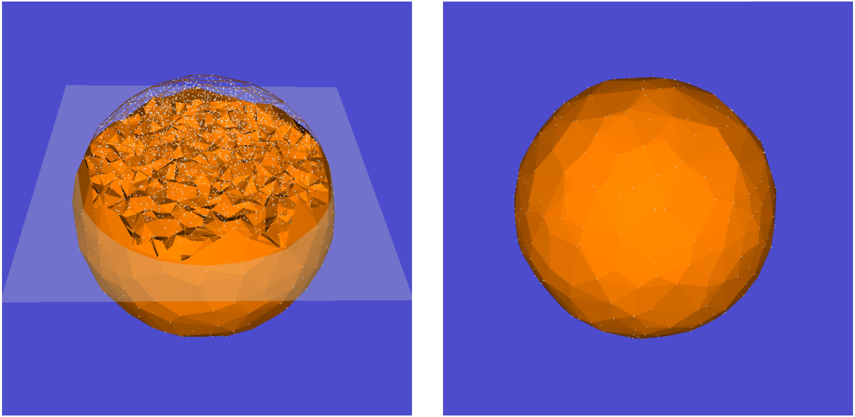


Figure: Convex hull computation process.

Instruction

Dependencies

1. ‘DartLib‘, a general purpose mesh library based on Dart data structure.
2. ‘freeglut‘, a free-software/open-source alternative to the OpenGL Utility Toolkit (GLUT) library.

Directory Structure

- 3rdparty/DartLib, header files for mesh;
- convex_hull/include, the header files for convex_hull;
- convex_hull/src, the source files for convex_hull;
- CMakeLists.txt, CMake configuration file;

Configuration

Before you start, read README.md carefully, then go three the following procedures, step by step.

1. Install [CMake](https://cmake.org/download/).
2. Download the source code of the C++ framework.
3. Configure and generate the project for Visual Studio.
4. Open the .sln using Visual Studio, and complie the solution.
5. Finish your code in your IDE.
6. Run the executable program.

Configure and generate the project

1. open a command window
2. cd Assignment_1_skeleton
3. mkdir build
4. cd build
5. cmake ..
6. open OTHomework.sln inside the build directory.

Finish your code in your IDE

- You need to modify the file: ConvexHull4D.cpp;
- search for comments "insert your code"
- Modify functions:
  1. $CConvexHull4D :: volume\_sign(cosntCTettest, constCPoint4p)$
  2. $CConvexHull4D :: \_init(std :: vectgor < CPoint4 > sites)$
  3. $CTetMesh :: create\_tet(constCDArray < int > test)$
  4. $CTetMesh :: remove\_tet(constinttet\_idx)$

Finish your code in your IDE

Try your best to improve the efficiency.