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Science Times

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The New York Times

An Elusive Proof and Its Elusive Prover

That rabbit is actually a sphere. (Read on.) But the man who proved it is missing.

By DENNIS OVERBYE

Grisha Perelman, where are you?

Three years ago, a Russian mathematician by the name of Grigory Perelman, a k a Grisha, in St. Petersburg, announced that he had solved a famous and intractable mathematical problem, known as the Poincaré conjecture, about the nature of space.

After posting a few short papers on the Internet and making a whirlwind lecture tour of the United States, Dr. Perelman disappeared back into the Russian woods in the spring of 2003, leaving the world's mathematicians to pick up the pieces and decide if he was right.

Now they say they have finished his work, and the evidence is circulating among scholars in the form of three book-length papers with about 1,000 pages of dense mathematics and prose between them.

As a result there is a growing feeling, a cautious optimism that they have finally achieved a landmark not just of mathematics, but of human thought.

"It's really a great moment in mathematics," said Bruce Kleiner of Yale, who has spent the last three years helping to explicate Dr. Perelman's work. "It could have happened 100 years from now, or never."

In a speech at a conference in Beijing this summer, Shing-Tung Yau of Harvard said the understanding of three-dimensional space brought about by Poincaré's conjecture could be one of the major pillars of math in the 21st century.

Quoting Poincaré himself, Dr.Yau said, "Thought is only a flash in the middle of a long night, but the flash that means everything."

But at the moment of his putative triumph, Dr. Perelman is nowhere in sight. He is an odds-on favorite to win a Fields Medal, math's version of the Nobel Prize, when the International Mathematics Union convenes in Madrid next Tuesday. But there is no indication whether he will show up.

Also left hanging, for now, is $1 million offered by the Clay Mathematics Institute in Cambridge, Mass., for the first published proof of the conjecture, one of seven outstanding questions for which they offered a ransom back at the beginning of the millennium.

"It's very unusual in math that somebody announces a result this big and leaves it hanging," said John Morgan of Columbia, one of the scholars who has also been filling in the details of Dr. Perelman's work.

Mathematicians have been waiting for this result for more than 100 years, ever since the French polymath Henri Poincaré posed the problem in 1904. And they acknowledge that it may be another 100 years before its full implications for math and physics are understood. For now, they say, it is just beautiful, like art or a challenging new opera.

Dr. Morgan said the excitement came not from the final proof of the conjecture, which everybody felt was true, but the method, "finding deep connections between what were unrelated fields of mathematics."

William Thurston of Cornell, the author of a deeper conjecture that includes Poincaré's and that is now apparently proved, said, "Math is really about the human mind, about how people can think effectively, and why curiosity is quite a good guide," explaining that curiosity is tied in some way with intuition.

"You don't see what you're seeing until you see it," Dr. Thurston said, "but when you do see it, it lets you see many other things."

Depending on who is talking, Poincaré's conjecture can sound either daunting or deceptively simple. It asserts that if any loop in a certain kind of three-dimensional space can be shrunk to a point without ripping or tearing either the loop or the space, the space is equivalent to a sphere.

The conjecture is fundamental to topology, the branch of math that deals with shapes, sometimes described as geometry without the details. To a topologist, a sphere, a cigar and a rabbit's head are all the same because they can be deformed into one another. Likewise, a coffee mug and a doughnut are also the same because each has one hole, but they are not equivalent to a sphere.

In effect, what Poincaré suggested was that anything without holes has to be a sphere. The one qualification was that this "anything" had to be what mathematicians call compact, or closed, meaning that it has a finite extent: no matter how far you strike out in one direction or another, you can get only so far away before you start coming back, the way you can never get more than 12,500 miles from home on the Earth.

In the case of two dimensions, like the surface of a sphere or a doughnut, it is easy to see what Poincaré was talking about: imagine a rubber band stretched around an apple or a doughnut; on the apple, the rubber band can be shrunk

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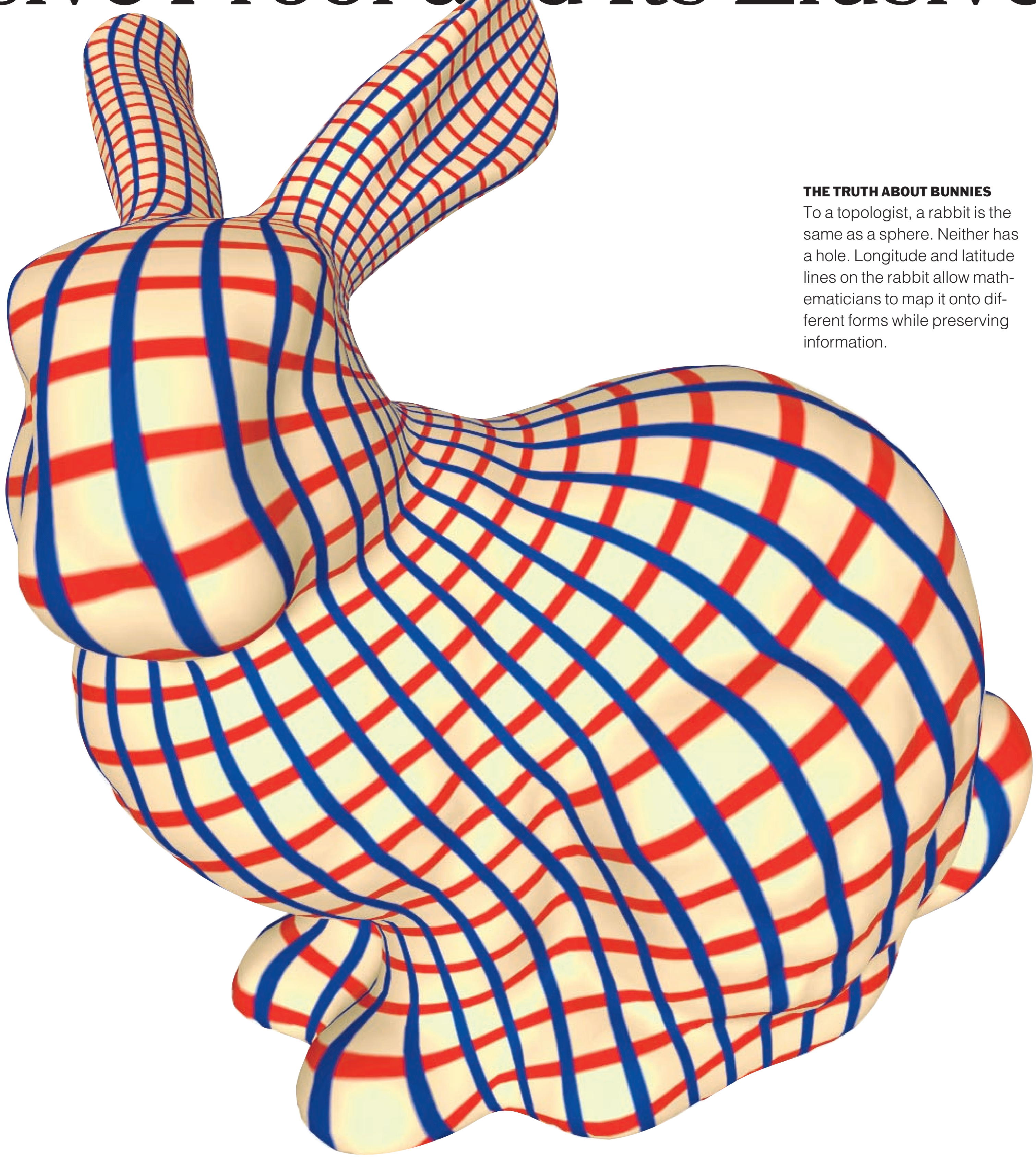
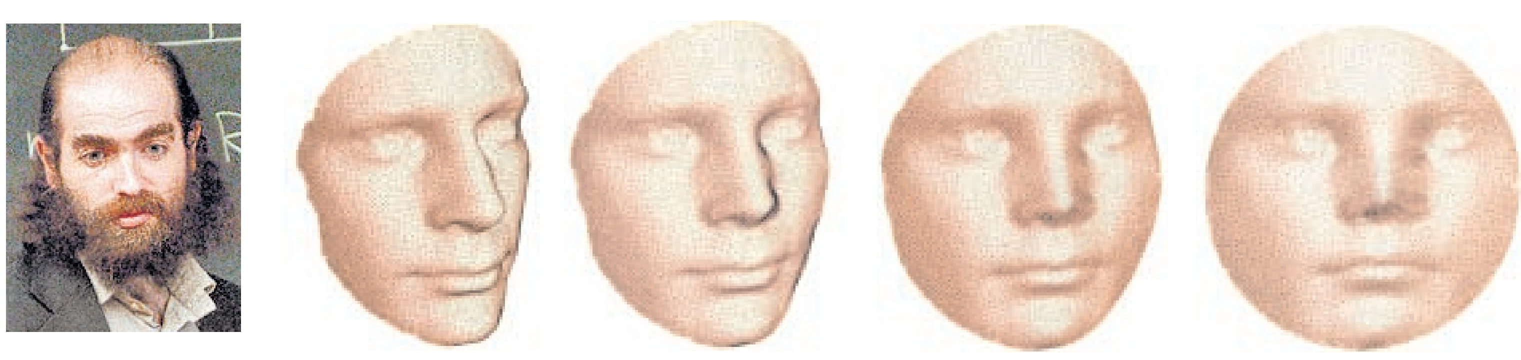


THE TRUTH ABOUT BUNNIES To a topologist, a rabbit is the same as a sphere. Neither has a hole. Longitude and latitude lines on the rabbit allow mathematicians to map it onto different forms while preserving information.



THE ESSENTIAL GRISHA A photo of the mathematician Grigory Perelman is altered by a technique known as the Ricci flow, becoming more and more spherical. Dr. Perelman used the technique, pioneered by Richard Hamilton, to solve a famous problem first posed by Henri Poincaré.

Renderings by Xianfeng David Gu and Shing-Tung Yau; photograph by Frances Roberts for The New York Times

Scientists Begin to Grasp the Stealthy Spread of Cancer

With new findings on metastasis, can new medicines be far behind?

By LAURIE TARKAN

The moment when a cancer begins to spread throughout the body — metastasis — has always been the most dreaded turning point of the disease.

Without metastasis, cancer would barely be a blip on the collective consciousness. Fewer than 10 percent of cancer deaths are caused by the primary tumor; the rest stem from metastasis to vital sites like the lungs, the liver, the bones and the brain.

Though chemotherapy and other treatments have lengthened the lives of people with metastasized cancer, no drugs have been specifically formulated to halt the process. That is because metastasis has remained something of a mystery until the last five years or so.

"In the last 30 years, we've learned all about identifying genes whose mutations initiate tumors," said Dr. Joan Massagué, chairman of the Cancer Biology and Genetics Program at Memorial Sloan-Kettering Cancer Center in New York. But these advances, he added, did not explain the metastatic process.

Now, knowledge of metastasis is beginning to accumulate to the point that new therapies are entering the pipeline.

"In terms of milestones or breakthroughs, most of them are about to be made," said Dr. Massagué.

Dr. Patricia S. Steeg, chief of the women's cancers section of the Laboratory of Molecular Pharmacology at the National Cancer Institute, said she was optimistic for the first time. "The trickle is close, the first agents are in early clinical testing or will be soon," she said. "I'm very enthusiastic, much more than I was five years ago."

The complexity of metastasis may well have discouraged research. To metastasize, cancer cells have to acquire several dozen genetic alterations — in contrast with the handful typically necessary to initiate a primary tumor, Dr. Massagué said. Further complicating matters, each case of metastasis — breast cancer that spreads to a lung, for instance, or prostate cancer that spreads to bone — is genetically and molecularly different from the rest.

Studying metastasis is expensive and

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Virgo Consortium

FINDINGS

Embryos, Half a Billion Years Old

More than 500 million years ago, tiny clumps of cells that had just begun to grow were suddenly buried in sediment and were fossilized instead.

Multicellular animal life had just appeared a few tens of millions of years before this, after about three billion years of single-cell organisms. And these cell clumps were growing into animals that had already evolved and diversified.

Now they are found in China and Siberia preserved as specks of rock — early animal life during embryonic development. Using intense X-rays generated by a particle accelerator, scientists have now been able to peer inside the fossils. The technique, called synchrotron X-ray tomographic microscopy, has produced sharp, three-dimensional images of the structure of the embryos.

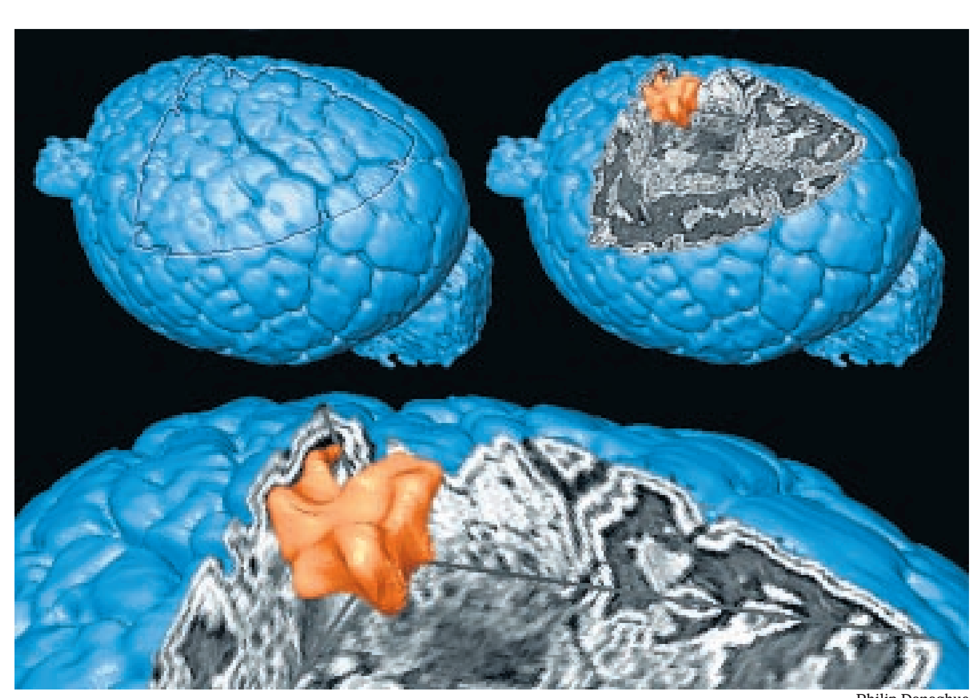
"What we can see inside are things we haven't seen before," said Philip C. J. Donoghue, a paleontologist at Bristol University in England and the lead author of a paper in the current issue of the journal Nature that describes fossil images of Markuelia, a wormlike animal. "You can only get so much looking at the outside. Embryology is about what goes on in the inside of embryos and not the outside."

For example, in some of the Markuelia embryos, scientists had seen a hollow space that had been interpreted as a nascent gut. With their closer, sharper look, they realized that the hollows were a result of incomplete fossilization, not a structure of the embryo.

In more developed embryos, the researchers — from China, Sweden, Britain and Switzerland — could make out the shape of the teeth and individual hairs one- or two-thousandths of a millimeter in length.

"We can see the very limits of fossilization," Dr. Donoghue said.

KENNETH CHANG



Philip Donoghue

FIRST IMPRESSIONS Scientists mistook the hollow part of the embryos for a nascent gut.

LETTERS

A Raw Deal

To the Editor:

Re "Tale of the Tapeworm" (Cases, Aug. 8):

My grandmother and my wife's grandmother made their own gefilte fish from freshwater species for 70 years, here and in "the old country." They also tasted continuously, but always had a pot of boiling water nearby on the stove so that they could periodically roll the mixture into a little ball and boil it for a few minutes before tasting.

This was the detail the mother of the subject failed to pass on to the next generation. One does not eat freshwater sushi.

ROBERT ROSENBERG, O.D.
Great Neck, N.Y.

Coping With Pain

To the Editor:

Re "Scientists Cast Misery of Migraine in a New Light" (Personal Health, Aug. 8): Like Jane E. Brody, I grew up in a migraine family. I started nursing my mother through chronic migraines when I was 5.

When migraines hit my brothers, cousins and me, we all discovered different triggering factors (mold, disco lights, MSG, sensory overload, stress, too much driving, neglecting meals and weather changes), and we all have different remedies (I drink fresh orange juice, strong tea and take a blistering hot shower).

Others find relief by pressing a bag of ice cubes around their necks and heads or by downing vitamin B. We've also found preventive help through acupuncture, shiatsu, regular exercise and heeding warning signs and triggering factors. Far better than reliance on prescription meds.

PAMELA ELLEN FERGUSON
Austin, Tex.

Stuart Bradford

To the Editor:

Re "Misery of Migraine": I am typing this letter in the midst of a migraine attack. What allows me to be functional is the miracle of triptans, which, after a lifetime of suffering (I am 47), I was prescribed about four years ago, shortly after my mother died.

Triptans control or kill outright about 80 percent of my migraines. I only wish that my mother had lived long enough to know there is life beyond migraine. How she, and we, suffered needlessly!

DONNA J. ANTON
Hayle, England

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To the Editor:

Re "Misery of Migraine": I suffered migraines from childhood, often misdiagnosed and misunderstood. (One doctor even suggested viral meningitis.) Prescriptions I received in adulthood added nausea to the pain but little else. Finally, frustrated with treating symptoms, I tried prevention. I started by looking at when I came down with them.

"Weather changes," as the column notes, were the key. Fronts coming in from the west bring pollens and mold spores with them. My seasonal migraines began to make sense. So, I asked my doctor for an allergy prescription.

Almost immediately, my migraines were reduced from perhaps a couple of dozen a year to almost none. That was 10 years ago.

DANIEL WARD
New York

Shortchanging Prevention

To the Editor:

Re "Talking About AIDS, With All the World Watching" ("The Doctor's World," Aug. 8): AIDS is something that can be prevented by behavior change, and it is sad to read that the emphasis of the AIDS conference will be almost entirely on cures. Yes, curing AIDS is important, but a comparable amount of money and brain power should be spent on prevention.

ROBERT W. LEACH
Swarthmore, Pa.

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To the Editor:

Re "Talking About AIDS": Dr. Lawrence K. Altman's overview of 15 international AIDS conferences is replete with noteworthy breakthroughs. Having attended each enclave until Durban and Bangkok, I welcome Dr. Altman's acumen and his ability to set the stage for the Toronto conference.

Although experts often find the conferences less than hospitable for meaningful professional dialogue, they've learned to explain and defend their findings to the core audience: people infected with H.I.V.

These conferences resemble family reunions, and as such have enabled the global H.I.V. family to understand and celebrate the breakthroughs, honor the departed and recommit to the hard work at home.

B. J. STILES
San Francisco

More letters: *nytimes.com/science*

Science Times welcomes letters from readers. Those submitted for publication must include the writer's name, address and telephone number. E-mail should be sent to **scitimes@nytimes.com.** *Send letters to Science Editor, The New York Times, 229 West 43rd Street, New York, N.Y. 10036.*

Elusive Proof, Elusive Prover: A New Mathematical Mystery

Continued From First Science Page

without limit, but on the doughnut it is stopped by the hole.

With three dimensions, it is harder to discern the overall shape of something; we cannot see where the holes might be. "We can't draw pictures of 3-D spaces," Dr. Morgan said, explaining that when we envision the surface of a sphere or an apple, we are really seeing a two-dimensional object embedded in three dimensions. Indeed, astronomers are still arguing about the overall shape of the universe, wondering if its topology resembles a sphere, a bagel or something even more complicated.

Poincaré's conjecture was subsequently generalized to any number of dimensions, but in fact the three-dimensional version has turned out to be the most difficult of all cases to prove. In 1960 Stephen Smale, now at the Toyota Technological Institute at Chicago, proved that it is true in five or more dimensions and was awarded a Fields Medal. In 1983, Michael Freedman, now at Microsoft, proved that it is true in four dimensions and also won a Fields.

"You get a Fields Medal for just getting close to this conjecture," Dr. Morgan said.

In the late 1970's, Dr. Thurston extended Poincaré's conjecture, showing that it was only a special case of a more powerful and general conjecture about three-dimensional geometry, namely that any space can be decomposed into a few basic shapes.

Mathematicians had known since the time of Georg Friedrich Bernhard Riemann, in the 19th century, that in two dimensions there are only three possible shapes: flat like a sheet of paper, closed like a sphere, or curved uniformly in two opposite directions like a saddle or the flare of a trumpet. Dr. Thurston suggested that eight different shapes could be used to make up any three-dimensional space.

"Thurston's conjecture almost leads to a list," Dr. Morgan said. "If it is true," he added, "Poincaré's conjecture falls out immediately." Dr. Thurston won a Fields in 1986.

Topologists have developed an elaborate set of tools to study and dissect shapes, including imaginary cutting and pasting, which they refer to as "surgery," but they were not getting anywhere for a long time.

In the early 1980's Richard Hamilton of Columbia suggested a new technique, called the Ricci flow, borrowed from the kind of mathematics that underlies Einstein's general theory of relativity and string theory, to investigate the shapes of spaces.

Dr. Hamilton's technique makes use of the fact that for any kind of geometric space there is a formula called the metric, which determines the distance between any pair of nearby points. Applied mathematically to this metric, the Ricci flow acts like heat, flowing through the space in question, smoothing and straightening all its bumps and curves to reveal its essential shape, the way a hair dryer shrink-wraps plastic.

Dr. Hamilton succeeded in showing that certain generally round objects, like a head, would evolve into spheres under this process, but the fates of more complicated objects were problematic. As the Ricci flow progressed, kinks and neck pinches, places of infinite density known as singularities, could appear, pinch off and even shrink away. Topologists could cut them away, but there was no guarantee that new ones would not keep popping up forever.

"All sorts of things can potentially happen in the Ricci flow," said Robert Greene, a mathematician at the University of California, Los Angeles. Nobody knew what to do with these things, so the result was a logjam.

It was Dr. Perelman who broke the logjam. He was able to show that the singularities were all friendly. They turned into spheres or tubes. Moreover, they did it in a finite time once the Ricci flow started. That meant topologists could, in their fashion, cut them off, and allow the Ricci process to continue to its end, revealing the topologically spherical essence of the space in question, and thus proving the conjectures of both Poincaré and Thurston.

Dr. Perelman's first paper, promising "a sketch of an eclectic proof," came as a bolt from the blue when it was posted on the Internet in November 2002. "Nobody knew he was working on the Poincaré conjecture," said Michael T. Anderson of the State University of New York in Stony Brook.

Dr. Perelman had already established himself as a master of differential geometry, the study of curves and surfaces, which is essential to, among other things, relativity and string theory Born in St. Petersburg in 1966, he distinguished himself as a high school student by winning a gold medal with a perfect score in the International Mathematical Olympiad in 1982. After getting a Ph.D. from St. Petersburg State, he joined the Steklov Institute of Mathematics at St. Petersburg.

In a series of postdoctoral fellowships in the United States in the early 1990's, Dr. Perelman impressed his colleagues as "a kind of unworldly person," in the words of Dr. Greene of U.C.L.A. — friendly, but shy and not interested in material wealth.

"He looked like Rasputin, with long hair and fingernails," Dr. Greene said.

Asked about Dr. Perelman's pleasures, Dr. Anderson said that he talked a lot about hiking in the woods near St. Petersburg looking for mushrooms.

Dr. Perelman returned to those woods, and the Steklov Institute, in 1995, spurning offers from Stanford and Princeton, among others. In 1996 he added to his legend by turning down a prize for young mathematicians from the European Mathematics Society.

Until his papers on Poincaré started appearing, some friends thought Dr. Perelman had left mathematics. Although they were so technical and abbreviated that few mathematicians could read them, they quickly attracted interest among experts. In the spring of 2003, Dr. Perelman came back to the United States to give a series of lectures at Stony Brook and the Massachusetts Institute of Technology, and also spoke at Columbia, New York University and Princeton.

But once he was back in St. Petersburg, he did not respond to further invitations. The e-mail gradually ceased.

"He came once, he explained things, and that was it," Dr. Anderson said. "Anything else was superfluous."

Recently, Dr. Perelman is said to have resigned from Steklov. E-mail messages addressed to him and to the Steklov Institute went unanswered.

In his absence, others have taken the lead in trying to verify and disseminate his work. Dr. Kleiner of Yale and John Lott of the University of Michigan have assembled a monograph annotating and explicating Dr. Perelman's proof of the two conjectures..

Dr. Morgan of Columbia and Gang Tian of Princeton have followed Dr. Perelman's prescription to produce a more detailed 473-page step-by-step proof of only of Poincaré's Conjecture. "Perelman did all the work," Dr. Morgan said. "This is just explaining it."

Both works were supported by the Clay institute, which has posted them on its Web site, claymath.org. Meanwhile, Huai-Dong Cao of Lehigh University and Xi-Ping Zhu of Zhongshan University in Guangzhou, China, have published their own 318-page proof of both conjectures in The Asian Journal of Mathematics (www.ims.cuhk.edu.hk/).

Although these works were all hammered out in the midst of discussion and argument by experts, in workshops and lectures, they are about to receive even stricter scrutiny and perhaps crossfire. "Caution is appropriate," said Dr. Kleiner, because the Poincaré conjecture is not just famous, but important.

James Carlson, president of the Clay Institute, said the appearance of these papers had started the clock ticking on a two-year waiting period mandated by the rules of the Clay Millennium Prize. After two years, he said, a committee will be appointed to recommend a winner or winners if it decides the proof has stood the test of time.

"There is nothing in the rules to prevent Perelman from receiving all or part of the prize," Dr. Carlson said, saying that Dr. Perelman and Dr. Hamilton had obviously made the main contributions to the proof.

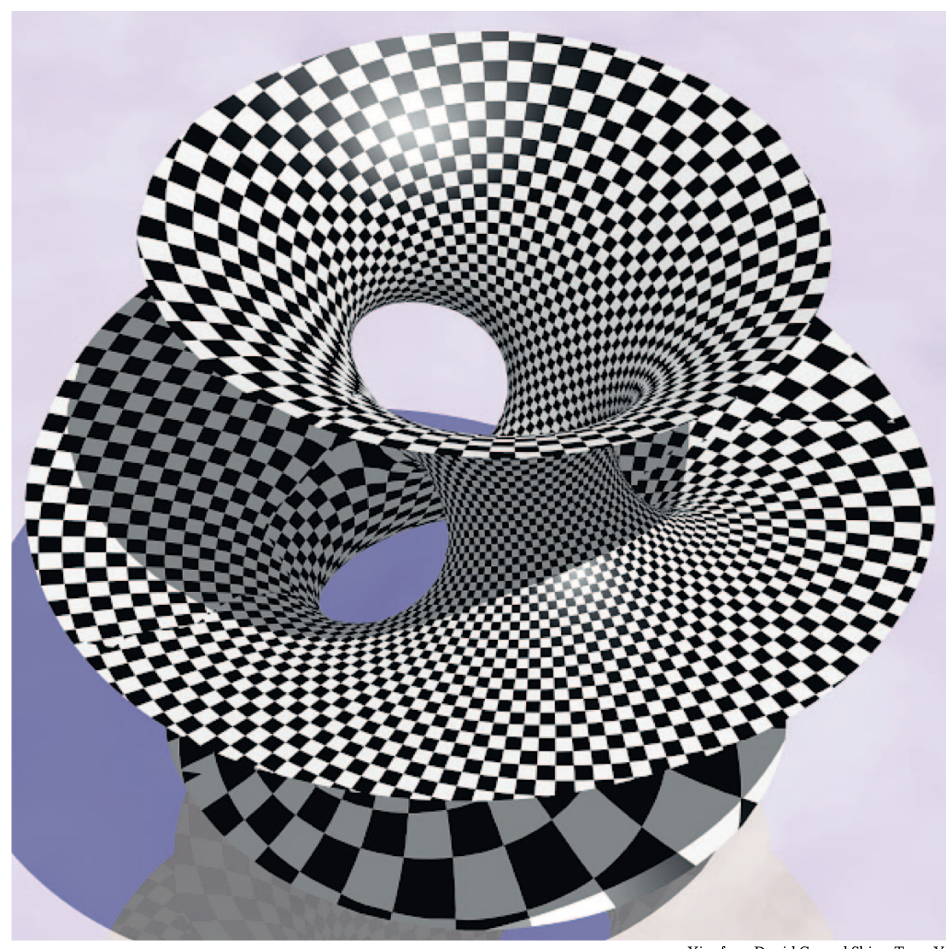
In a lecture at M.I.T. in 2003, Dr. Perelman described himself "in a way" as Dr. Hamilton's disciple, although they had never worked together. Dr. Hamilton, who got his Ph.D. from Princeton in 1966, is too old to win the Fields medal, which is given only up to the age of 40, but he is slated to give the major address about the Poincaré conjecture in Madrid next week. He did not respond to requests for an interview.

Allowing that Dr. Perelman, should he win the Clay Prize, might refuse the honor, Dr. Carlson said the institute could decide instead to use award money to support Russian mathematicians, the Steklov Institute or even the Math Olympiad.

Dr. Anderson said that to some extent the new round of papers already represented a kind of peer review of Dr. Perelman's work. "All these together make the case pretty clear," he said. "The community accepts the validity of his work. It's commendable that the community has gotten together."

Left, Hulton-Deutsch Collection/Corbis; top right, Bill Wingell for The New York Times; above, Allison Evans/Clay Mathematics Institute

THE MATHEMATICIANS Henri Poincaré, above left, posed his vexing problem in 1904. In 1986, William Thurston, top right, of Cornell won a Fields Medal for expanding on it. Richard Hamilton, above right, of Columbia invented a way to help solve it.



Xianfeng David Gu and Shing-Tung Yau

NOT A BUNNY Even topologists don't think this soap film can be made into a sphere.

A century-old conundrum and two of math's most prestigious prizes hang in the balance.

ONLINE: ASK THE WRITER

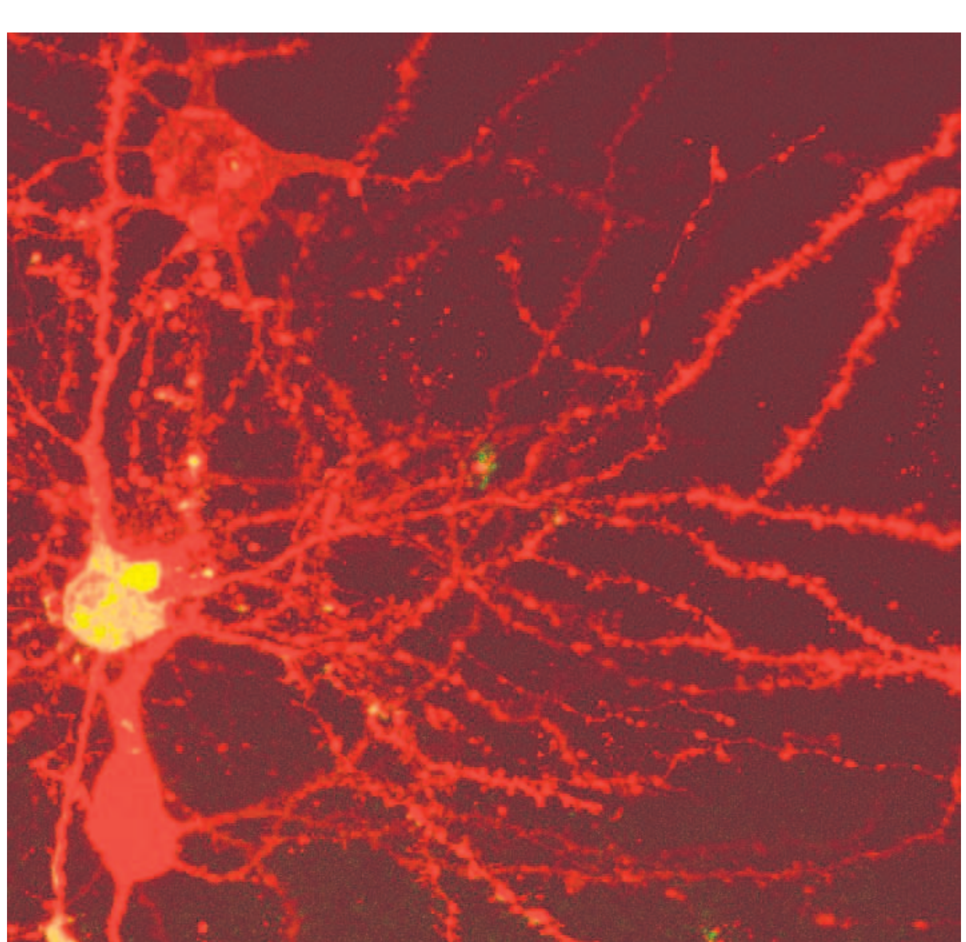
Dennis Overbye answers questions from readers.

askscience@nytimes.com

SCIENCE ILLUSTRATED

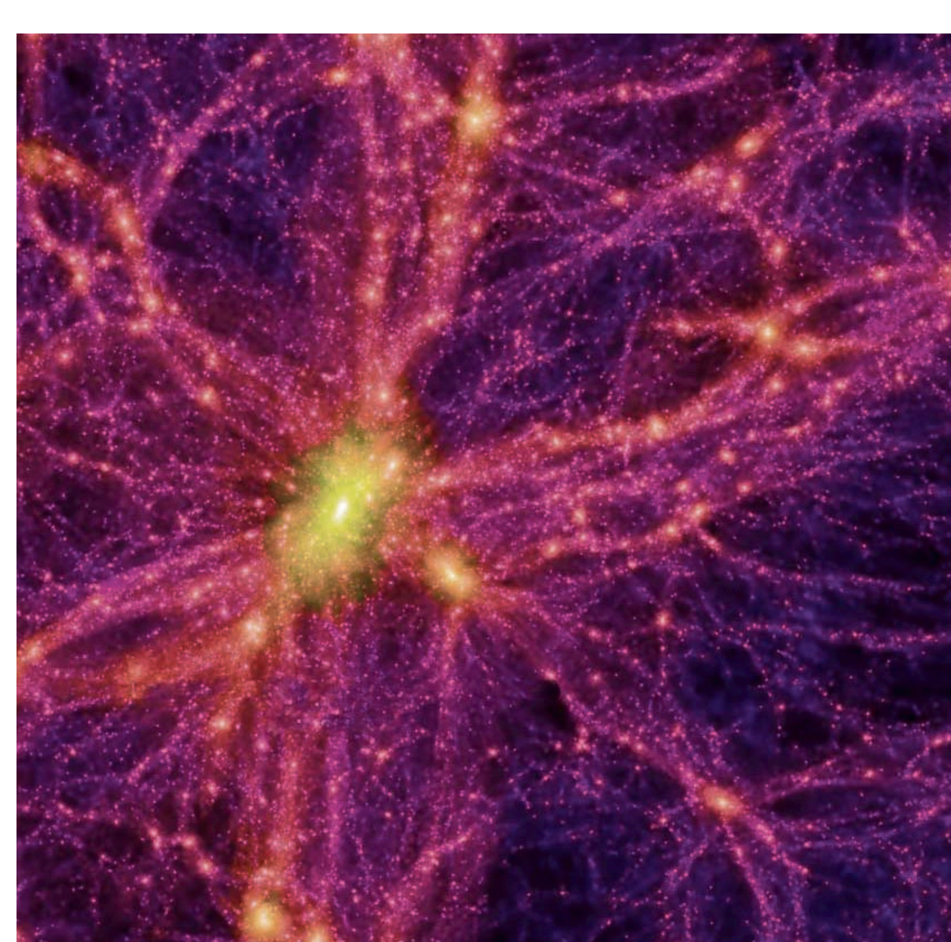
They Look Alike, but There's a Little Matter of Size

One is only micrometers wide. The other is billions of light-years across. One shows neurons in a mouse brain. The other is a simulated image of the universe. Together they suggest the surprisingly similar patterns found in vastly different natural phenomen. *DAVID CONSTANTINE*



Mark Miller

Mark Miller, a doctoral student at Brandeis University, is researching how particular types of neurons in the brain are connected to one another. By staining thin slices of a mouse's brain, he can identify the connections visually. The image above shows three neuron cells on the left (two red and one yellow) and their connections.



Virgo Consortium

An international group of astrophysicists used a computer simulation last year to recreate how the universe grew and evolved. The simulation image above is a snapshot of the present universe that features a large cluster of galaxies (bright yellow) surrounded by thousands of stars, galaxies and dark matter (web).

Source: Mark Miller, Brandeis University; Virgo Consortium for Cosmological Supercomputer Simulations; www.visualcomplexity.com

The New York Times