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Hybrid Sequence Charts

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Abstract

We introduce Hybrid Sequence Charts (HySCs) as a visual description technique for communication in hybrid system models. To that end, we adapt a subset of the well-known MSC syntax to the application domain of hybrid systems. The semantics of HySCs is different from standard MSC semantics. Most notably, we use a shared variables communication model and assume the existence of a continuous, global clock. Similar to their classic counterpart HySCs can be advantageously used in the early phases of the software development process. In particular, in the requirements capture phase, they improve the dialog between customers and application experts. They complement existing formalisms like hybrid automata by focusing on the interaction between the system's components. We outline the key concepts and the usage of HySCs along an example, the specification of an electronic height control system. Then we define their formal semantics.

Contents

1 Introduction

In recent years a considerable number of description techniques has been developed for the specification of hybrid systems. Some of them are based on Petri nets [DA92, Wie96], others use logic [Lam93] and yet others are based on some kind of automata [ACH$^+$95, LSVW96, GSB98]. However, little work has been done to visualize the behavior of a hybrid system together with the communication between its components. Yet, a thorough integration of interaction-based and state-based description techniques is essential if we wish to support and improve today's development processes for hybrid and, more generally, embedded systems.

We regard a hybrid system as consisting of a set of time-synchronously operating components, each encapsulating a private state and communicating with the other components over directed channels. The behavior of a component is characterized, as intuitively shown in Figure 1, top left, by periods where the values on the channels change smoothly and by time instants at which there are discontinuities. In our approach the discontinuities are caused by discrete actions. The smooth periods are caused by analog activities. Two attempts at visualizing the evolution of the values of a hybrid system's channel- and private variables are trajectories and timing diagrams. Their deficiencies motivate our introduction of Hybrid Sequence Charts, below.

Trajectories. *Trajectories* are a straightforward visualization approach that directly depicts the evolution of a system's variables over time (Figure 1, top left). While this approach is simple and effective it can only depict one special case, namely the one in which all variables evolve as in the diagram. It cannot highlight qualitative differences between system states. Visualization by trajectories is supported by development tools like MATLAB [TMI99].

Timing diagrams. A first step from single trajectories to an abstract description of sets of trajectories is obtained by partitioning for each variable the time period under consideration into qualitatively equivalent intervals and by only giving a predicate specifying the variable's evolution within the respective interval. In the diagram of Figure 1, bottom left, for example, it is only important to know whether variable *fHeight* is inside or outside a given tolerance interval. Therefore, the concrete trajectory *fHeight*(t) from Figure 1, top left, can be abstracted to the sequence of intervals with the predicates *greater*, meaning that *fHeight* is outside the tolerance interval, *inside*, which is abbreviated by *i.* in the figure and means that *fHeight* is inside the tolerance interval, the unlabeled interval, meaning that the value of *fHeight* is arbitrary, and *inside* again.[1] Note that the resulting diagram has some similarity with timing diagrams [ABHL97, FJ97], which are widely used in hardware design, and the *constraint diagrams* intro-

[1]Label *c.* is used as abbreviation for *constant* in the figure.

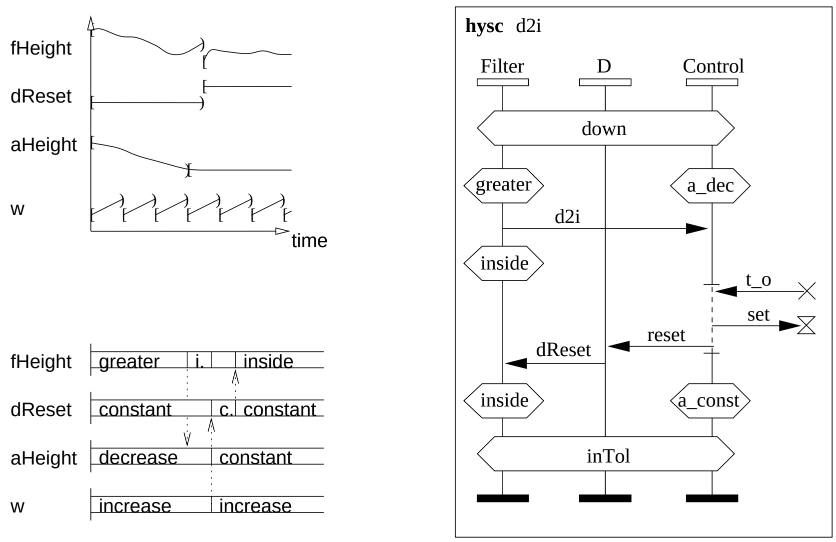


Figure 1: Description techniques for the behavior of hybrid systems.

duced in [Die96]. Causality can be indicated in the diagram by drawing vertical arrows between the abstract time axes of two variables if a change in the first variable is relevant, i.e. may provoke a qualitative change, for the evolution of the second one.

Hybrid Sequence Charts. In this paper we go a step further and also abstract from the individual variables in the graphical representation of system behavior. Thus, instead of partitioning and giving predicates for individual variables, we project the trajectories of all variables of one system component on a single abstract time axis. One axis for each component is appropriate, because we are interested in the sequence of qualitative states each component traverses. Such a qualitative state of a component is usually characterized by a predicate over all its variables (see Figure 1, right). This projection was motivated by notations for component interaction that have gained increasing popularity in the domain of telecommunication systems (cf. [IT96]), and, more generally, in object-orientation (cf. [Rat97, BMR$^+$96, SHB96, BHKS97]). We are aware, of course, that the semantic models – if existent – of such notations do not necessarily match the time-synchronous hybrid system model with communication proceeding over shared channels that we have sketched above. Yet, we believe that by adapting notation from, say, MSCs (cf. [IT96]) to the application domain we consider here, we can carry over much of the intuition that has contributed significantly to the popularity of sequence charts in general. In fact, we consider capturing interaction sequences among system components an important step of *any* development process. Therefore, we borrow a subset of the syntax of MSC-96 (cf. [IT96]) for the specification of interaction sequences within hybrid systems[2];

[2]This has the further advantage that developers can use standard syntax-directed graphic

we call the resulting notation "Hybrid Sequence Charts (HySCs)". In particular, we use arrows to denote events; arrows are directed from the originator of the event to its destination. Angular boxes denote conditions on the component's variables; they may span a single instance axis (local conditions), or multiple axes (non-local condition), and even all component axes (global condition). The remaining syntactic elements in Figure 1, right, are introduced later. Every HySC specifies a typical evolution, or *scenario*, of the system under consideration in connection with its environment over some finite time interval. If the environment does not behave as depicted in the HySC, no statement is made about the system's evolution. By composing such typical evolutions appropriately, we can achieve a specification of the system's behavior upon different inputs from the environment. Even a complete specification covering all possible inputs is possible. We use *High-level HySCs* (HHSCs), whose syntax we also borrow in part from MSC-96, to specify the composition of HySCs. To make HHSCs applicable in the context of hybrid systems we provide notation for expressing preemption, which is an important concept for embedded systems.

HySCs in the development of hybrid systems. Just as MSCs [IT96] or sequence diagrams [Rat97] in the discrete case, HySCs can be used for requirements specification, interface specification, test-case specification, validation, and documentation. Due to their intuitive appearance they are particularly well-suited for capturing and specifying system requirements in the dialog among engineers from different disciplines, as well as among engineers and customers.

Overview. The rest of this paper is organized as follows. In Section 2 we introduce HySCs informally and explain our understanding of them. In Section 3 we present an example hybrid system; in particular, we discuss the key parts of its formal specification with HySCs in Section 3.2. Section 4 contains the formal semantics of HySCs. We summarize our work, and draw conclusions in Section 5.

2 Hybrid Sequence Charts - HySCs

We start with a short introduction to the syntax and informal semantics of basic HySCs that consist of interactions, conditions, and coregions only. Then we cover HHSCs, which allow us to specify hierarchic "roadmaps" through sets of HySCs.

Basic HySCs. Basic HySCs contain one vertical axis, an abstract time axis, for each component, or *instance*, under consideration. Time advances from top to bottom. Sequences of incoming and outgoing arrows partition the time axis of each component into intervals. According to our view of hybrid systems, which we have sketched in Section 1, we require the existence of a global clock, and assume that communication occurs without delay (therefore, all arrows in our

editors for their specifications.

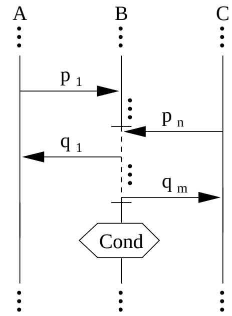


Figure 2: Basic segment of a HySC.

HySCs are horizontal). We assume further that the *components* occurring in the HySC are connected by *channels* along which message exchange occurs. Hence, a HySC is built up from sequences of segments of the form given in Figure 2. Each such segment denotes the execution of an *action* by component B. The action is triggered by the occurrence of all events p_1 through p_n; we say that the *action guard* becomes true. The result of executing the *action body* is that B simultaneously emits the events q_1 through q_m, and changes its state to the one specified in the condition labeled *Cond* in Figure 2. Actions in hybrid systems usually depend on the values of continuous variables; therefore, we consider action guards and action bodies carefully, below.

Before we regard the actions in detail, it is necessary to explain our classification of variables. In our view each component has a set of *input* variables, which are written by the environment or by other components and a set of *controlled* variables that are written by the component itself. The set of controlled variables of a component is further partitioned into a set of *private* variables, whose elements are only visible to the component, and a set of *output* variables, whose elements may be read by the other components or the environment. The input and the output variables are the *observable* variables.

The *action guard* $p_1 \wedge \ldots \wedge p_n$ is a conjunction of predicates p_i. Each predicate p_i that labels an arrow from a component, say A, to B may depend on the old and current values of the output variables of A that are input by B and optionally on the old values of some other private variables of B[3]. The arrow indicates the moment of time (the event) when p_i becomes true. A similar arrow must be drawn if p_i becomes false again, before the action is executed. However, no second arrow needs to be drawn if the predicate possibly only holds for a single point in time, i.e. if the predicate depends on the occurrence of an event or on the exact value of a continuous variable.

The *action body* $q_1 \wedge \ldots \wedge q_m$ is also a conjunction of predicates q_i. Each predicate

[3]Actually, the old values of the output variables of A that are input by B are kept in private variables of B.

q_i that labels an arrow from B to, say, A specifies the current values for the output variables of B that are input by A. These values may depend on the current value of all input variables and on the old and current value of all controlled variables of sender B.

As soon as all parts from the action guard are true, the action body is executed. All the changes that it causes on the output variables simultaneously become visible to those other components which read these variables. Simultaneity is expressed graphically by a *coregion*, i.e. by drawing a region of the time axis of one component as a dashed line; all the predicates in this coregion are evaluated simultaneously (see Figure 2).

We allow the use of predicates as condition labels to indicate a component's state, and adopt the convention that no new condition symbol is drawn if the control-state does not change. Conditions ranging over a set of components are also allowed, and express a global state of the referenced components. A local as well as such a *hierarchic* condition $Cond$ remains valid up to the next condition symbol that references the same or a superset of the components referenced by $Cond$.

Events can be expressed in terms of (event) predicates by toggling boolean variables. For example, we write e?! for $e' = \neg e$ meaning that the current value of e (denoted by e' in the predicate) is the negation of the old value (denoted by e in the predicate) [AH96, GSB98]. The old value of a variable e at a time t is defined as the limit from the left $lim_{u \nearrow t} e(u)$ for this variable, i.e. as the value just before t.

Note that an arrow from A to B can in general be labeled with the conjunction of a part of an action body q_i of A and a part of an action guard p_j of a different action of B. This may be the case if the current values specified for the output from A to B are relevant for p_j.

A qualitative state in a hybrid system is characterized by a set of trajectories that are allowed for the variables in that state. Therefore, the condition after an action in a HySC not only determines the next qualitative state, but it also specifies how input and controlled variables of the component are expected to evolve in this qualitative state. Controlled variables may only evolve continuously, because in our view discontinuities may only be caused by qualitative changes, which in turn result from actions.

HySCs can also be used to specify timing requirements like "at least time t_s passes between the arrows a and b", as proposed in [Sch98] for timed MSCs. The way to specify these requirements is to add an *observer component* that synchronizes with the observed component and that has a private variable, which evolves in pace with global time, as specified in the component's conditions. This private variable is used to measure the length of time intervals between certain events used for synchronization.

A timeout can be specified by using a private variable, which also evolves in pace with global time, and an action guard that becomes true when the variable has reached a certain threshold. Setting the variable to a certain value corresponds to resetting the timer. In our example we therefore use the set-timer and timeout symbols borrowed from MSC-96 to denote this.

High-level HySCs (HHSCs). HySCs can be used within HHSCs to specify the *complete* behavior of a system. For this complete behavior description HHSCs provide operators for the concatenation of HySCs, the choice between HySCs and the iteration of HySCs. The choice is controlled by *global* conditions, i.e. by conditions ranging over all components. A branch of a choice in the HHSC may be taken iff the condition guarding it is currently true. The system behavior is then determined by the HySC following the branch operator. It must start with the same condition as the selected branch. Syntactically, the starting point in an HHSC is represented by an outlined, downward triangle, an end-point (if it exists) by a filled, upward rectangle. References to other HySCs appear in rounded boxes. Conditions are depicted as in basic HySCs. Lines (or arrows) determine the “road-map”, i.e. the sequence in which the interactions appearing in the referenced HySCs may occur. Choice is represented by multiple outgoing edges in the HHSC (see Section 3.2 for examples).

In this paper we introduce the additional concept of preemption to HySCs. Graphically preemption is depicted as a labeled, dashed arrow between two HySC references in an HHSC. Its meaning is that the system behavior is as determined by the HySC reference that is the arrow’s source, as long as the *preemptive predicate*, to which the arrow’s label refers, is false. As soon as the predicate becomes true, the system behavior is as specified by the HySC reference to which the arrow is pointing. Preemption is widely used in the programming of embedded systems. We believe that this is a highly important concept. The example in the next section underlines this. Note, however, that none of the popular graphical notations for component interaction, such as [IT96] or [Rat97], offers adequate syntax for the specification of preemption.

3 HySCs in Practice

To explain the capabilities and usage of HySCs, we formally specify a non-trivial example system and discuss the key parts of this specification.

3.1 An Electronic Height Control System

As example we use an electronic height control system (EHC), taken from a former case study carried out together with BMW. The purpose of this system

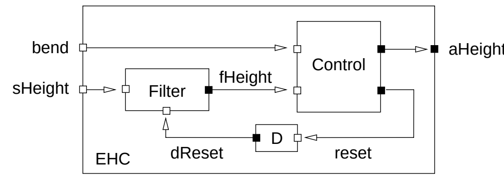


Figure 3: Architecture of the EHC.

is to control the chassis level of an automobile by a pneumatic suspension. The abstract model of this system, which regards only one wheel was first presented in [SMF97]. It basically works as follows: whenever the chassis level *sHeight* is below a certain lower bound, a *compressor* is used to increase it. If the level is too high, air is blown off by opening an *escape valve*. The chassis level is measured by *sensors* and *filtered* to eliminate noise. The filtered value *fHeight* is read periodically by the *controller*, which operates the compressor and the escape valve and resets the filter when necessary. A further sensor *bend* informs the controller whether the car is going through a curve. Periodical sampling of *fHeight* occurs in dependence of a timer, which is local to the controller. Besides the environment, the basic components of the system are the filter and the controller (see Figure 3). The escape valve and the compressor are modeled within the controller. The component labeled D introduces a delay and ensures that the feedback between the filter and the controller is well-defined.

A specification of the EHC with HyCharts, a state-based description technique for hybrid systems, can be found in [GSB98].

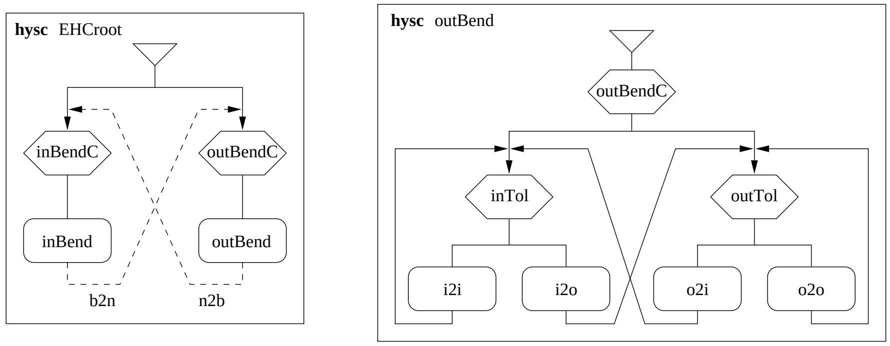
3.2 Specification with HySCs

We specify behavior required by the EHC by using HySCs. First, we present HHSCs for the top-level requirements. Then, we consider two of the basic HySCs in detail.

3.2.1 High-level HySCs (HHSCs)

The top-level description of the EHC is given by a HHSC, as shown in Figure 4, left. On this abstraction level, we distinguish between two scenarios: the car is either inside a curve or going straight. The behavior inside a curve is characterized by the HySC `inBend`. The behavior outside a curve is characterized by the HySC `outBend`.

Preemption. The EHC switches between these two behaviors each time the boolean value provided by the variable *bend*, which is controlled by the environment, is toggled. In other words, toggling *bend* is a *preemption* event. To describe

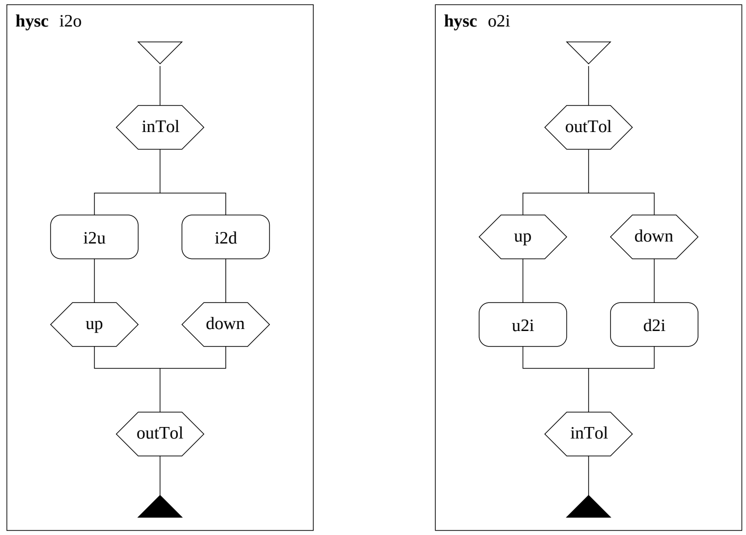


`inBendC`	$\equiv$	$bend = True$	`inTol`	$\equiv$	$\frac{d}{dt} aHeight = 0$
`outBendC`	$\equiv$	$bend = False$	`outTol`	$\equiv$	$\frac{d}{dt} aHeight \neq 0$
`b2n`	$\equiv$	`n2b` $\equiv$ $bend?!$			

Figure 4: The HySCs `EHCroot` and `outBend`.

this situation we use the preemption mechanism that we have introduced in Section 2. Recall that we use a special kind of arrows, *preemption arrows*, to denote preemption in HHSCs. As explained above, they are represented visually by a *dashed arrow* connecting a source HySC reference to a destination HySC reference, and are labeled by the *preemptive predicate*. Their semantics is given in Section 4. Intuitively, any prefix of the traces described by the source HySC reference may be followed by a time instant at which the *preemptive predicate* is true and then by a trace of the destination HySC reference. The labels `inBend` and `outBend` in the HySC boxes, i.e. the boxes with the rounded edges, refer to further HySCs. The labels `inBendC` and `outBendC` in the angular condition boxes refer to the condition predicates $bend = True$ and $bend = False$, where variable *bend* signals whether the car is in a curve. The labels `b2n` and `n2b` both stand for the event predicate `b2n` $\equiv$ `n2b` $\equiv bend?!$, i.e. for the occurrence of an event which toggles the value of *bend* (see Section 2). Note that for easier reference we also give the definition of the condition and event predicates in a box below the HySCs in Fig. 4 and the following figures.

(Nondeterministic) choice. The HHSC `outBend` describes the behavior of the EHC as long as the car is outside a curve (Fig. 4, right). On this level we use the nondeterministic choice operator, graphically depicted as branching arrows, to distinguish between two cases. In the first case, the compressor and the escape valve are off, because the value of *fHeight*, which was read last, was inside the tolerance interval. A further choice operator splits this case into two sub-cases: If *fHeight* remains inside the interval, then the behavior is given by the HySC `i2i`. If the chassis level gets outside the interval, then we have a behavior as described by the HySC `i2o`. The second case describes the behavior



`inTol`	$\equiv$	$\frac{d}{dt}aHeight = 0$	`up`	$\equiv$	$\frac{d}{dt}aHeight > 0$
`outTol`	$\equiv$	$\frac{d}{dt}aHeight \neq 0$	`down`	$\equiv$	$\frac{d}{dt}aHeight < 0$

Figure 5: The HySCs `i2o` and `o2i`.

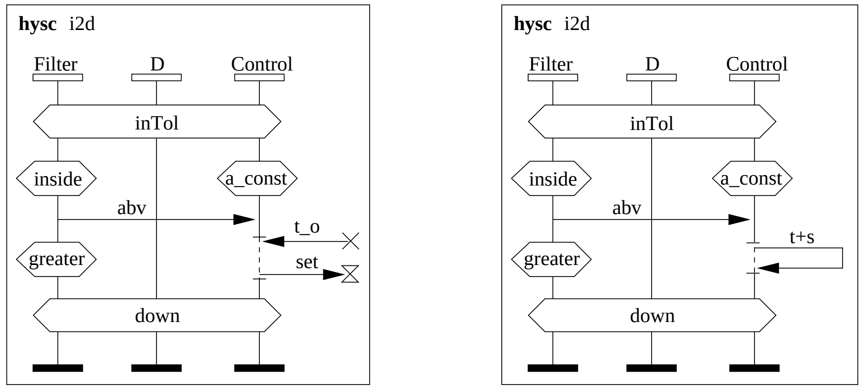
if compressor or escape valve are on, because of the last value of *fHeight* being outside the tolerance interval. This part of the HySC is symmetric to the first one.

The labels `inTol` and `outTol` in the HySC refer to the predicates $\frac{d}{dt}aHeight = 0$ and $\frac{d}{dt}aHeight \neq 0$, respectively, which characterize global states of the system. Variable *aHeight* (actuator height) models how the chassis level is influenced by the compressor and the escape valve. If the derivative of *aHeight* is zero, i.e. *aHeight* remains constant then the chassis level is not modified by the two actuators, the compressor and the escape valve.

Feedback. After the behavior specified by the HySCs `i2i`, `i2o`, `o2i` and `o2o` is finished, a new cycle starts in which we again have to distinguish the cases from above. This is modeled by the *feedback arrows* in the HySC leading from the bottom of it up to those points in the HySC from where the following behavior must continue. Thus, feedback allows us to specify infinite behavior.

Finite Behavior. The HHSCs `i2o` and `o2i` in Fig. 5 are examples for HySCs that do not specify infinite behavior. Instead of feedback arrows, an arrow leading to a black triangle is drawn in them to mark their end.

This completes the exposition of the basic features of HHSCs. Now, we continue with the description of basic HySCs.



`a_const`	$\equiv$	$\frac{d}{dt}aHeight = 0 \ \wedge \ w \leq w_s \ \wedge \ \frac{d}{dt}w = 1$			
`inTol`	$\equiv$	$\frac{d}{dt}aHeight = 0$	`abv`	$\equiv$	$fHeight' \geq ub$
`inside`	$\equiv$	$fHeight \in [lb, ub]$	`t_o`	$\equiv$	$w = w_s$
`greater`	$\equiv$	$fHeight \geq ub$	`set`	$\equiv$	$w' = 0$
`down`	$\equiv$	$\frac{d}{dt}aHeight < 0$	`t + s`	$\equiv$	$w = w_s \ \wedge \ w' = 0$

Figure 6: The HySC `i2d` and its reduction without timeout arrows.

3.2.2 Basic HySCs

All the basic HySCs referenced directly or indirectly by HHSC `outBend` describe the behavior of the EHC in the interval between two expirations of the Controller's timer. In the following we will analyze HySC `i2d` in detail. Furthermore, we will explain HySC `inBend`.

The HySC `i2d` describes the scenario in which the chassis level increases from within the tolerance interval to a value above the upper bound (Fig. 6, left). It appears in the right branch of HHSC `i2o` (Fig. 5, left).

Condition predicates. The HySC starts with the condition box labeled `inTol` (see Fig. 6, left). As mentioned in the previous section this label refers to predicate $\frac{d}{dt}aHeight = 0$. Because the condition box ranges over all components of the diagram it is a *global* condition. The following conditions `inside` and `a_const` range over only one component. Hence, they are local conditions. They add some more detail on the evolution of the variables. Label `inside` refers to predicate $fHeight \in [lb, ub]$, where lb and ub are constants denoting the lower and upper bound of the tolerance interval. Label `a_const` stands for $\frac{d}{dt}aHeight = 0 \ \wedge \ w \leq w_s \ \wedge \ \frac{d}{dt}w = 1$. The first conjunct of this condition means that the chassis level is not modified by $aHeight$, the second conjunct means that variable w is less than constant w_s, the sampling period, and the third conjunct provides that w evolves in pace with the global time, i.e. it is a clock variable or a timer. No local predicate is given for component D. By convention this means that it implicitly has local predicate *True*.

Events. The very moment *fHeight* reaches the upper bound of the tolerance interval is given by the horizontal arrow labeled by `abv`, which stands for event predicate $fHeight' \geq ub$.

After the event `abv` has occurred, the chassis level is above the tolerance interval. Again, this property (or interval invariant) is given by a local condition predicate, the condition predicate `greater`, which stands for $fHeight \geq ub$.

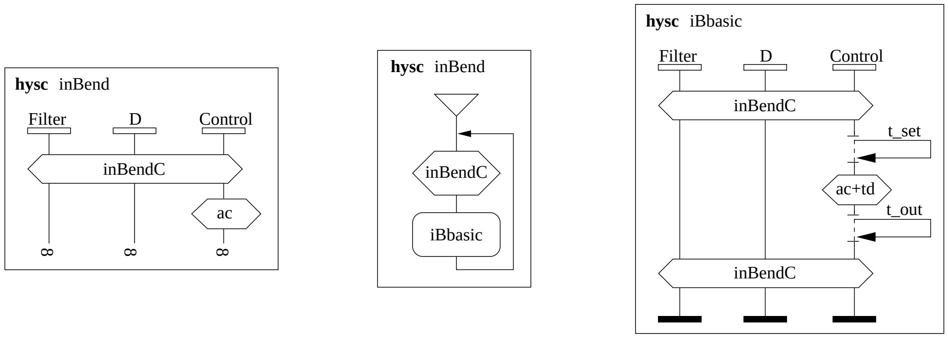
Timers. The control component senses that the chassis level is too low, only when the timer has expired, i.e., with some delay. As a consequence, neither the escape valve, nor the compressor are actuated before the expiration. Correspondingly, the local condition `a_const` continues to hold for the controller.

In the diagram we draw the timeout and set-timer arrows `t_o` and `set` borrowed from MSC-96 to represent an event the control component sends to itself. Predicate `t_o` stands for $w = w_s$, i.e. the timer has reached the threshold, and `set` stands for $w' = 0$ which starts a new sampling period by resetting the timer.

On the level of semantics these arrows can be reduced to a single arrow labeled `t+s` pointing from the axis of the control component to itself (see Fig. 6, right). The label refers to event predicate $w = w_s \ \wedge \ w' = 0$.

Scoping of conditions. As mentioned previously, conditions remain valid until the next condition on the same or on a higher level of hierarchy is given. Thus, before the timer has expired, the overall behavior of the EHC still has to satisfy the global condition `inTol`, because no other global condition occurred up to that point. Correspondingly, the set of behaviors characterized by the conjunction of the predicates `inside` $\wedge$ `a_const` and by `greater` $\wedge$ `a_const` is a subset of the behaviors characterized by `inTol`.

Infinite continuous behavior. In the context of hybrid systems it is sometimes necessary to specify analog behavior that lasts forever. For instance, the behavior specified by HySC `inBend` which is referenced by HHSC `EHCroot` (Fig. 4, left) may last forever, if the car remains in a curve forever. To allow the specification of infinite continuous behavior we do not add a new construct, but introduce a macro that allows to specify it comfortably and that is reduced to primitive constructs. Fig. 7, left, shows the HySC `inBend` with the macro ∞ to denote that it lasts forever. The macro is a notational shorthand for a HHSC with feedback that iterates a finite but arbitrarily long basic HySC with the required continuous behavior. The HHSC for the example is given in Fig. 7, middle. The iterated HySC is depicted in Fig. 7, right. The two events `t_set` and `t_out` result from introducing a new private variable t to component *Control* which is not used elsewhere and which is used to specify a non-deterministically set timeout. Of course the variable could also have been introduced to any of the other components.



$$\begin{array}{lcl@{\qquad}lcl} \texttt{inBendC} & \equiv & bend = True & & & \\ \texttt{ac} & \equiv & \frac{d}{dt} aHeight = 0 & \texttt{t\_set} & \equiv & t' > 0 \\ \texttt{ac} + \texttt{td} & \equiv & \texttt{ac} \wedge \dot{t} = -1 & \texttt{t\_out} & \equiv & t = 0 \end{array}$$

Figure 7: The HySC `inBend` with macro (left) and its reduction to primitives (middle and right).

Note that the HySC specification we have given is not complete for the EHC. Instead it defines a set of *required behaviors*. To extend it to a complete specification we would furthermore have to consider scenarios in which *fHeight* leaves and enters the tolerance interval several times within one sampling interval. Using HHSCs with choice and feedback this is straightforward.

4 Semantics of HySCs

Suppose we are given a set of HySCs with the components (or instances) $C_1, \ldots,$ C_n. For each component C_i, we assume its interface, i.e. the set of input and controlled variables, to be given.

In the following let S_i be the data space associated with the controlled variables of component C_i. For uniformity, let S_0 be the data space associated with the variables controlled by the environment and $S = S_0 \times \ldots \times S_n$. Then we define the semantics of a HySC M to be a set $[\![M]\!] \subseteq S^{\mathbb{R}_+} \times \mathbb{R}_+^\infty$ of pairs (φ, t) where $\varphi \in \mathbb{R}_+ \to S$ is a *piecewise smooth* function (also called *a dense communication history* or *dense stream*) that exhibits the behavior required by M inside the time interval $[0, t]$. If $t = \infty$ then the behavior of φ is constrained by M along the whole time axis, i.e., the HySC M never terminates. Such HySCs may be defined by using, for example, feedback.

We say that a function $f \in \mathbb{R}_+ \to Q$ is piecewise smooth iff every finite interval on the nonnegative real line $\mathbb{R}_+$ can be partitioned into *finitely* many left closed and right open intervals such that on each such interval f is infinitely differentiable (i.e., f is in C^∞) for $Q = \mathbb{R}$ or f is constant for $Q \neq \mathbb{R}$. Infinite differentiability is required for convenience. It allows us to assume that all differentials of f are well-defined. A tuple of functions is infinitely smooth iff all its components are. We write $Q^{\mathbb{R}_+}$ to denote the set of piecewise smooth functions from $\mathbb{R}_+$ to the set Q. Furthermore, we write Q^A for the set of functions from A to Q that are piecewise smooth on the interval A. Intuitively, a dense communication history is obtained by pasting together smooth pieces. The time instants at which the pieces are pasted together are those at which events occur.

Let O be the projection of $S_1 \times \ldots \times S_n$ on the output variables, i.e. the data space of the output variables, and let P be the projection of $S_1 \times \ldots \times S_n$ on the private variables, i.e. the data space of the private variables of the system. With this bit of structure on the data-space, we can also interpret the semantics of a HySC $[\![M]\!]$ as a relation between the dense histories of the input variables, the dense histories of the private and output variables and the considered time intervals, i.e., $[\![M]\!] \subseteq S_0^{\mathbb{R}_+} \times (P \times O)^{\mathbb{R}_+} \times \mathbb{R}_+^\infty$. To model analog behavior in a *well behaved way,* the relation $[\![M]\!]$ has to be *time guarded*, i.e. for any moment of time $u \in \mathbb{R}_+$, the values of the variables controlled by the components are completely determined by the values of the input variables until that moment. Formally, for all $\varphi_1, \varphi_2 \in S_0^{\mathbb{R}_+}$ and $u \in \mathbb{R}_+$ if $\varphi_1{\downarrow_{[0,u)}} = \varphi_2{\downarrow_{[0,u)}}$ then:

$$\{\psi_1 \mid (\varphi_1, \psi_1) \in \pi_{12}\, [\![M]\!]\}{\downarrow_{[0,u)}} = \{\psi_2 \mid (\varphi_2, \psi_2) \in \pi_{12}\, [\![M]\!]\}{\downarrow_{[0,u)}}$$

where by $\varphi{\downarrow_\delta}$ we denote the restriction of a dense stream to the time interval δ. Restriction is extended to tuples and sets of dense streams in a componentwise and pointwise style, respectively. By π_{12} we denote the projection of a tuple (or set of tuples) on the first two components. Note that we do not demand that the relation given by $[\![M]\!]$ is total in the set of input streams $S_0^{\mathbb{R}_+}$. This takes into account the fact that a single HySC describes a system's response to a particular input from the environment. Only if an HHSC is used to specify the behavior of a system completely, i.e. for all possible inputs, it must result in a relation that is total in the input streams.

A note on zenoness. Specifications which demand that a system performs infinitely many discrete moves within a finite interval are called *zeno*. Like with other powerful description techniques for hybrid systems, such as hybrid automata [ACH+95], it is possible to write down zeno specifications with HySCs. For instance, zenoness can result from specifying that the system always reacts discretely when a continuous input signal crosses a boundary value. In a high-level specification technique we do not want to exclude such specifications which certainly make sense for many input signals. Hence, zeno behavior has to be ruled out later in the design process. Note that on the level of semantics zeno behavior is excluded, since streams containing infinitely many discontinuities within a finite interval are not piecewise smooth.

4.1 Predicates

Condition predicates. Before we turn to the definition of the semantics of HySCs, some thoughts about the semantics of the condition and event predicates are necessary. The semantics of a condition predicate p_K ranging over the components C_k, $k \in K$, for a set K of indices, is a relation $[\![p_K]\!] \subset \bigcup_{A \in Int} I_K^A \times (P_K \times O_K)^{A_c}$, where I_K is the data space of the input variables of the components in K, without those variables that are output by other components in K, O_K is the data space of their output variables and P_K is the data space of their private variables. Int is the set of possibly infinite right-open intervals starting from zero, $Int = \{[0,t) \mid t \in \mathbb{R}_+ \setminus \{0\}\} \cup \mathbb{R}_+$ For a set X, the notation X^{A_c} denotes the set of piecewise smooth functions X^A which furthermore are continuous, hence $X^{A_c} \subset X^A$.

This type of the predicates' semantics permits discontinuities in the input, while the controlled variables must still evolve continuously. This reflects that discrete jumps in the evolution of the controlled variables are interpreted as events, hence they are only allowed when an event arrow is drawn in the HySC. Furthermore, the type allows that a condition predicate specifies finite behavior of varying length. For instance, this is useful to model timeout conditions depending on a skewed clock, like in the condition $c \leq 1 \;\wedge\; \dot{c} \in [0.9, 1.1]$. Note that condition predicates may constrain the evolution of the input variables. This is justified, because a HySC only specifies a system's behavior for those cases in which the environment behaves as expected.

The condition predicate that holds in a certain section of the abstract time axes of all the components in a HySC can be derived as the conjunction of all, local, and hierarchic condition predicates that are valid in this section. The derived condition ranges over all the components, therefore its semantics is a relation over the evolution of the input variables from the environment and all the controlled variables of the system.

Event predicates. The semantics of the event predicates e which label the arrows is a relation between the old and the new values of the variables $[\![e]\!] \subset S \times S$, where we demand that $[\![e]\!]$ is topologically closed. This is necessary to guarantee that there exists a minimal time t at which the predicate becomes true for the first time. The semantics of simultaneous events, which are graphically denoted by arrows emanating from or pointing to a dashed region of the abstract time axis of a component in a HySC, is defined as the conjunction of the individual predicates of all the simultaneous events within the dashed region under consideration. Those variables for which the event predicates do not specify new values remain constant. The timeout and set-timer symbols are reduced to event predicates over private variables in the way explained in Section 2 and in the example of Section 3.

4.2 Basic HySCs

The basic idea behind the semantics of a HySC M is that it defines a set $[\![M]\!]$ of tuples such that for each $(\varphi, t) \in [\![M]\!]$ the dense history φ behaves inside the time interval $[0, t]$ as required by M and arbitrarily outside of $[0, t]$. In the definition of $[\![M]\!]$ it is quite useful to generalize the lower bound 0 to an arbitrary value $u \in \mathbb{R}_+$ and to work with sets $[\![M]\!]_u$ where the dense histories φ are constrained inside the time interval $[u, t]$. However, we have to take care to maintain the quite natural assumption of HySCs that the time's origin is at the top of their vertical time axis. In the following paragraphs we define $[\![M]\!]_u$ inductively on the structure of M. Then obviously the semantics of a HySC M is $[\![M]\!] \stackrel{\mathsf{def}}{=} [\![M]\!]_0$.

Note that the semantics definition we will give is compatible to the formalism of HyCharts, defined in [GSB98]. HyCharts are a graphical formalism for the state-based specification of hybrid systems. Thus, HySCs, which allow interaction- or event-based specifications, can be applied in conjunction with HyCharts in the development process.

Neutral HySC. HySCs without events act as the neutral elements with respect to our semantics:

$$[\![M]\!]_u \stackrel{\mathsf{def}}{=} \{(\varphi, u) \mid \varphi \in S^{\mathbb{R}_+}\}$$

Hence, all the conditions in the HySCs are ignored, and no time elapses in a neutral HySC.

Single event HySC. Suppose p is the condition predicate that results from the conjunction of all the condition predicates that are valid in the section of the HySC before event e happens. Note that e may be the conjunction of a set of simultaneous events. $[\![M]\!]_u$ is defined as follows:

$$\begin{aligned} [\![M]\!]_u \stackrel{\mathsf{def}}{=} \quad & \{(\varphi, t) \in S^{\mathbb{R}_+} \times \mathbb{R}_+ \mid \\ & \qquad t = min\{v > u \mid (lim_{x \nearrow v} \varphi(x), \varphi(v)) \in [\![e]\!]\} \wedge \\ & \qquad \varphi_u{\downarrow}_{[0,t-u)} \in [\![p]\!]{\downarrow}_{[0,t-u)} \} \end{aligned}$$

where $min\,\emptyset \stackrel{\mathsf{def}}{=} \infty$ and $\varphi_u(x) \stackrel{\mathsf{def}}{=} \varphi(u + x)$. To constrain φ inside $[u, t]$ without violating the time's origin assumption we constrain the translation φ_u of φ by the condition predicate p inside the interval $[0, t - u)$. Note that the restriction of $[\![p]\!]$ to $[0, t - u)$ only contains streams that are defined on $[0, t - u)$. If $[\![p]\!]$ only contains shorter streams, the restriction is empty. Longer streams are cut at $t - u$.

The definition requires that a finite, non-zero amount of time passes before the event becomes true. The HySC then terminates at the first time instant t at which e is true. Provided e does not hold initially, this first time instant, defined as the minimum of a set, is guaranteed to exist, because $[\![e]\!]$ is topologically closed.

(See [GS98] for a proof under similar assumptions.) Demanding that some time passes before the event occurs is motivated by the visual representation. If we wanted to specify that no time passes between two consecutive events, we would have to use simultaneous events, graphically indicated by a coregion.

Note that $\infty \notin \mathbb{R}_+$ and therefore if $t = \infty$ then $[\![M]\!] = \emptyset$. Thus, the semantics requires that the event eventually occurs, which is also motivated by the visual representation. The event arrow in the diagram would be misleading, if we allowed it to never occur.

Sequential composition. The sequential composition of the HySCs M_1 and M_2, textually denoted as $M_1; M_2$, is syntactically well formed only if M_1 ends with the global condition with which M_2 starts. In particular, this includes the case that M_1 and M_2 are successive parts of a single, larger HySC. The semantics is given only for well formed terms.

$$[\![M_1; M_2]\!]_u \stackrel{\text{def}}{=} \{(\varphi, t) \in S^{\mathbb{R}_+} \times \mathbb{R}_+^\infty \mid \exists v \in \mathbb{R}_+ .\ (\varphi, v) \in [\![M_1]\!]_u \ \wedge \ (\varphi, t) \in [\![M_2]\!]_v\}$$

Note that whereas the HySC $M_1; M_2$ may describe an infinite computation ($t \in \mathbb{R}_+^\infty$) any of its prefixes exhibiting the behavior required by M_1 has to be finite ($v \in \mathbb{R}_+$).

4.3 HHSCs

Nondeterministic choice. The semantics of a ramification of the HySCs M_1 and M_2, textually written as $M_1 \vee M_2$, is given by the *union* of the semantics of each alternative:

$$[\![M_1 \vee M_2]\!]_u \stackrel{\text{def}}{=} [\![M_1]\!]_u \cup [\![M_2]\!]_u$$

Feedback. The semantics of a feedback arrow in an HHSC is defined as the greatest fixed point of the following equation:

$$[\![M\!\uparrow]\!]_u = [\![M; (M\!\uparrow)]\!]_u$$

where $M\!\uparrow$ textually denotes the feedback of HySC M. The fixed point is well-defined, because the monotonicity of the defining equation ensures its existence.

Preemption. Suppose the HySC M_1 may be preempted by the event e and continued by the HySC M_2, textually written as $M_1\, e\, M_2$. To define its semantics, let $[\![M]\!]_u^v$ be the set obtained by "cutting" the histories $(\varphi, t) \in [\![M]\!]_u$ at time point $v \leq t$ such that φ is constrained by M within the halfopen interval $[u, v)$. Formally:

$$[\![M]\!]_u^v = \{(\varphi, v) \mid \exists (\psi, t) \in [\![M]\!]_u .\ \varphi{\downarrow_{[u,v)}} = \psi{\downarrow_{[u,v)}} \wedge v \leq t\}$$

Then the associated semantics is defined as follows:

$$\begin{aligned}\llbracket M_1\, e\, M_2 \rrbracket_u \stackrel{\mathsf{def}}{=} \quad & \{(\varphi, t) \in S^{\mathbb{R}_+} \times \mathbb{R}_+^\infty \mid \exists v \in \mathbb{R}_+^\infty . \\ & \quad v = min\{y > u \mid (lim_{x \nearrow y} \varphi(x), \varphi(y)) \in \llbracket e \rrbracket\} \wedge \\ & \quad (\varphi, v) \in \llbracket M_1 \rrbracket_u^v \wedge (\varphi, t) \in \llbracket M_2 \rrbracket_v\}\end{aligned}$$

where for any M we define $\llbracket M \rrbracket_\infty \stackrel{\mathsf{def}}{=} S^{\mathbb{R}_+} \times \{\infty\}$. The definition constrains the behavior according to HySC M_1 as long as e does not hold. Starting from the first time instant where e is true, the behavior is as specified by HySC M_2. Note that the behavior at the first time instant where e holds is no longer constrained by M_1. This is reasonable, because preemptive events typically falsify the current condition predicate of M_1. As in the semantics of single event HySCs some time must pass before e holds. In contrast to the semantics of single event HySCs it is allowed that e does not occur. In this case the semantics specifies that the behavior is according to M_1 forever.

Preemption with feedback. Suppose that a HySC M is restarted by an event e, textually written as $M{\Uparrow_e}$.[4] Its corresponding semantics is given by the greatest fixed point of the following equation:

$$\llbracket M{\Uparrow_e} \rrbracket_u = \llbracket M e (M{\Uparrow_e}) \rrbracket_u$$

Again, the fixed point is well-defined, because of the monotonicity of the defining equation.

5 Conclusion

Borrowing from the standardized syntax of MSC-96, we have introduced a description technique that allows the system developer to specify the communication between the components of a hybrid system graphically. Basically, this is achieved by giving precise meaning to the conditions and events in HySCs. Motivated by the specific needs of embedded systems we have, furthermore, included a construct into our definition of HHSCs that allows us to specify preemption. We demonstrated the usage of HySCs along a non-trivial example and defined their formal semantics. HySCs are more abstract than drawing trajectories of the system variables, and are more detailed than other forms of graphical interaction specifications that do not handle continuous variables, e.g. [IT96, Rat97]. Thus we believe they are a good supplement to state-based hybrid techniques like hybrid automata or HyCharts [ACH+95, GSB98], just as ordinary sequence diagrams are beneficial in the development of discrete systems. In particular,

[4] This construct is necessary to give a semantics to HySC `EHCroot` from our example in Section 3.

they seem to be well-suited for bridging the gaps between requirements capture, specification, and later phases of system development. Note that, apart from their syntax, HySCs are substantially different from standard MSCs.

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References

[ABHL97] T. Amon, G. Borriello, T. Hu, and J. Liu. Symbolic timing verification of timing diagrams using presburger formulas. In *Proc. of the 34th Design Automation Conference*. ACM, 1997.

[ACH+95] R. Alur, C. Courcoubetis, N. Halbwachs, T.A. Henzinger, P.-H. Ho, X. Nicollin, A. Olivero, J. Sifakis, and S. Yovine. The algorithmic analysis of hybrid systems. *Theoretical Computer Science*, 138:3–34, 1995.

[AH96] R. Alur and T.A. Henzinger. Reactive modules. In *Proc. of the 11th Annual Symposium on Logic in Computer Science*. IEEE Computer Society Press, 1996.

[BHKS97] M. Broy, C. Hofmann, I. Krüger, and M. Schmidt. A graphical description technique for communication in software architectures. Technical Report TUM-I9705, Technische Universität München, 1997.

[BMR+96] F. Buschmann, R. Meunier, H. Rohnert, P. Sommerlad, and M. Stal. *A System of Patterns. Pattern-Oriented Software Architecture*. Wiley, 1996.

[DA92] R. David and H. Alla. *Petri Nets and Grafcet: Tools for modelling discrete event systems*. Prentice Hall, 1992.

[Die96] C. Dietz. Graphical formalization of real-time requirements. In *Proc. Formal Techniques in Real-Time and Fault-Tolerant Systems (FTRTFT'96)*, LNCS 1135. Springer Verlag, 1996.

[FJ97] K. Feyerabend and B. Josko. A visual formalism for real time requirement specifications. In *Proc. AMAST Workshop on Real-Time Systems and Concurrent and Distributed Software (ARTS'97)*, LNCS 1231. Springer Verlag, 1997.

[GS98] R. Grosu and T. Stauner. Modular and visual specification of hybrid systems - an introduction to HyCharts. Technical Report TUM-I9801, Technische Universität München, September 1998.

[GSB98] R. Grosu, T. Stauner, and M. Broy. A modular visual model for hybrid systems. In *Proc. Formal Techniques in Real-Time and Fault-Tolerant Systems (FTRTFT'98)*, LNCS 1486. Springer-Verlag, 1998.

[IT96] ITU-TS. Recommendation Z.120 : Message Sequence Chart (MSC). Geneva, 1996.

[Lam93] L. Lamport. Hybrid systems in TLA+. In R.L. Grossman, A. Nerode, A.P. Ravn, and H. Rischel, editors, *Hybrid Systems*, LNCS 736. Springer-Verlag, 1993.

[LSVW96] N.A. Lynch, R. Segala, F.W. Vaandrager, and H.B. Weinberg. Hybrid I/O automata. In R. Alur, T.A. Henzinger, and E.D. Sontag, editors, *Hybrid Systems III*, LNCS 1066. Springer-Verlag, 1996.

[Rat97] Unified modeling language, version 1.1. Rational Software Corporation, 1997.

[Sch98] I. Schieferdecker. Proposal for time and performance in MSCs. In *Proc. ITU-T Meeting SG10*, Geneva, 1998.

[SHB96] B. Schätz, H. Hußmann, and M. Broy. Graphical Development of Consistent System Specifications. In J. Woodcock and M.-C. Gaudel, editors, *FME'96:Industrial Benefit and Advances in Formal Methods*, volume 1051 of *LNCS*. Springer, 1996.

[SMF97] T. Stauner, O. Müller, and M. Fuchs. Using HyTech to verify an automotive control system. In *Proc. Int. Workshop on Hybrid and Real-Time Systems (HART'97)*, LNCS 1201. Springer-Verlag, 1997.

[TMI99] The MathWorks Inc. MATLAB. `http://www.mathworks.com/products/matlab/`, 1999.

[Wie96] R. Wieting. Hybrid high-level nets. In *Proc. of the 1996 Winter Simulation Conference, Coronado, California, USA / Charnes*, pages 848–855, 1996.